NETWORKED PREDICTIVE CONTROL OF SYSTEMS WITH RANDOM COMMUNICATION DELAY

G. P. Liu*,†, J. X. Mu* and D. Rees*

*School of Electronics, University of Glamorgan, Pontypridd CF37 1DL, UK, email: gpliu@glam.ac.uk
†Institute of Automation, Chinese Academy of Sciences, Beijing 100080, China

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Abstract

This paper proposes a novel networked predictive control strategy and analyses the stability of closed-loop networked predictive control systems with random communication delay. The proposed networked predictive controller consists of a networked control predictor and a conventional predictive controller. The networked control predictor compensates the network communication delay. The conventional predictive controller achieves the required control performance. The stability criteria of a closed-loop networked predictive control system are analytically derived for both the fixed communication delay and random communication delay. The simulated results confirm the analytical results given in the paper.

1 Introduction

Since the first networked device Cambridge Coffeepot appeared on the Internet (http://www.cl.cam.ac.uk/coffee), the rapid growth of the WWW over the past several years has resulted in a growing number of web accessible devices/systems over the Internet, for example, the multimedia education system using the Internet (Nemoto et al., 2000), Internet-based control engineering laboratory (Overstreet and Tzes, 1999), and web-based telerobots (Taylor and Dalton, 2000). Clearly, the Internet has provided a powerful tool for distributed collaborative work. The emerging network technologies do have the potential to apply the advantages of this way of working to the high-level control systems. The advantages include the following: 1) allow remote monitoring and tuning of control systems; 2) allow large (or global) area distributed control; 3) allow collaboration between skilled system designers and operators situated in geographically diverse locations. These are not achievable by the use of design methodologies for conventional control systems.

Recently, more and more attention has been paid to various issues of network based control systems, for example, the stability problem (Zhang et al., 2001) in the presence of network delays and data packet drops, the design and implementation problem of the networked control system (Yang et al., 2003; Zhigoglyadov and Middleton, 2003) and the network traffic congestion problem (Wong and Brockett, 1999). Although the network (for example, Internet) is being applied to the control system area, most control methods based on networks simply use the network as a data transmission device to transmit the control status of a system to a website. This is not much different from the normal Internet information services. Much research work is needed to develop systematic design methods using this technology for the design of such network-based real-time distributed control systems. Much attention needs to be paid to the network environment issues for control systems, such as the network communication delay and the stability of closed-loop networked control systems. This paper presents a novel networked predictive control strategy and studies its closed-loop stability, which are different from those in conventional (or non-networked) control systems.

2 Networked predictive control

In networked control systems, the control sequence can be transmitted at the same time through a network, which is not done in tradition control systems. Since there is an unknown network communication delay, a new networked predictive controller is proposed. It consists of two parts: a networked control predictor and a conventional predictive controller. The former is used to compensate the unknown random communication delay. The latter is designed by conventional predictive control methods, for example, the general predictive control. This networked predictive control system (NPCS) structure is shown in Fig. 1. The communication delay in the feedback channel is not considered in this paper.

Consider a plant described by

$$A(z^{-1})y(t+d) = B(z^{-1})u(t)$$

where $y(t)$ and $u(t)$ are the output and control input of the plant, $d$ is the time delay, and $A$ and $B$ are the system polynomials, i.e.
\[ A(z^{-1}) = 1 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_n z^{-m} \]
\[ B(z^{-1}) = b_0 + b_1 z^{-1} + b_2 z^{-2} + \cdots + b_m z^{-m} \]

From now on, the coefficients of a polynomial denoted by an upper case are denoted by its corresponding lower case with a numbered subscript, for example, \( A_k(z^{-1}) = a_{1_k} z^{-1} + a_{2_k} z^{-2} + \cdots + a_{n_k} z^{-m} \). In this paper, the general predictive control method is employed to design the conventional predictive controller. As usual, the system performance function without communication delay (i.e., \( \pi(t) = u(t) \)) is

\[ J(u(t)) = \sum_{k=0}^{N_i-1} (r(t+k) - \hat{y}(t+k))^2 + \rho \sum_{k=1}^{M} (\Delta u(t+k-1))^2 \]

where \( N_I \) and \( N-J \) are the minimum and maximum prediction horizon, respectively, \( M \) the control horizon, \( \rho \) the weight factor, and \( \Delta = 1 - z^{-1} \). For \( k = 1, 2, \ldots, N-I \), there exists the following Diophantine equation:

\[ \Delta A(z^{-1}) E_k(z^{-1}) + z^{-d} F_k(z^{-1}) = 1 \]

where \( E_k(z^{-1}) \) and \( F_k(z^{-1}) \) are \((k-I)\)-th and \( n \)-th order polynomials, respectively. Let

\[ G(z^{-1}) = [G_1(z^{-1}), G_2(z^{-1}), \ldots, G_n(z^{-1})] \]
\[ F(z^{-1}) = [F_1(z^{-1}), F_2(z^{-1}), \ldots, F_n(z^{-1})] \]
\[ C_k(z^{-1}) = B(z^{-1}) E_k(z^{-1}) \]
\[ G_k(z^{-1}) = z^{-d_k} \bar{c}_k(z) = \sum_{j=0}^{d_k+N_k} \bar{c}_j z^{-j} \]

Minimising the performance function gives the following predictive control sequence at time \( t \):

\[
\begin{bmatrix}
  u(t | t) \\
  u(t+1 | t) \\
  \vdots \\
  u(t+M-1 | t)
\end{bmatrix} = \begin{bmatrix}
  1 \\
  1 \\
  \vdots \\
  1
\end{bmatrix} \begin{bmatrix}
  H_1 \\
  H_2 \\
  \vdots \\
  H_M
\end{bmatrix} \times \begin{bmatrix}
  (R(t) - G(z^{-1}) \Delta u(t-1 | t-1) - F(z^{-1}) y(t))
\end{bmatrix}
\]

where \( u(t+\hat{t}) \) is the \( \hat{t} \)-th step ahead control prediction at time \( t \), and the matrix \( H \) is

\[
[ H_1 ] = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 1 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \cdots & 1 \end{bmatrix}, \quad \left( \Gamma^T \Gamma + \rho I_{M \times M} \right)^{-1} \Gamma^T \in \mathbb{R}^{M \times N} 
\]

\[
\begin{bmatrix}
  c_{10} & 0 & \cdots & 0 \\
  c_{21} & c_{10} & \cdots & 0 \\
  c_{31} & c_{21} & \cdots & 0 \\
  \vdots & \vdots & \ddots & \vdots \\
  c_{N-2d,M-Nd-2} & c_{N-2d,M-Nd-2} & \cdots & c_{N-2d,M-Nd-2} \\
\end{bmatrix} 
\]

and the vector \( R(t) \) is given by

\[
R(t) = [r(t+N_1), r(t+N_1+1), \ldots, r(t+N_1+N-1)]^T \in \mathbb{R}^{N \times 1} 
\]

To compensate the network communication delay, a networked control predictor is proposed. Since the network can transmit a set of data at the same time, it is assumed that the all predictive control sequence at time \( t \) is packed and sent to the plant. The networked control predictor takes the latest control value from the predictive control sequences available on the plant side. For example, if the following predictive control sequences are available on the plant side:

\[
\begin{bmatrix}
  u(t-k-2 | t-k-2) \\
  u(t-k-1 | t-k-1) \\
  \vdots \\
  u(t+M-k-3 | t-k-2) \\
  u(t+k | t-1) \\
  \vdots \\
  u(t+M-k-2 | t-k-1)
\end{bmatrix}
\]

where there are two control values \( u(t+j-2) \) and \( u(t+j-1) \) available at time \( t \), then, the control input of the plant at time \( t \) will be taken as

\[
\bar{u}(t) = u(t | t-k-1) 
\]

which is the latest predictive control value at time \( t \).

3 Stability of NPCS with fixed delay

The most important issue in control systems is the stability of the closed-loop system. This section discusses the stability of closed-loop networked predictive control systems and assumes that the network communication delay is fixed, given by \( k \). From the predictive control sequence, it can be obtained that

\[
u(t | t) = u(t-1 | t-1) + H_1 R(t) - H_1 G(z^{-1}) \Delta u(t-1 | t-1) - H_1 F(z^{-1}) y(t)
\]

Define \( u(t-1 | t-1) = z^{-1} u(t | t) \). Then

\[
u(t | t) = \frac{H_1 R(t) - H_1 F(z^{-1}) y(t)}{(1 + H_1 G(z^{-1})) \Delta}
\]

The \( k \)-step ahead predictive control at time \( t \) is expressed by

\[
u(t+k | t) = u(t+k-1 | t-1) + H_{k+1} R(t) - H_{k+1} G(z^{-1}) \Delta u(t+k-1 | t-1) - H_{k+1} F(z^{-1}) y(t)
\]

which leads to
As there exists a fixed communication delay \( k \), the above predictive control signal at time \( t \) will reach the plant side at time \( t+k \). The plant to be controlled can be rewritten by

\[
A(z^{-1}) y(t+d+k) = B(z^{-1}) u(t+k)
\]

To compensate the network communication delay, the control input of the plant is given by the following networked control predictor:

\[
u(t+k) = u(t+k|t)
\]

which leads to

\[
A(z^{-1}) y(t+d+k) = B(z^{-1}) \left[ (1+H_k G(z^{-1})^{-1})A(z^{-1})e^{z^{-d}k} \right] y(t) + B(z^{-1}) H_k z^{-1} R(t) = B(z^{-1}) (H_k z^{-1} + H_k \Delta) R(t)
\]

Clearly, the closed-loop characteristic equation is given by

\[
(1 + H_k G(z^{-1})^{-1})A(z^{-1}) + z^{-d} B(z^{-1}) (H_k z^{-1} + H_k \Delta) F(z^{-1}) = 0
\]

Therefore, if the roots of the above characteristic polynomial are within the unit circle, the closed-loop networked predictive control system is stable.

### 4 Stability of NPCs with Random delay

In practical network communications, there exist a number of uncertainties which affect the transmission time during the data information exchange, for example, communication traffic congestion. This results in the communication delay being random. This section studies how this random communication delay affects the stability of closed-loop networked predictive control systems. Here, it assumes that the random communication delay is bounded with \( d_N \).

The plant can be written as

\[
A(z^{-1}) y(t) = z^{-d} B(z^{-1}) \| u(t) \|
\]

Since the communication delay is random, to effectively compensate for this delay the networked control predictor is designed to be

\[
u(t-i) = u(t-i|t-i-k_i), \quad \text{for } i = 0, 1, 2, \ldots, m,
\]

subject to \( k_i < k \), where \( u(t-i|t-i-k_i) \) is the latest predictive control at time \( t-i \) which is available at the plant side and \( k_i \in \{0, 1, \ldots, d_N \} \) is a random number.

Following the fixed delay case, the conventional predictive control \( u(t-i|t-i-k_i) \) is calculated by

\[
u(t-i|t-i-k_i) = \frac{(H_k z^{-1} + H_k \Delta) (R(t-i-k_i) - F(z^{-1}) y(t-i-k_i))}{(1 + H_k G(z^{-1}))}
\]

As a result, the closed-loop control system is

\[
A(z^{-1}) y(t) = z^{-d} \sum_{i=0}^{m} b_i u(t-i|t-i-k_i)
\]

which can be simplified to be in the form of

\[
A(z^{-1}) (L + H_k G(z^{-1})^{-1}) y(t) + z^{-d} \sum_{i=0}^{m} b_i (H_k z^{-1} + H_k \Delta) F(z^{-1}) z^{-d} R(t) = 0
\]

Therefore, the closed-loop characteristic equation is

\[
A(z^{-1}) (L + H_k G(z^{-1})^{-1}) A(z^{-1}) + z^{-d} \sum_{i=0}^{m} b_i (H_k z^{-1} + H_k \Delta) F(z^{-1}) z^{-d} R(t) = 0
\]

subject to \( k_i < k \). Since the communication delay \( k \) varies randomly, there are a huge number of possibilities for the above characteristic equation. This makes it very difficult to determine the stability of the closed-loop networked predictive control system by the direct use of the above equation. Now, let

\[
P_i(z^i) P_i(z^{-i}) = A(z^{-i}) (L + H_k G(z^{-i})^{-1}) A(z^{-i})
\]

\[
Q_i(z^{-i}, k_i) = b_i (H_k z^{-i} + H_k \Delta) F(z^{-i}) z^{-d_i}
\]

\[
P_i(z^i) = P_i(z^{-i}) = P_i(z^i) z^{-d_i} + \sum_{i=0}^{m} Q_i(z^{-i}, k_i)
\]

where

\[
P_i(z^i) = p_{i,0} z^{d_i} + p_{i,1} z^{d_i-1} + \cdots + p_{i,d_i} z^{d_i-d_i}
\]

\[
P_i(z^i) = p_{i,0} z^{d_i} + p_{i,1} z^{d_i-1} + \cdots + p_{i,d_i} z^{d_i-d_i}
\]

\[
Q_i(z^{-i}, k_i) = q_{i,0}(k_i) + q_{i,1}(k_i) z^{-1} + q_{i,2}(k_i) z^{-2} + \cdots
\]

\[
P_i(z^i) = p_{i,0} + p_{i,1} z + p_{i,2} z^{2} + \cdots
\]

Clearly, the coefficients of the closed-loop characteristic polynomial \( P(z^i) \) are calculated by

\[
p_i = p_{i,d_i} \quad j = 0, 1, 2, \ldots, d_i-1
\]

\[
p_j = p_{i,d_i} + \sum_{i=0}^{m} q_{i,d_i-j}(k_i) \quad j = d_i, d_i+1, d_i+2, \ldots
\]
Both the upper and lower bounds of the coefficients of the polynomial $P(z^{-1})$ are

$$p_{j}^{+} = p_{j}^{-} = p_{j,i}, \quad \text{for } j = 0, 1, 2, \ldots, d - 1$$

$$p_{j}^{+} = \max_{k_{i},k_{j-1}} p_{j} = \max_{k_{i},k_{j-1}} \left( p_{j,i} + \sum_{i=0}^{d} q_{j,i}(k_{i}) \right),$$

$$p_{j}^{-} = \min_{k_{i},k_{j-1}} p_{j} = \min_{k_{i},k_{j-1}} \left( p_{j,i} + \sum_{i=0}^{d} q_{j,i}(k_{i}) \right),$$

subject to $k_{i,j} < k_{j}$ and $k_{i} \in \{0, 1, 2, \ldots, d_{x}\}$. Using Kharitonov theory (1978), the stability of the polynomial $P(z^{-1})$ can be guaranteed by the following four polynomials:

$$K_{1}(z^{-1}) = p_{0}^{+} + p_{1}^{+} z^{-1} + p_{2}^{+} z^{-2} + p_{3}^{+} z^{-3} + p_{4}^{+} z^{-4} + \cdots$$

$$K_{2}(z^{-1}) = p_{0}^{-} + p_{1}^{-} z^{-1} + p_{2}^{-} z^{-2} + p_{3}^{-} z^{-3} + p_{4}^{-} z^{-4} + \cdots$$

$$K_{3}(z^{-1}) = p_{0}^{+} + p_{1}^{+} z^{-1} + p_{2}^{+} z^{-2} + p_{3}^{+} z^{-3} + p_{4}^{+} z^{-4} + \cdots$$

$$K_{4}(z^{-1}) = p_{0}^{-} + p_{1}^{-} z^{-1} + p_{2}^{-} z^{-2} + p_{3}^{-} z^{-3} + p_{4}^{-} z^{-4} + \cdots$$

Therefore, if the roots of the above four polynomials are stable, the closed-loop networked predictive control system with a random communication delay is stable, which is a sufficient criterion. However, the closed-loop system may be stable or unstable if the above four polynomial are not stable.

### 5 Simulated examples

Consider the control of a chemical heat process through a network. This process can be modelled in the s-domain as follows:

$$G(s) = \frac{0.7}{s^{2} + 1.7s + 0.25}$$

The input is the flow of chemical reactant and the output, the temperature of a chemical process. For the system sampling rate 0.1s, the discrete model of the process is

$$G(z) = \frac{0.003309 z^{-1} + 0.003127 z^{-2}}{1 - 1.841 z^{-1} + 0.8437 z^{-2}}$$

with the GPC setting as $N_{i} = 1, N = 31, M = 30, \rho = 0.5$.

<table>
<thead>
<tr>
<th>$k=0$</th>
<th>$k=1$</th>
<th>$k=2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-0.4788</td>
<td>0.7914</td>
</tr>
<tr>
<td>0.7907</td>
<td>0.7910</td>
<td>-0.5437+0.3644i</td>
</tr>
<tr>
<td>0.8794+0.1517i</td>
<td>0.7888+0.1517i</td>
<td>-0.5437-0.3644i</td>
</tr>
<tr>
<td>0.8794-0.1517i</td>
<td>0.8788-0.1517i</td>
<td>0.6249+0.5041i</td>
</tr>
<tr>
<td>0.3196+0.4603i</td>
<td>0.3196+0.4603i</td>
<td>0.8776+0.1515i</td>
</tr>
<tr>
<td>0.8776+0.1515i</td>
<td>0.8776+0.1515i</td>
<td>0.8776-0.1515i</td>
</tr>
</tbody>
</table>

If the constant communication delay is over 4 steps, the closed-loop networked predictive control is unstable, as shown in Fig. 3. Clearly, the simulation results confirm the theoretical results.

![Fig. 2. The step response with 4-step fixed delay.](image1)

![Fig. 3. The step response with 5-step fixed delay.](image2)
When the communication delay is random, the closed-loop system is stable for the upper bound of the random delay is not greater than 4 steps, as shown in Fig. 4. But, the closed-loop system may be stable or unstable if the upper bound of the random delay is greater than 4 steps, which is shown in Figs. 5-6.

6 Conclusions

This paper has proposed a novel networked predictive control scheme and studied the stability of closed-loop networked predictive control systems. To overcome the effects of the network communication delay, the networked predictive controller consists of two parts: a networked control predictor that compensates the network communication delay and a conventional predictive controller that achieves the required control performance. It has considered two cases of the network communication delay: one is that it is fixed and the other is that it is random. For the former case, a stability criterion is derived for the closed-loop system and is expressed by the stability of a single polynomial. For the latter case, the system stability criterion is much more complicated and is expressed by the stability of four Kharitonov polynomials. Those analytical results provide a novel design of networked control systems.

References