AN IMPROVED MATHEMATICAL MORPHOLOGY FILTER FOR FAULT LOCATION IN POWER TRANSMISSION LINES

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Abstract

This paper presents a novel mathematical morphology (MM) Erosion-Dilation (ED) filter. The two basic operators of MM, Erosion and Dilation, can be regarded as the special cases of the ED filter which is formulated in a unified mathematical framework to provide filtering effects between that offered by Erosion and Dilation filters. The ED filter performs much better in noise reduction compared with Dilation and Erosion filters. It is applied to reduce noise in the transient signals and improve the performance of the power transmission line fault location which was detected using Multi-resolution Morphology Gradient (MMG) [1]-[3]. The effectiveness of the ED filter for noise reduction is evaluated by a comprehensive simulation study.

1 Introduction

Mathematical Morphology has been widely studied for signal / image processing. It is largely used for shape detection but rarely for noise reduction. In this paper, a novel MM Erosion-Dilation (ED) filter is proposed to effectively reduce the noise. This proposed filter has parameters \((m, k)\) and it extracts the \(k\)th element of an input signal sequence in which its elements are sorted in ascending order according to the structure element with the length of \(m\). Usually, Dilation and Erosion filters are considered as the two basic MM filters. These two filters can be regarded as two special cases of the ED filter because they extract the minimum and maximum values of the input signal sequence, which are the 1st and \(m\)th elements in the sorted input signal respectively. Dilation and Erosion filters can be formulated in a unified mathematical framework defined by the ED filter assuming the structure element is symmetric with respect to its origin.

The statistical properties of the proposed ED filter are discussed in this paper in order to further understand the output distribution of the filter with inputs polluted by noise, which are the most important for a nonlinear filter. The analysis and computation of the output bias and variances for noisy signals processed by the ED filter give many significant results. These results show that ED filter with parameters \((m, (m + 1)/2)\), which is namely a Median filter, performs the best in noise reduction. Compared with other noise reduction methods, the ED filter has the advantage of simple and fast computation, which is useful in many applications.

The MM technique has been applied for an ultra-high-speed directional relay and fault location[1]-[3]. A Multi-resolution Morphology Gradient (MMG) operator was proposed to extract transient features from fault-generated voltage and current wave signals propagating along power transmission lines during a post-fault period. The simulation results showed that the performance of MMG is satisfactory under various fault conditions. However, the performance will inevitably deteriorate when various disturbances are imposed on the transient current signals. These disturbances can be considered as noise. Therefore, a noise reduction method should be integrated into the MMG scheme in order to improve the accuracy of fault location.

The efficiency of noise reduction offered by the ED filter for the transient current signals is verified by the simulation studies considering a variety of the scenarios of transient voltage and current signals. By this novel MM filter, the MMG-based fault location scheme is improved to be noisy tolerant considerably.

2 The Definition of Morphological Filter ED

The multi-level morphology [4] is widely used in signal and image processing. The corresponding multi-level morphological filters are expressed by:

\[
\text{Dilation} : \quad (f \oplus G)(x) = \max\{f(x - z) | z \in G\}
\]

\[
\text{Erosion} : \quad (f \ominus G)(x) = \min\{f(x + z) | z \in G\}
\]

where \(f(x) \in \mathbb{E}\) is a 1-D signal. Assuming the structure element \(G\) is symmetric with respect to its origin, the definition of Dilation becomes:

\[
\text{Dilation} : \quad (f \oplus G)(x) = (f \oplus \tilde{G})(x) = \max\{f(x + z) | z \in G\}
\]
A novel multi-level morphological filter, named ED filter, is defined as follows:

\[
\text{ED} : \quad (f \odot_{m,k} G)(x) = \tilde{f}(x_k)
\]

where \( \odot \) denotes an ED operator and its subscript \( \{m, k\} \) denotes that the ED filter has a flat structure element \( G \) with a length \( m \) and the order of ED filter is \( k \). In the ED, \( \{f(x + z) | z \in G\} \) is sorted in ascending order and expressed as \( \{\tilde{f}(x_i), i = 1, \ldots, m\} \). The ED filter selects the \( k \)th element in the \( m \) samples of the sorted data, which is the segment of the input signal \( \{f(x + z) | z \in G\} \) according to the structure element \( G \). The output of the ED filter, when \( k \) is selected to be \( m \) or 1, can be regarded as that of the Erosion or Dilation filter in these special cases. Their relationship is given as follows:

\[
\begin{align*}
\text{Erosion} : & \quad (f \odot G)(x) = (f \odot_{m,1} G)(x) \\
\text{Dilation} : & \quad (f \oplus G)(x) = (f \odot_{m,m} G)(x)
\end{align*}
\]

This is because that the first and last element in the sorted data \( \{\tilde{f}(x_i)\} \) are the minimum and maximum values in \( \{f(x + z) | z \in G\} \) respectively. In these cases, the ED filter works as the Erosion and Dilation filters respectively. In general, the ED can provide filtering effects between that offered by the the Erosion and Dilation filters. The well-known Median filter firstly proposed by Tukey [5] can be denoted as \( (f \odot_{m, \frac{m+1}{2}} G)(x) \). Therefore, the Dilation, Erosion and Median filters can be regarded as the special cases of the ED filter which is formulated in a unified mathematical framework in terms of order \( k \). Here, \( G \) must be assumed to be symmetric to its origin, which also means the length \( m \) should be an odd number.

### 3 Noise Reduction Ability of ED Filter

#### 3.1 Output Distribution of ED Filter

Nonlinear morphological filters provide as a useful tool for signal analysis, especially for processing noisy signals. The study of the statistical properties of morphological filters is necessary in order to further understand their principles and noise influences[6]. In this paper, the statistical properties of ED filter are discussed and the output distribution of the ED filter is observed with respect to the independent non-identically distributed inputs. This output distribution is analysed to illustrate the noise reduction performance.

Let \( f(x) \) denote a multi-level signal independent of every \( x \) and it can be expressed as \( f_x \) for simplicity. Assuming that \( f_x \) comply with a continuous distribution function \( F(f_x) \) at position \( x \), the output distribution of the ED filter is obtained as follows:

\[
F_{\odot_{m,k}}(f_x) = 1 - \sum_{i=k}^{m} \binom{m}{i} (1 - F(f_x))^i F(f_x)^{m-i}
\]

An input of the ED filter is assumed \( f_x \) considered as the uniform distribution \( U[0, 1] \) and the normal Gaussian distribution \( N(0, 1) \) on the interval \([0, 1]\). The mean and variance of \( U[0, 1] \) are 0.5 and 1/12 and \( N(0, 1) \) are 0 and 1 respectively. Figure 1 illustrates the continuous distribution function \( F_{\odot_{m,k}}(f_x) \), when the input is \( U[0, 1] \) and \( N(0, 1) \) respectively, and the structure element length \( m \) is 5, given different \( k \).

![Figure 1: The output distribution function \( F_{\odot_{5,k}}(f_x) \) of ED filter](image)

It should be mentioned that the output distribution \( F_{\odot_{m,k}}(f_x) \) can be more easily calculated by the numerical computation than the analytical solution. This can be applied for other types of inputs to evaluate \( F_{\odot_{m,k}}(f_x) \).

#### 3.2 Performance of Noise Reduction

##### 3.2.1 Performance With Different Order \( k \)

From the output distribution function of the ED filter, the output mean and variance can be derived. If the input is \( N(0, 1) \) noise with zero mean and unit variance, the output mean and variance are shown in Figure 2(a) and (b) respectively, where the structure element length, \( m \), is set to 5. The following facts have been observed:

- For all \( k \), the output variances are less than 1, which means the noise is effectively reduced.
- When the order \( k \) is at the centre of interval \([1, m]\), the ED filter produces the least variance.
output, i.e. the best performance of noisy reduction. In other words, Median filter performs better than the Dilation and Erosion filters in noise reduction.

• The ED filter produces an output bias. The amplitude of output bias is determined by the output mean. Median filter results in a bias close to zero while Dilation and Erosion filters produce positive and negative output biases respectively.

Because the input variance is 1, the noise reduction can be theoretically measured in decibel by:

\[ ND = 10 \log_{10}(1/v_{m,k}) \]  

(3)

where \( v_{m,k} \) is the output variance of the ED filter with parameters \((m, k)\). The following table gives the result.

<table>
<thead>
<tr>
<th>( k )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( ND )</td>
<td>3.49dB</td>
<td>5.06dB</td>
<td>5.42dB</td>
<td>5.06dB</td>
<td>3.49dB</td>
</tr>
</tbody>
</table>

Table 1: Noise reduction performance for different order \( k \) and \( m=5 \)

3.2.2 Performance With Different Structure Element Length \( m \)

The structure element length \( m \) is the critical factor in morphological filters. Figure 3 shows the ED performance with different \( m \). From this figure, the following characters of the ED filter can be observed:

• The output variance of the ED filter decreases when \( m \) increases, which means that the ability of noise reduction is enhanced. It must be mentioned that more signal details will be removed simultaneously when \( m \) increases to a larger value. The tradeoff between reducing noise and reserving signal details must be taken into consideration when the ED filter is designed.

• The performance of Median filter is superior to Dilation or Erosion filters with any length \( m \).

• With increasing \( m \), the output bias of Dilation and Erosion filters increases correspondingly. The output bias of Median filter is close to zero.

Figure 3: The output mean and variance of ED filter \((m = 3, \cdots, 13)\) for input \( N(0,1) \)

4 Principles of Fault Location Using Mathematical Morphology

The MM technique has been applied for an ultra-high-speed directional relay and fault location[1]-[3]. The MMG technique was used to extract transient features from fault-generated voltage and current wave signals propagating along power transmission line during a post-fault period. In this work, the PSCAD/EMTDC software is employed to simulate a power transmission system based on a frequency-dependent transmission line model [7]. All parameters are given with reference to a 400kV EHV vertical constructed line [8], which is typically used in the UK SupreGrid system, as indicated in Figure 4. The transient current and voltage waves generated by a fault at point F travel away from the fault point at velocity \( v \). The fault of Type A is single-ended technique in which the transient sequences are captured at one end of the transmission line and the transient current or voltage signals are propagated as shown in Figure 4.

Figure 4: A solid phase-A-earth fault of Type A in 400kV power transmission lines system.

Type A fault locator is developed on the successive identification of the travelling high frequency transients arriving at the measurement points. The distance to the fault can be calculated based on the travelling wave propagation theory and the following
formulæ are given:

\[ L_R = \frac{(T_{r3} - T_{r1})L}{((T_{r2} - T_{r1}) + (T_{r3} - T_{r1}))} \] (4)

\[ L_S = \frac{(T_{s2} - T_{s1})L}{((T_{s2} - T_{s1}) + (T_{s3} - T_{s1}))} \] (5)

where \( L_r \) and \( L_s \) indicate the measured distance between the fault and the busbar R and S respectively, \( T_{r1}, T_{r2}, T_{r3} \) and \( T_{s1}, T_{s2}, T_{s3} \) are time-tags at which the captured transient sequences are observed.

The MMG technique is developed in order to extract the transient waves and exactly detect the arrival time of wavefronts at the measurement point. A quadratic MMG technique with a structure element of 5 sampling points along a flat line is utilized to process the transient current signal in order to extract and locate the transient sequences. The time-tags \( T_{r1}, T_{r2} \) and \( T_{r3} \) can be extracted accurately as shown in Figure 5.

\[ s(t) \]
\[ \rho(t) \]
\[ \frac{\text{Amplitude}}{\text{Time (ms)}} \]

Figure 5: The performance of MMG scheme without noise disturbance

\[ \frac{s(t)}{\rho(t)} \]
\[ \frac{\text{Time (ms)}}{\text{Time (ms)}} \]

Figure 6: The performance of MMG scheme with noise disturbance

In Figure 6, \( s(t) \) represents the transient current signal disturbed with noise and \( \rho(t) \) is the quadratic MMG. The influence of noise cannot be ignored because the maxima in \( T_{r1}, T_{r2} \) and \( T_{r3} \) are readily confused with the fluctuations caused by noise. Because the maxima at point A is very difficult to be distinguished from the one at point B, the correct recognition of the time-tag \( T_{r2} \) at point A becomes difficult and the fault location could be wrongly determined if the MMG is simply applied. Therefore, the noise reduction method must be developed and integrated into the MMG protection schemes.

5 Simulation Study and Results

5.1 Simulation of Noise Reduction

Figure 7(b) shows the random noise imposed on the original signal in Figure 7(a). The noise can be significantly reduced by the ED filter, which is shown in Figure 7(c), (d) and (e) respectively, where the performance of the ED filter with \( (m = 15, k = 1) \) is obviously the best. It cannot be ignored that the ramp is observably shifted when the ED filter with \( (m = 15, k = 1) \) and \( (m = 15, k = 15) \) are applied. This shift will inaccurately extract the position of ramp and it is not desirable for the extraction of time-tags in the fault location scheme.

\[ \frac{s(t)}{\rho(t)} \]
\[ \frac{\text{Time (ms)}}{\text{Time (ms)}} \]

Figure 7: The performance comparison of ED filters

5.2 Performance Comparison with Fourier Transform and Wavelet Transform

Figure 8: The performance of noise reduction with a 7th order Butterworth filter

A 7th-order Butterworth lowpass filter, which is a typical digital filter based on Fourier Transform, is applied for noise reduction and the result is shown.
in Figure 8. The normalized cutoff frequency is set to 0.1, which means only the lowest ten percent frequencies in the whole bandwidth of the input signal are kept as the output. The calculation of the Butterworth filter involves fourteen times multiplications and additions for each output sample and is comparable with the one of the ED filter \((m=15)\). The time shift for the ramp is obvious in Figure 8 and it will cause both the inaccurate extraction of time-tags in transient signals and delays in response to signal changes.

The simulations in [3] show that the Wavelet Transform method has a delay of 100\(\mu s\) in response to the transient signal changes, which is caused by the large data window required to cover the period of the Wavelet. However the ED filter can provide a rapid response simultaneously to the signal changes, since it requires a much shorter data window for calculation.

6 The Improvement of MMG-based Fault Location Scheme

The noise reduction method is developed and integrated into the MMG-based fault location scheme. The improvement of the MMG scheme can be achieved when the ED filter is used to reduce the noise in transient signals before MMG is applied. The significant improvement is shown in Figure 9.

![Figure 9: The improvement of MMG scheme with noise reduction](image)

The signal \(s(t)\) in Figure 6 is fed into an ED filter with parameters \((m = 5, k = 3)\). The output is denoted by \(s_{de}(t)\). The signal \(\rho_{de}(t)\) represents the output of the MMG filter applied to \(s_{de}(t)\). The noise is effectively reduced and the unprofitable maxima at point B is removed magically. Therefore, the accurate extraction of the time-tag \(T_{r2}\) can be achieved easier and more reliably.

A variety of fault scenarios have been simulated to evaluate the noise reduction with different fault types and different fault locations. The results show that the performance of ED filter is very promising.

7 Conclusion

In this paper, a novel MM ED filter is proposed to effectively reduce noise of the transient signals and improve the performance of the MMG-based fault location. The generic statistical characteristics of the proposed morphological filter have been discussed. The results show that the ED filter with parameters \((m, (m + 1)/2)\), functioned as a Median filter, performs better in noise reduction, in comparison with Dilation and Erosion filters. The efficiency of the noise reduction, offered by the ED filter, has been verified by the simulation studies. The results show that the MMG-based fault location scheme has been improved significantly, using the ED filter.

References


