SLIDING MODE IMPLEMENTATION OF A RATE COMMAND FLIGHT CONTROL SYSTEM FOR A HELICOPTER IN HOVER

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Abstract

This paper presents an investigation into the design of a flight control system, using a decoupled non-linear sliding mode control structure, designed using a linearised, 9th order representation of the dynamics of a PUMA helicopter in hover. The controllers are tested upon a higher order, non-linear helicopter model, called RASCAL. This design approach is used for rate command flight control implementation, and the control performance is assessed in the terms of handling qualities through the Aeronautical Design Standards for Rotorcraft (ADS-33). In this context a linearised approximation of the helicopter system is used to design an SMC control scheme. These controllers have been found to yield a system that satisfies the Level 1 handling qualities set out by ADS-33.

1 Introduction

The issue of helicopter flight control has been discussed extensively in the relevant literature [11,15]. Due to the complexity of helicopter dynamics, the design and implementation of controllers is difficult. Helicopters are highly coupled systems, not only due to the fact that the rotor, provides propulsion and is the main control actuator, therefore is the source of much of the complexity. The level of detail used to represent the rotor dynamics is often an important factor during the design of the controller and the selection of the associated parameters [9,12]. As the flight conditions change, these dynamics change, often resulting in controllers that will only perform to specification within the operation margin for which they are designed. All these factors have stimulated this area of research and resulted in a study of various control strategies being applied to this application.

One area of control that has had little application to the helicopter problem is that of non-linear, variable structure methods, such as Sliding Mode control, (SMC) [5,16,17]. SMC is comprised of two parts: a linear equivalent term and a non-linear switching term. This non-linear term is the unique attribute for this type of control scheme. It provides much of the controllers’ actuation power and provides high robustness to model uncertainty and external disturbance. However it is also often the source of concern for this controller structure as the non-linear term tends to switch around the zero error region giving a high frequency input to the control actuator, called chattering [4], which can be avoided by employing a soft switching control structure. The application of a SMC scheme to a helicopter system is presented in this paper. The controller will be evaluated using the Aeronautical Design Standard performance specification of handling qualities requirements for military rotorcraft, ADS-33E [1].

2 Helicopter Model

The RASCAL model (Rotorcraft Aerodynamics Simulation for Control Analysis) [8] is used to represent the helicopter dynamics in this study. This is a high order, nonlinear, individual rotor blade representation of the helicopter dynamics. This differs from many other helicopter models in that it uses an individual blade representation of the rotors, and not a disc. This means that the high-order dynamics of the rotor are captured, instead of the assumption that the rotor tilt is quasi-steady [8]. Although linearising models involve omitting important high order, non-linear dynamics, generally control engineering utilizes linear models of systems, which they base the controller designs upon. It is assumed that a linear model can represent the important dynamics needed for adequate controller design. This means that a linear model must be formed from the non-linear model described in [8], accomplished using a numerical method [7]. This defines a state space system given by:

\[
\dot{x} = Ax + Bu \quad (1)
\]

\[
x = [u \ v \ w \ p \ q \ r \ \phi \ \theta \ \psi]^T \quad (2)
\]

\[
u = [\theta_0 \ \theta_{1s} \ \theta_{1c} \ \theta_{0r}]^T \quad (3)
\]

where, \(A\) is the system derivative matrix, \(B\) is the control derivative matrix, \(u\) (surge), \(v\) (sway) and \(w\) (heave) are the velocities in the body referenced \(x\), \(y\), and \(z\) axes respectively, with the rotational velocities \(p\) (roll rate), \(q\) (pitch rate), and \(r\) (yaw rate), the attitudes \(\phi\) (roll), \(\theta\) (pitch) and \(\psi\) (yaw) about those axes. \(\theta_{0}\) is the main rotor collective, \(\theta_{1c}\) is the main rotor longitudinal cyclic, \(\theta_{1s}\) is the main rotor lateral cyclic, and \(\theta_{0r}\) is the tail rotor collective. This model is used for the controller design, but the full representation of the helicopter system is used for testing and evaluating the controllers. For this a PUMA helicopter in a hovering flight mode is used [8].
3 Sliding Mode Control

SMC is a non-linear control methodology. It has the advantages over linear controls in that it can be more robust to matched, unmodelled, uncertain system dynamics, and disturbances [4]. The controllers developed in this paper are of individual decoupled controllers [6, 13] that have their total control effort comprised of two parts: a linear equivalent term, \( u_{\text{equivalent}} \) and a non-linear switching term, \( u_{\text{switching}} \):

\[
\begin{align*}
\begin{cases}
  u = u_{\text{equivalent}} + u_{\text{switching}} & \\
  u_{\text{equivalent}} = -k^T x' & \\
  u_{\text{switching}} = \left( h^T b \right) \left[ h^T x_{\text{cmd}} - h^T f(x) - sgn(\sigma(\Delta x')) \phi_{BL} \right] 
\end{cases}
\end{align*}
\]

The closed loop system dynamics are represented by a sliding manifold [4]. This sliding manifold is a hyper plane representing zero steady state error, which the controller strives to converge the system toward. The switching term is effective when the system diverges from the zero sliding surface, and causes the system to converge back towards it. The equivalent controller is effective when upon the sliding manifold, representing the desired closed-loop system dynamics.

\[
\begin{align*}
  &u_{\text{equivalent}} = -k^T x' \\
  &k \text{ is the decoupled feedback gain vector found from pole placement [9] and } x' \text{ represents the decoupled system states. The switching term drives the systems when subjected to disturbance or commands, defined by the sliding surface, } \sigma, \text{ the sliding surface used here is represented thus [6, 13]:} \\
  &\sigma(\Delta x') = h^T \Delta x' = h^T (x' - x_{\text{cmd}}) \\
  &x_{\text{cmd}} \text{ is the desired trajectory. } \sigma \text{ is a function of the state error, } \Delta x', \text{ and } h \text{ is the right eigenvector [6] of the desired decoupled close loop system matrix, } A_c, \text{ found from:} \\
  &A_c = A' - b' a^T \\
  \text{where } A' \text{ is the decoupled system matrix, } b' \text{ is the decoupled input distribution vector. This leads to an appropriate controller function to represent the switching action [6, 13]:} \\
  &u_{\text{switching}} = \left( h^T b \right) \left[ h^T x_{\text{cmd}} - h^T f(x) - sgn(\sigma(\Delta x')) \right] \\
  \text{where, } f(x) \text{ represents the unmodelled dynamics. However,} \\
  \text{(8) is not very practical, due to system noise and actuator dynamics, resulting in chattering [4]. This is due to small} \\
  \text{values of } \sigma \text{ causing the switching term to add a magnitude of } \eta \text{ to the control action. Consequently, this} \\
  \text{overcompensates for the small error and thus causes the input signal to oscillate around } \sigma = 0. \text{ This may result in high actuator wear and excite any} \\
  \text{high frequency modes of the system. For this reason, other switching regimes that incorporate a "boundary layer", } \\
  \phi_{BL}, \text{ around the sliding surface can be used [13]. For this} \\
  \text{application, a saturation function is employed. This is similar to that of a } sgn \text{ function in that when } \sigma/\phi_{BL} > 1, \text{ or } \sigma/\phi_{BL} < -1, \text{ the} \\
  \text{output of the sat function is the same as the } sgn \text{ function.} \\
  \text{However when within the boundary layer, } |\sigma| \leq \phi_{BL}, \text{ the output is equal to } \sigma/\phi. \text{ This is known as } \text{pseud}(\sigma) \text{ (or soft) switching as it removes the hard transition between the sudden transitions from } -1 \text{ to } 1 \text{ [13]. This is shown below:} \\
  &\text{sat}(\sigma/\phi_{BL}) = \begin{cases} 
  1 & \sigma/\phi_{BL} > 1 \n  -1 & \sigma/\phi_{BL} < -1 \n  \sigma/\phi_{BL} & \text{otherwise} 
\end{cases}
\]

However, when within this region, there is no guarantee that the sliding surface will be reached [4]. Hence, there must be a trade-off in terms of robustness, performance and chattering. This gives the total control effort as:

\[
\begin{align*}
  u = -k^T x' + \left( h^T b \right) \left[ h^T x_{\text{cmd}} - h^T f(x) - sgn(\sigma(\Delta x')) \phi_{BL} \right] 
\end{align*}
\]

4 ADS-33

The ADS-33 document outlines the desired handling qualities of military rotorcraft (See ADS-33 and Cooper Harper for more information [1, 2]). This paper centers upon the design and implementation of a rate command response type, (offers the highest level of agility but tends to offer the lowest level of stabilisation). This means that the pilot is commanding the attitude rate (°/sec), and the pitch, roll and yaw rates will follow the trajectories of any pilot input. E.g. For a step lateral cyclic input, a step increase to a constant roll rate will result, proportional to the magnitude of the step input, and roll attitude will increase linearly at a gradient equal to the commanded roll rate. The assessment criteria for rate response types can be broken down into the following areas: small amplitude, moderate amplitude, large amplitude, and inter axis coupling.

Small Amplitude inputs are defined in two parts; Short-term and Mid-term response. The short-term response is defined by bandwidth and phase delay parameters. The bandwidth, \( \omega_{BW} \), is defined to be equal to the phase limited bandwidth, \( \omega_{BWphase} \) (frequency giving 45° phase margin) [1]. The phase delay parameter is defined as:

\[
\tau_p = \frac{\Delta \Phi}{57.3 \Delta \omega_{180}} 
\]

where \( \Delta \Phi/\omega_{180} \) is the difference in phase between the 180° frequency, \( \omega_{180} \), and twice the 180° frequency, \( 2\omega_{180} \). The mid-term response concerns the damping ratio, \( \varsigma \), at frequencies below the bandwidth frequency, \( \omega_{BW} \), found above. It is a measure of the controller’s ability to reject unwanted oscillations caused by disturbances, and high order dynamics. ADS-33 states that for Level 1, \( \varsigma < 0.35 \) [1], found by measuring the maxima and minima of the impulse response [3].

The moderate amplitude requirements are also known as Attitude Quickness. This is because it is a ratio of peak achievable rate to the peak attitude change. The above ratio can be compared to the ADS-33 criteria for different magnitudes of change in attitude. The criteria is structured so that the over and under shoot characteristics of the attitude response are detrimental. This measure is particularly relevant for rate response types, as they tend to offer the highest level of agility at the sacrifice of stability.

Large Amplitude response is important as it helps to assess the craft’s ability to retain high levels of handling at attitudes where the non-linearities are most severe [14]. As controllers are typically designed with linear, small-amplitude,
approximations of the real system, it is necessary to test the system outside the range where these approximations are valid. The requirements for Aggressive Target, Acquisition and Track maneuvers (highest specification), for hover and low speed flight are ±30° for pitch, ±50°/sec for roll, and ±60°/sec for yaw. The manner in which pitch is affected by roll, and vice-versa is the inter-axis coupling. For pitch and roll coupling the ratio of roll attitude due to pitch attitude commanded change following a fast input should not exceed ±0.25 for Level 1 [1], and vice versa.

5 Control System

The control structure employed to implement SMC is shown below.

![Control Structure Diagram](image)

The system incorporates a model reference block in the form of a low pass pre-filter. This takes the pilot commands, filtering out any high frequency inputs, providing demanded attitudes and rates (6 states). The 3 individual controllers provide the control action. However, as the controllers are decoupled, assuming no cross coupling of the dynamics or actuators, this needs to be taken into account. The unmodelled system dynamics (which include the cross coupling, off axis terms in the system matrix A), are represented by the $\tilde{j}(\omega)$ term in the controller given in (10).

However, for the cross coupling caused by the actuator dynamics (off axis terms in input distribution matrix B), a pre-compensation matrix is employed. The effect of this is to diagonalise the B matrix by selecting the matrix as follows:

$$
\begin{bmatrix}
\theta_{1c} \\
\theta_{1s} \\
\theta_{0vr}
\end{bmatrix} = 
\begin{bmatrix}
1 & -b_{4,2}/b_{4,3} & -b_{4,4}/b_{4,3} \\
-b_{5,2}/b_{5,3} & 1 & -b_{5,4}/b_{5,3} \\
-b_{6,2}/b_{6,4} & -b_{6,3}/b_{6,4} & 1
\end{bmatrix}
\begin{bmatrix}
\theta_{1cSMC} \\
\theta_{1sSMC} \\
\theta_{0vrSMC}
\end{bmatrix}
$$

(11)

where $\theta_{SMC}$ are the respective outputs from the 3 decoupled SMC controllers. The controllers themselves are designed as multi-state systems, with each controller design incorporating the appropriate rate and attitude states.

It has been found that testing the controller using the state space matrices $A$ and $B$, yields a very high closed loop gain system. However, when testing upon the full helicopter model, the resulting gain had to be greatly reduced. The high order dynamics of the system become troublesome, and consequently noise in the form of angular rate transferred from main rotor vibration, results in a large reduction of gain, and a large increase in boundary layer. A hard switching controller is not possible in this system due to these high frequency elements. Therefore a soft switching $sat$ regime is used. Finally the controllers are designed with closed loop poles at 0 and -4, indicating a desired critically damped system. These poles are not influenced by the systems dynamics due to the robust properties of the non-linear switching term. Although choosing real poles specifies a critically damped system, the system’s response can exhibit higher order characteristics due to the influence of the switching term, and the unmodelled, higher order dynamics of the system. This has the effect of decreasing the damping and the increased phase roll-off at high frequency reduces the bandwidth and increases the phase delay. The switching gain and boundary layer are chosen to give stable responses at high amplitude.

6 Pitch Control

The first requirement is that of small amplitude bandwidth and phase delay. Figure 2 shows the resulting bandwidth and phase delay in relation to the ADS-33 requirements. This indicates that the Level 1 requirement can be satisfied.

![ADS-33 requirements for Pitch Rate](image)

The mid-term requirement is a minimum damping factor of $\zeta=0.35$ in order to satisfy Level 1 handling [1]. The pitch damping is found to be 0.68, well above the 0.35 requirement.

The attainable attitude quickness and the ADS-33 requirements are shown in Figure 3. It can be seen that the Level 1 requirements can be satisfied. The large amplitude requirement demands for Rate control, a minimum of ±30°/sec [1]. This is shown in Figure 4 for a positive step in pitch. It should be noted that there is overshoot present in the response, and the coupling between pitch and roll and yaw.

![Moderate amplitude Pitch Rate](image)
7 Roll Control

The phase delay in roll is the most stringent, with a required phase delay of under 0.12sec for Level 1. Figure 5 shows the resulting bandwidth and phase delay in relation to the ADS-33 requirements, and that the Level 1 requirements are met. For mid-term response, the roll damping is found to be 0.70. The moderate amplitude requirements in roll are the most stringent. The attainable attitude quickness and the ADS-33 requirements are shown in Figure 6 and that Level 1 is met.

8 Coupling

Interaxis Coupling defined as a ratio of roll due to pitch, and vice-versa. For coupling of pitch due to roll, a ratio of 0.04 is attained, and for roll due to pitch, a ratio of 0.12 is found. This is well below the 0.25 Level 1 requirement [1].

9 Yaw Control

The yaw bandwidth requirement is the highest, with a required bandwidth of at least 3.5rad/sec. Figure 8 shows the resulting bandwidth and phase delay in relation to the ADS-33 requirements. This shows that the Level 1 requirements are satisfied. For mid-term response, the yaw damping has been found to be 0.9. The attainable attitude quickness and the ADS-33 requirements are shown in Figure 9. It can be seen that the Level 1 requirements are satisfied.

The large amplitude requirement demands for Rate control, a minimum of $\pm 50^\circ$/sec [1]. Figure 7 shows a positive step command in roll. It should be noted that there is overshoot present in the response, some high frequency oscillation, and the coupling between roll, pitch and yaw. As with pitch this coupling can be improved upon by increasing the gain of the off-axis controllers at the expense of the commanded responses in those axes.
It must be noted here that these results are for negative yaw commands. Although the resulting attitude quickness is very similar for positive steps up to 40°/sec, at inputs over 40°/sec the system can become unstable. This is likely to be due to the asymmetry of the aircraft from the direction of rotation of the main rotor, placement of the tail rotor, and the effect of wake caused by the former on the latter. Large roll rates are often found, as can also be seen in the large amplitude input in the negative direction, in Figure 10. The large amplitude requirement demands for Rate control, a minimum of ±60°/sec [1]. The response in yaw shows a high overshoot in the positive direction, while convergence of all axes if fast, there is high peak rates in pitch, and especially in yaw, implicating that decoupling of these dynamics has not been entirely successful. Further investigation of this is required to be able to improve this, and to enable high positive commanded rates.

8 Conclusions

In this paper, the successful implementation of a sliding mode control system has been presented. The assessment of these controllers to the ADS-33 requirements has shown that Level 1 handling can be achieved for pitch, roll and yaw, creating a high bandwidth, highly agile and stable control platform. Despite the use of a decoupled control strategy, the coupling between the channels is well within the acceptable levels. Also, although the high order dynamics are not taken into account in the controller design, which are often used in other control schemes, the testing of the controllers upon a model where these factors are included, the SMC can cope with such dynamics. However, there is room for improvement in this system. Many of the high order dynamics can be observed on the responses, with the effect of reducing bandwidth and damping, and removal of these would be desirable, particularly those between the roll rates caused by high amplitude yaw rate commands.

9 References