CLOSED LOOP CONTROL OF GAS JET FLAMES DISTRIBUTION USING
PROBABILITY DENSITY FUNCTION SHAPING TECHNIQUES

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Abstract

This paper presents a closed-loop control algorithm for the 2D Flame Temperature Distribution (FTD) of gases using the recently developed stochastic distribution control techniques [11]. At first, the temperature distribution of the flame along the cross section is approximated. Under the assumption that the flames are symmetrical with respect to the vertical axis, the shape control of FTD is transferred into the control of the Flame Energy Distributions (FED). A control algorithm is then designed using the output probability density function control concept, where an equivalent static model is established using the B-spline functions. Simulations have been performed where the mechanical models of the flames and the FED controller are embedded into a closed-loop control structure. The results show that the FED can be made to effectively track a given shape.

1 Introduction

The efficiency of the boiler is mostly determined by the flame status of each burner [1–3]. The fuel of the boiler can be either blast furnace gas or pulverized coals. In the blast furnace gas boiler, the flame at each burner is in a form of gas turbulent jet flame when the fuel jet velocity is higher than a certain threshold value, and in the form of gas flamelet jet flame when the velocity is lower. In pulverized coal boilers, the flame at each burner is formed by two-phase turbulent reactive flows.

According to Bilger [4], for turbulent reactive flows, there are mainly two modelling methods, namely the probability density function (PDF) based modelling [5] and the conditional moment closure (CMC) based modelling [6]. The former calculates the properties of turbulent reactive flow fields, where at each point in the flow field, a complete statistical description of the state of the fluid is established for the velocity-composition joint PDF. On the other hand, the CMC method predicts the conditional averages and higher moments of quantities such as species mass fractions and enthalpy. Based on these models, numerical simulation can be performed to produce the FTD of the flame, leading to a time-consuming process.

For the gas flamelet jet flame, a static model can be established [7] and used in the closed-loop control system. For example, Hans et al [8] designed a closed-loop control system of the gas jet flame, where, oxygen and combustible gas are used as the control inputs and their combustion flame is the output. CCD cameras are used to capture multi-spectral images that are processed to produce flame features (brightness and height). Zhou et al [9] realized a closed-loop control of the boiler flame through the radiant energy obtained from the temperature of the flame. However, since traditional boilers control the input fuels according to the oxygen ratio in the flue gas, it is generally difficult to obtain a desired closed-loop performance due to the existence of a long time delay. As such, serial control is employed [9] to solve the large inertia problem by taking the radiant energy as an intermediate variable.

In the analysis of flames, the FTD is generally used as a parameter to estimate the situation of the boiler. In many cases, FTDs can be “one-to-one” mapped into the Flame Energy Distributions (FED) that can be regarded as a PDF. This indicates that the recently developed stochastic distribution control theory (that aims at controlling the shape of systems PDFs) can be directly applied to formulate a closed-loop control that controls on-line the shape of the FED.

In recent years, the shape control of the PDFs of system variables has been proposed [11 – 12], where the aim of the control input design is to guarantee the tracking of output PDF to a given PDF. It has been shown that these groups of methods can be widely used in chemical processing and papermaking systems. As the flames energy distribution can be regarded as a PDF, in this paper a PDF shaping method will be used to control the gas flamelet jet flame based on the static model. In this closed-loop control system, the jetting rate of the fuel is used as the control input and the FED is the output. It is assumed that instruments such as CCD cameras are available to capture the flame images that can be further processed to extract the FED information, which is taken as the feedback signal for the closed-loop control.

2 Flame model presentation and simulation

2.1 Flame model presentation

In practice, the gas flamelet jet flame is a 3-D flame just like a torch flame. Because the flamelet jet flame is an axially...
symmetrical flame, a 2-D temperature distribution model can be well used in most cases to represent the flame temperature distribution as shown in Figure 1, where \(d_0\) is the diameter of the fuel injection nozzle, \(u_0\) is the speed of the fuel injection at the nozzle, \(x\) is the vertical coordinate, \(r\) is the horizontal coordinate, and \(b(x)\) is the flame boundary. The two arcs in Figure 1 are the isotherms of the jet flame.

Figure 1: Flame temperature distribution demonstration

2.2 Boundary conditions and conservation equations

To obtain a static model the flame is assumed to be an uncompressible steady flow and the columned free jet flame. All the variables concerned are defined as follows:

- \(r\) = Horizontal coordinate
- \(x\) = Vertical coordinate
- \(u\) = Vertical velocity
- \(v\) = Horizontal velocity
- \(T\) = Temperature
- \(T_0\) = Temperature at \(x\), that is, temperature in environment
- \(Y_i\) = Mass fraction for component \(i\), where \(\sum Y_i = 1\)
- \(\rho\) = \(\sum \rho_i\), Desity of the mixture
- \(c_p\) = Specific heat capacity of the fuel
- \(w_i\) = Reaction rate of the component \(i\)
- \(Q_i\) = Reaction heat of the component \(i\)
- \(\gamma, \alpha, D\) = Movement viscosity coefficient, diffusion coefficient, heat diffusion coefficient of laminar flow respectively
- \(\gamma_i, \alpha_i, D_i\) = Movement viscosity coefficient, diffusion coefficient, heat diffusion coefficient of turbulent flow respectively

Using these notations, the following continuity, momentum, and energy and mass balance equations can be obtained

\[
\frac{\partial (ru)}{\partial x} + \frac{\partial (rv)}{\partial r} = \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) \tag{1}
\]

\[
ru \frac{\partial u}{\partial x} + rv \frac{\partial u}{\partial r} = (\gamma + \gamma_\tau) \frac{\partial}{\partial r} \left( r \frac{\partial u}{\partial r} \right) \tag{2}
\]

where \(i (= 1, 2, 3)\) stand for fuel (F), oxidant (OX), and the product of combustion. These equations are sufficient to present the evolution of the flame.

To obtain an analytical solution, the following boundary conditions are needed:

- When \(x = 0\) and \(0 \leq r \leq \frac{d_0}{2}\):
  \[
u = u_0\]
  \[T - T_e = T_0 - T_e\]
  \[Y_{OX} - Y_{OX,mb} = -Y_{OX,mb}\]
  \[Y_F = Y_{F,mb}\]

- When \(r = 0\) and \(0 \leq x \leq \infty\):
  \[
  \frac{\partial}{\partial r} \left( ru \frac{\partial T}{\partial r} \right) = 0 \quad \frac{\partial}{\partial r} \left( ru \frac{\partial T}{\partial r} \right) = 0 \\
  \frac{\partial}{\partial r} \left( r (ruv) \right) = 0 \quad Y_{OX} = Y_{OX,mb} = Y_{OX,mb} - Y_{OX,mb}\]

The suffix \(\infty\) represents the value infinite

The suffix 0 represents the value at the nozzle

The suffix \(m\) represents the value at central axis

2.3 Solution of the equations (1)-(4)

Because the distribution of velocity, temperature and mass fraction in any cross section is similar, a Gaussian type function in Figure 2 can be used to describe such a distribution, leading to

\[
T = \exp\left[-K\left(\frac{r}{b(x)}\right)^2\right] \tag{5}
\]

where \(K\) is a coefficient between 82.0 and 92.0. To simplify the model, the following double biases distribution is used to approximate the Gaussian distribution

\[
T = 1 - \frac{r}{b(x)}, \quad |r| \leq b(x) \tag{6}
\]

A satisfactory solution can be obtained from the conservation equations (1)-(4) after such an approximation.
Indeed, the initial conditions of these conservation equations when \( x = 0 \) and \( 0 \leq r \leq d_n / 2 \) are given as follows:

\[
\begin{align*}
\text{Velocity: } & \quad u = u_0 \\
\text{Temperature: } & \quad T - T_a = T_0 - T_a \\
\text{Flame Temperature: } & \quad Y_f = Y_{f,0} \\
\end{align*}
\]

\[
\begin{align*}
\text{Initial Conditions: } & \quad Y_{ox} - Y_{ox,\infty} = -Y_{p,\infty} \\
& \quad \frac{\partial Y_{ox} - Y_{ox,\infty}}{\partial r} = 0 \\
& \quad \frac{\partial Y_{p} - Y_{p,\infty}}{\partial r} = 0
\end{align*}
\]

On the other hand, when \( r = 0 \) and \( 0 \leq x \leq \infty \), it can be shown that

\[
\begin{align*}
\text{Velocity: } & \quad \frac{\partial u}{\partial r} = 0 \\
\text{Temperature: } & \quad \frac{\partial (T - T_a)}{\partial r} = 0 \\
\text{Flame Temperature: } & \quad \frac{\partial Y_{ox} - Y_{ox,\infty}}{\partial r} = 0 \\
\end{align*}
\]

Based on the temperature distribution approximation and the initial conditions, a solution can be obtained from the conservation equations. For this purpose, define

\[
F(x, r) = \frac{B_{FT}}{B_{FT,0}} = \frac{Q_f}{Q_f} + Y_f
\]

As such, the FTD can be formulated to give:

\[
T(x, r) - T_a = \left( [T_0 - T_a] - \frac{Q_f}{c_p} \right) F(x, r) - \frac{Y_f Q_f}{c_p}
\]

For turbulent flows, the definition of \( F(x, r) \) in (10) is given by

\[
F(x, r) = \left( 1 + 8 \frac{\left( \frac{c_f c}{d_o} \right)^{1 - 1} \left( \frac{2 c}{d_o} \right)^{1 - 1} \left( \frac{3 c}{d_o} \right)^{1 - 1} \right) \right)
\]

Whilst for the flamelet flows, \( F(x, r) \) in (10) is given by:

\[
F(x, r) = \left( 1 + \frac{8 \left( \frac{c_f c}{d_o} \right)^{1 - 1} \left( \frac{2 c}{d_o} \right)^{1 - 1} \left( \frac{3 c}{d_o} \right)^{1 - 1} \right) \right)
\]

\[
\Re_e = \frac{u_0 d_o}{\nu}
\]

From (13) it can be seen that the control input \( u_0 \) controls the value of \( F(x, r) \). Using (11), it can be seen that \( u_0 \) directly controls the FTD (i.e., \( T(x, r) \)). As such, model (10) – (13) can be regarded as a static model for the flame system.

2.4 Simulation of the FTD model

Based on the solutions (10)-(13), an FTD model can be constructed as shown in Figure 3.

In the simulation, the input is the fuel jet velocity \( u_0 \) and the output is the FTD, namely \( T(x, r) \). The process parameters are selected as follows:

\[
\begin{align*}
\text{Fuel - CO} & \quad c_p = 2078.6 \ J / \ K g \cdot K \\
Q_f & = -282.84 \ K J / \ m o l \\
d_o & = 0.01m \\
u_0 & = 1 m / s \\
\nu & = 3.695 \times 10^{-5} \\
T_0 & = 500K \\
K & \text{is the absolute thermometric scale} \\
0^\circ C & = 273.15K
\end{align*}
\]

Using model (10)-(13), different FTDs can be obtained when \( u_0 \) changes. If different temperature values are represented by different colour, then a coloured temperature distribution image can be obtained as shown in Figure 4. In Figure 4, the height of the simulated flame is 80cm, the width is 3cm and the highest temperature is 2000K when the CO jet velocity is \( u_0 = 1 m/s \). Because the Reynolds number of CO is between 4800 and 5000, the flamelet flame changes to a turbulent flame when \( u_0 \geq 17.7 m/s \), leading to the following parameters used to simulate the turbulent flame FTD as shown in Figure 5.

\[
\begin{align*}
\text{Fuel - CO} & \quad c_p = 2078.6 \ J / \ K g \cdot K \\
Q_f & = -282.84 \ K J / \ m o l \\
d_o & = 0.01m \\
u_0 & = 30 m / s \\
c & = 0.0128 \\
T_0 & = 500K
\end{align*}
\]
3 System modelling and control

The process to be controlled is the static jetting flame model as shown in (10)-(13), where the output is the flame temperature distribution, which is a 2D distribution along (x, r) directions (see Figure 1). However, since the flame is symmetric, another distribution, namely the Flame Energy Distribution, can be used as a feedback for the closed-loop control. Physically, the FED is the sum of all the temperature values in each horizontal cross section and is therefore calculated from

\[
\gamma(x, u_0(k)) = \frac{\int_{0}^{\infty} T(x, r) dr}{\int_{0}^{\infty} dx \int_{0}^{\infty} T(x, r) dr}
\]

(16)

Such a \(\gamma(x, u_0(k))\), that corresponds to the \(T(x, r)\) as shown in the left hand side of Figure 6, is displayed in the right hand side of Figure 6. In this case, the input is the fuel jet velocity \(u_0(k) \in [0, 2]\) (where \(k\) stands for the current sample time). As the FTD can be retrieved from \(\gamma(x, u_0(k))\) using the symmetrical nature of the flames, the aim of the control design is to make \(\gamma(x, u_0(k))\) as close as possible to a given flame energy distribution.

As \(\gamma(x, u_0(k))\) is a ratio of the energy in each cross section to the total energy as shown in (16), it can also be regarded as a probability density function, where the stochastic distribution theory [11] can be directly applied to control the shape of \(\gamma(x, u_0(k))\). A closed-loop flame distribution control system can be established as shown in Figure 7.

\[
\eta = \int_{0}^{1} (g(x) - L(x))^2 dx
\]

(21)

then the control input can be calculated from

\[
u_0(k) = u_0(k-1) - 2\mu(V^T(u)\Sigma - \eta)\frac{\partial V(u)}{\partial u} | u = u_0(k-1)
\]

(22)

where \(\mu > 0\) is a pre-specified step length.

4 Simulation results and solutions

Forty 2nd order basis functions are used to approximate the flame energy distribution curve. For one input can generate one FED curve, the given FED curve is determined by a given input \(u_0\). Let \(\mu = 0.01\) in equation (22). Figure 8 shows the response of the control input as calculated from (22), and Figure 9 gives the response of the performance function in (20). The 3D plots in Figures 10-11 show how the closed-loop control can be realized as so as to control the distribution of FED towards its target distribution.
Conclusions

In this paper a static model of flames distribution systems is derived and used for the closed-loop control design. Under the assumption that the flame is symmetric with respect to the vertical axis, the flame energy distribution function is used to realise the control of the flames temperature distribution. As such a distribution can now be measured through video camera and image processing, the proposed control can be used to construct an effective closed-loop control for the flames. A simulated example has been given where desired results have been obtained.

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References