SESELF-OPTIMISING CONTROL FOR ECONOMIC VARIATION

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Abstract

Optimal plant operation problems are multiobjective in nature because contributions from utility costs, material costs and production costs may vary due to rapidly changing market. Traditionally, re-optimisation of plant operating point will be necessary when market price change significantly. Applying recently developed techniques of self-optimising control, a new method to compensate the market price change to achieve multiobjective self-optimising control has been proposed. The new approach extends the techniques of controlled variable synthesis using linear combinations of process measurements proposed in [1] by treating changes of market price as virtual measurements. According to the proposed approach, the effect of market price change on optimality of plant operation can be quickly compensated by adjusting setpoints calculated using virtual measurements. The evaporator case study shows that the new approach is effect and efficient.

1 Introduction

Competitive market and restrictive regulations on environmental emissions are pushing process industry to implement optimal plant operation. Generally, optimal operation is a dynamic optimisation problem. However, for continuous processes, using the concept of “quasi-steady-state” [2], the optimal operating point can be obtained via static optimisation. Nevertheless, due to inevitable disturbances and uncertainties (modelling and measurement), the nominal operating point may not be optimal at all in real situations. Traditionally, this problem is solved by irregular re-optimisation of the operating point according to new disturbance conditions. Recently, due to the rapid increase of computing power, the topic of real-time optimisation (RTO) has attracted significant amount of research efforts. In contrast with the increasing interest of research in RTO, another solution to the robust optimal plant operation problem, the self-optimising control has received surprisingly less attention.

Self-optimising control is a control strategy where by maintaining certain carefully selected variables at constant setpoint, the plant operation will be automatically optimal or suboptimal without re-optimisation even in the presence of plant disturbances. The idea was initially proposed in early 1980’s [3, 2] and was recalled recently [4]. Several techniques to select controlled variables and to configure self-optimising control systems have been proposed, for example [1, 5, 6, 7]. The main difference between the RTO approaches and self-optimising control techniques is that the former are feed-forward solutions whilst the latter are feedback ones. Hence, plant operation might be more robustly optimal under self-optimising control than using RTO approaches.

On the other hand, optimal plant operation problems are multiobjective in nature because the cost function is normally a weighted combination of various utility costs, material costs, production costs and penalties of environmental emissions. Due to the rapid change of market those weights in the cost function may have to be modified regularly to reflect the new market conditions. Therefore, the originally obtained operating point may need to be updated to reflect such changes. It is desired to use the self-optimising control techniques to indirectly drive the process to the new optimal operating point. However, since the weights of cost function has no direct effect on process variables, using the controlled variables selected by current self-optimising control techniques will not be able to adjust the plant operation to reflect the change of the cost function. In order to regain the “self-optimising” feature, the whole procedure to select controlled variables may have to be repeated and the control system has to be re-configured. This inefficient approach is obviously undesirable.

In this work, the techniques of controlled variable synthesis using linear combinations of process measurements proposed in [1] has been extended by introducing “virtual measurements”, i.e., the weights of sub-costs are treated as “virtual measurements” and used to synthesise controlled variables. This is equivalent to introduce an extra part of setpoints, which are determined by weights of the cost function multiplying by a set of coefficients obtained during controlled variable synthesis. With these coefficients of “virtual measurements”, the new setpoints to compensate the change of cost function can be quickly calculated and implemented in the system. An evaporator case study has been included to show the effectiveness and efficiency of the new approach to achieve multiobjective self-optimising control.

The paper is organised as follows: after a brief introduction of existing self-optimising control theory in section 2, the extended approach using “virtual measurements” is
proposed in section 3 and is demonstrated with the evaporator example in section 4. Some concluding remarks are presented in section 5.

2 Self-optimising control

For convenience of presentation, previous results of self-optimising control, mainly based on [1] are summarised in this section.

Consider the following optimisation problem:

$$\min \ J = \phi_0(x, u, d)$$

subject to

$$f(x, u, d) = 0$$
$$g_0(x, u, d) \leq 0$$

where $x \in \mathbb{R}^{n_x}$, $u \in \mathbb{R}^{n_u}$ and $d \in \mathbb{R}^{n_d}$ are state, input and disturbance variables respectively. For a given nominal disturbance, $d^*$, the solution of the above optimisation problem is denoted as, $x^*$ and $u^*$. Assume that at the optimal point, the following equalities hold:

$$F(x^*, u^*, d^*) = \begin{bmatrix} f(x^*, u^*, d^*) \\ g(x^*, u^*, d^*) \end{bmatrix} = 0$$

where $f(\cdot)$ and $g(\cdot)$ are vector-valued functions with dimensions of $n_f$ and $n_g$ respectively. If $m = (n_x + n_u) - (n_f + n_g) \neq 0$, then the optimisation problem has $m$ unconstrained degrees of freedom. Denote $u^* = [u_0^* \ v^*]^T$ with $v \in \mathbb{R}^m$ and $z = [x^T \ u_0^*]^T$. Assume constraints (2) remain active for $d$ within a small area around $d^*$. Then, in the area, constraints (2) can be denoted as $F(z, v, d) = 0$, which implicitly defines $z = z(v, d)$. Using this implicit function, the optimisation problem (1) can be simplified to an $m$-dimension unconstrained optimisation problem:

$$\min _v J = \phi(v, d)$$

Denote $J^*$ and $v^*$ the minimum cost and the corresponding argument obtained under nominal disturbance, $d^*$, respectively. For general disturbance conditions, the minimum cost and argument are functions of $d$, i.e. $J_{opt}(d)$ and $v_{opt}(d)$. For simplicity of notation, the argument $d$ will be omitted occasionally.

The optimality condition of problem (3) is $\phi_{v} = 0$, which can be linearised around the nominal point leads to

$$\delta v_{opt} = -\phi_{vv}^{-1}\phi_{vd}\delta d$$

Now consider the situation where $m$ controlled variables, $c$ are maintained at constant setpoints. The controlled variables considered here are linear combinations of process measurements, i.e.

$$c = Hy$$

where $H$ is a constant matrix and $y$ the process measurements. Assume at the nominal point, $(v^*, d^*)$, $y$ can be linearised as

$$\delta y = G^y\delta v + G^y_d\delta d$$

Hence,

$$\delta c = G^v\delta v + G^v_d\delta d = HG^y\delta v + HG^y_d\delta d$$

Under perfect control, $\delta c = 0$, i.e. $c = c^*$. Hence, the deviation of manipulated variables, $\delta v$ is a function of the deviation of disturbances, $\delta d$ around the nominal point,

$$\delta v|_{c=c^*} = -G^{-1}G^v_d\delta d$$

Therefore, the difference of manipulated variable moves between the optimal case and constant $c$ case can be approximated as

$$\Delta v = \delta v|_{c=c^*} - \delta v_{opt} = (G^v_{vv} - G^v_d - G^v_{vd})\delta d$$

Applying Taylor expansion to the cost function, the economic loss due to constant $c$ control can be estimated as follows:

$$L := J|_{c=c^*} - J_{opt} = \frac{1}{2} (\Delta v)^T \phi_{vv} \Delta v$$

If $c$ is not exactly maintained at $c^*$ due to any measurement noise, $n = HW_n y_n$, where $y_n$ is a normalised noise vector associated with the measurement vector, $y$, $W_n$ is a weighting matrix to normalise measurement noise within $\pm 1$ level, then $\Delta v$ becomes

$$\Delta v = \delta v|_{c=c^*+n} - \delta v_{opt} = M_d \delta d + M_n y_n$$

where

$$M_d = \phi_{vv}^{-1}\phi_{vd} - (HG^y)^{-1}HG^y_d$$

$$M_n = -(HG^y)^{-1}HW_n$$

Then, the optimal coefficient matrix, $H$ can be obtained by solving the optimisation problem defined as:

$$\min _H \sigma \left( \phi_{vv}^{1/2} [M_d \ M_n] \right)$$

where $\sigma$ denotes the maximum singular value of a matrix.

3 Setpoint compensation

One way to solve a multiobjective problem is to represent multiple objectives as a scalar weighted-sum cost function. By varying weights, a family of solutions, the Pareto set can be obtained through single objective optimisation. Therefore, in the following analysis, it is assumed that the cost function of a multiobjective problem in (3) is a linear combination of $r$ sub-objective functions,

$$J = \phi(v, d, \alpha_1, \ldots, \alpha_r) = \sum _{i=1}^{r} \alpha_i \phi_i(v, d)$$

Then,

$$\phi_v = \sum _{i=1}^{r} \alpha_i [\phi_i]_v \quad \phi_{vd} = [\phi_i]_v$$

$$\phi_{vv} = \sum _{i=1}^{r} \alpha_i [\phi_i]_{vv} \quad \phi_{vd} = \sum _{i=1}^{r} \alpha_i [\phi_i]_{vd}$$
It is clear that the optimal manipulated variable move defined in (4) is a function of the multiobjective weights, $\alpha_1, \ldots, \alpha_r$, whilst the move implied by constant control of $c$ is independent of these weights. Therefore, the effect of multiobjective weight change on economic loss in (10) cannot be compensated by any measurement combination. To restore the self-optimising feature, an obvious solution would be re-optimising the nominal operating point according to changed weights, then re-calculate the measurement coefficient matrix, $H$ based on the new operating point. Certainly, such an inefficient approach is undesired. An efficient alternative is much sought-after. The main problem of the above method is that the process measurements are independent of multiobjective weights. To overcome this difficult, the multiobjective weights are treated as “virtual measurements” so that they can be included in the measurement set. More specifically, augment measurement vector, $y$, coefficient matrix, $H$ and disturbances vector $d$ as follows:

$$\begin{align*}
\overline{y} &= [y^T \alpha_1 \cdots \alpha_r]^T \quad (16) \\
\overline{H} &= [H \beta_1 \cdots \beta_r] \quad (17) \\
\overline{d} &= [d^T \alpha_1 \cdots \alpha_r]^T \quad (18)
\end{align*}$$

where $\beta_i \in \mathbb{R}^m$, $i = 1, \ldots, r$. Then the following augmented matrices are obtained:

$$\begin{align*}
\overline{HG}^\alpha &= HG^\alpha \\
\overline{HG}_d &= [HG_d^\alpha \beta_1 \cdots \beta_r] \\
\phi_{\overline{y}} &= [\phi_{vd} \phi_{y1}^T \cdots \phi_{yr}^T] \\
M_d &= \phi_{\overline{y}}^{-1} \phi_{vd} - (HG_d^\alpha)^{-1} HG_d^\alpha \\
&= [M_d \ M_1 \cdots \ M_r]
\end{align*}$$

where $M_i = \phi_{\overline{y}}^{-1}[\phi_{yi}]^T - (HG_i^\alpha)^{-1} \beta_i$, $i = 1, \ldots, r$.

Thus, matrix $H$ and vectors, $\beta_1, \ldots, \beta_r$ can be synthesised simultaneously via solving the optimisation problem:

$$\min_{H, \beta_1, \ldots, \beta_r} \overline{y} \left( \phi_{\overline{y}}^{1/2} [M_d \ M_1 \cdots \ M_r \ M_u] \right) \quad (19)$$

However, by noticing the fact that each $\beta_i$ is only associated with $M_i$, a simplified procedure to get $H$ and $\beta_i$ vectors are derived as follows:

**Step 1** Calculate $H$ by solving the optimisation problem in (14).

**Step 2** Calculate $\beta_i = (HG_i^\alpha)\phi_{\overline{y}}^{-1}[\phi_{yi}]^T$ for $i = 1, \ldots, r$.

Let $\overline{r}$ the augmented controlled variable, i.e.

$$\overline{r} = \overline{Hy} = Hy + \sum_{i=1}^{r} \beta_i \alpha_i = c + c_\alpha \quad (20)$$

where $c_\alpha$ is a constant vector for a given set of $\alpha_i$’s. For nominal weights, $\alpha_\alpha^*$, the setpoint of $\overline{r}$ is $\overline{r}^* = c^* + c_\alpha^*$. Thus, the control equation is $\overline{r} = c + c_\alpha = \overline{r}^* = c^* + c_\alpha^*$. This is equivalent to

$$c = c^* - (c_\alpha - c_\alpha^*) = c^* - \sum_{i=1}^{r} (\alpha_i - \alpha_i^*) \quad (21)$$

Hence, the second term defines an extra part of setpoint of controlled variable $c$ to compensate any multiobjective weight changes. The new part of setpoint can be easily calculated and implemented instantly without re-optimisation of new operating point and new coefficient matrix, $H$.

### 4 Evaporator case study

The new multiobjective setpoint compensation approach is applied to an evaporation process [8], shown in Figure 1.

![Evaporator System](image)

**Figure 1. Evaporator System**

This is a “forced-circulation” evaporator, where the concentration of dilute liquor is increased by evaporating solvent from the feed stream through a vertical heat exchanger with circulated liquor. The process variables are listed in Table 1 and model equations are given in Appendix A.

The economic objective is to minimise the operational cost [$/h$] related to steam, cooling water and pump work [9, 10]

$$J = \alpha F_{100} + 0.6 F_{200} + 1.009 (F_2 + F_3) \quad (22)$$

where $\alpha$ is determined by steam price. The process has the following constraints related to product specification, safety and design limits:

- $X_2 \geq 35.5\% \quad (23)$
- $40 \text{kPa} \leq P_2 \leq 80 \text{kPa} \quad (24)$
- $P_{100} \leq 400 \text{kPa} \quad (25)$
- $F_{200} \leq 400 \text{kg/min} \quad (26)$
- $0 \text{kg/min} \leq F_3 \leq 100 \text{kg/min} \quad (27)$

Note a 0.5% back-off has been enforced on $X_2$ to ensure the variable remaining feasible for all possible disturbances.

The process model has three state variables, $L_2$, $X_2$ and $P_2$ with eight degrees of freedom. Four of them are disturbances, $F_1$, $X_1$, $T_1$ and $T_{200}$. The rest four degrees


\[
\begin{array}{|c|c|c|c|}
\hline
\text{Var.} & \text{Description} & \text{Value} & \text{Units} \\
\hline
F_1 & \text{Feed flowrate} & 10 & \text{kg/min} \\
F_2 & \text{Product flowrate} & 1.41 & \text{kg/min} \\
F_3 & \text{Circulating flowrate} & 23.05 & \text{kg/min} \\
F_4 & \text{Vapor flowrate} & 8.59 & \text{kg/min} \\
F_5 & \text{Condensate flowrate} & 8.59 & \text{kg/min} \\
X_1 & \text{Feed composition} & 5 & \% \\
X_2 & \text{Product composition} & 35.5 & \% \\
T_1 & \text{Feed temperature} & 40 & \degree \text{C} \\
T_2 & \text{Product temperature} & 91.22 & \degree \text{C} \\
T_3 & \text{Vapor temperature} & 83.61 & \degree \text{C} \\
L_2 & \text{Separator level} & 1 & \text{meter} \\
P_2 & \text{Operating pressure} & 56.42 & \text{kPa} \\
T_{100} & \text{Steam temperature} & 151.52 & \degree \text{C} \\
P_{100} & \text{Steam pressure} & 10.02 & \text{kg/min} \\
Q_{100} & \text{Heat duty} & 366.63 & \text{kW} \\
F_{200} & \text{Cooling water flowrate} & 230.54 & \text{kg/min} \\
T_{200} & \text{Inlet C.W. temperature} & 25 & \degree \text{C} \\
T_{201} & \text{Outlet C.W. temperature} & 45.5 & \degree \text{C} \\
Q_{200} & \text{Condenser duty} & 330.77 & \text{kW} \\
\hline
\end{array}
\]

Table 1. Variables and Optimal Values

of freedom are manipulable variables, \(F_2, P_{100}, F_3\) and \(F_{200}\). The optimisation problem of (22) with process constraints, (23) to (27) has been solved with \(\alpha = 600\), which corresponds to steam price at 10 \([\$/\text{kg}]\) and under nominal disturbances:

\[
d = (F_1 \quad X_1 \quad T_1 \quad T_{200})^T = (10 \quad 5 \quad 40 \quad 25)^T
\]

(28)

The minimum cost is 6178.2 \$/h. Corresponding values of process variables are shown in Table 1.

At the optimal point, there are two active process constraints, \(X_2 = 35.5\%\) and \(P_{100} = 400\) [kPa]. These two constraints will keep active within whole disturbance region, which is defined as \(\pm 20\%\) of the nominal disturbances. Physically, the first active constraint is because a higher outlet composition requires more solvent to be evaporated, therefore needs more steam, cooling water and pump cost. For the second constraint, since heater duty, \(Q_{100}\) is determined by both steam pressure, \(P_{100}\) and circulating flowrate, \(F_3\), reducing \(P_{100}\) will increase \(F_3\) due to energy balance. However, the sensitivity to steam cost of \(P_{100}\) is much lower than that of \(F_3\). Hence, an optimal operation should keep \(X_2\) at its lower bound and \(P_{100}\) at its higher bound.

These two active constraints plus the separator level, which has no steady-state effect on the plant operation, but must be stabilised at its nominal value, consume three degrees of freedom. Therefore, the optimal condition has one degree of freedom, \(i.e.\) an extra controlled variable needs to be selected to achieve self-optimising control.

There are several options to select the extra controlled variable. The gradient of one degree of unconstrained freedom has been derived [7], which is theoretically optimal, however may not be realisable due to the difficulty to get all necessary variables online. The multiobjective weight, \(\alpha\) can be explicitly included in the gradient. Thus, in the case study, the gradient control will be treated as a benchmark for other options. Using flow ratio, \(F_{200}/F_1\) as the extra controlled variable proposed in [11] and using temperature difference, \(T_{201} - T_{200}\) are two of the best choices [6]. However, both two options require disturbance measurements. Another option using flow ratio, \(F_{200}/F_4\) as the controlled variable is proposed in this work, which does not require any disturbance measurable. The setpoint compensation approach will be applied to these three configurations to demonstrate the effectiveness of the method.

By scaling disturbances by their maximum range (\(\pm 20\%\) of nominal values) and scaling measurement noise by 5% of nominal values, the loss index, \(\text{i.e.}\) the maximum singular value of \(\phi_{\text{sv}}/M_d\) around the nominal operating point and the setpoint compensating coefficient \(\beta\) for the above four schemes are calculated and shown in Table 2.

It is shown that for the nominal objective, all schemes can achieve self-optimising performance, \(\text{i.e.}\) all economic losses are very small. For example, for \(F_{200}/F_1\) configuration, the predicted loss in the worst situation is 0.0831\(^2/2\) = 0.0035 [\$/h]. However, all three alternative schemes are relatively sensitive to objective weight change. Without setpoint compensation, the loss due to \(\alpha\) change is estimated as \(\phi_{\text{sv}}(M_d\delta\alpha)^2/2 = (0.013\delta\alpha)^2/2\). Suppose the steam price varies within 5–15 [\$/kg], which corresponds to 300 \(\leq\) \(\alpha\) \(\leq\) 900. Then, in the worst situation, the loss could reach 7.585 [\$/h]. Although it is still relatively small comparing to the nominal cost 6178.2 [\$/h], it is much larger than the loss due to disturbances and measurement noise. Thus, without setpoint compensation, the benefit gained by using self-optimising control could be lost completely when market price changes.

The importance of setpoint compensation is demonstrated through dynamic simulation. Firstly, each control configuration listed in Table 2 with other control loops is implemented in a decentralised control system, where \(L_2\) is controlled by \(F_3\), \(X_2\) is controlled by \(F_2\), and \(P_2\) is controlled by \(F_{200}\) whilst the setpoint of \(P_2\) is used to control one of the controlled variables listed in Table 2. The setpoint \(P_2\) is linked with a saturation block to cope with the conditionally active constraints. Details of the control configuration are referred to [6]. Random disturbances within \(\pm 20\%\) of their nominal values are applied to the systems all the time. With properly tuned PI controllers, all configurations are stabilised and able to maintain all constraints within the specified range. During 100-hour simulated operation, the operating costs of all configurations for \(\alpha = 300, 600\) and 900 are calculated and listed in Table 3. To compare the efficiency of the compensation, the optimal setpoints for all three \(\alpha\) cases are calculated via optimisation and the opera-
tion costs under optimal setpoints are shown in Table 3 as well. The results show that the setpoint compensation can reduce economic loss in an order of magnitude for each configuration. Compare with the optimal setpoint cases, the percentages of economic loss due to weight change recovered by setpoint compensation are over 83% for \( T_{201} - T_{200} \) case, over 97% for other two cases. Therefore, the self-optimising feature is almost recovered by setpoint compensation without re-optimisation and re-configuration.

5 Conclusions

The multiobjective issue related to self-optimising control is investigated. The effect of multiobjective weight change on the economic loss can be compensated via a proper setpoint adjustment. By introducing “virtual measurements”, the existing self-optimising control techniques can be directly applied to calculate the setpoint compensation. The evaporator case study shows that the propose approach is efficient and effect.

References


A Model equations

\[
\frac{dL_2}{dt} = F_1 - F_4 - F_2 
\]

\[
\frac{dX_2}{dt} = \frac{F_1X_1 - F_2X_2}{20} 
\]

\[
\frac{dP_2}{dt} = \frac{F_4 - F_5}{4} 
\]

\[
T_2 = \frac{0.5616P_2 + 0.3126X_2 + 48.43}{38.5} 
\]

\[
T_3 = \frac{0.507P_2 + 55.0}{38.5} 
\]

\[
T_4 = \frac{Q_{100} - 0.07F_1(T_2 - T_1)}{38.5} 
\]

\[
T_{100} = \frac{0.1538P_{100} + 90.0}{38.5} 
\]

\[
Q_{100} = \frac{0.16(F_1 + F_3)(T_{100} - T_2)}{38.5} 
\]

\[
F_{100} = \frac{Q_{100}/36.6}{14F_{200} + 6.84} 
\]

\[
F_{200} = \frac{0.9576F_{200}(T_3 - T_{200})}{0.14F_{200} + 6.84} 
\]

\[
T_{201} = \frac{T_{200} + 13.68(T_3 - T_{200})}{0.14F_{200} + 6.84} 
\]

\[
F_5 = \frac{Q_{200}}{38.5} 
\]