Keywords: model predictive control, LQ-MPC, hot rolling mill, dynamic rolling simulator, looper height control, looper and tension control.

Abstract

The looper and tension control is important in a hot strip mill because it affects the strip quality as well as strip threading. There are tension disturbances, which might cause constraint violations in the transient state resulting in severe defects of strip quality and instability of operation. Therefore, constraints in controller design should be considered. Another challenge in the controller design comes from the mutual interaction between the looper angle and tension. The mutual interaction can make system response slow and degrades the control performance. Up to now, main design issue was focused to reduce mutual interaction and reject disturbance quickly. This paper investigates the efficacy of an MPC scheme for this control problem, both in handling the mutual interaction and also the constraints.

1 Introduction

The tension and looper control system affects the strip threading as well as dimensional accuracy such as thickness and width of a strip through mass flow and inter-stand strip tension. Fig. 1 shows the detail of the tension and looper control in the finishing mill. To control inter-stand strip tension the rotating speed of the upstream stand’s mill motor or torque of the looper motor can be manipulated. The looper between the stands reduces tension variations by changing its angle, and can make the process operate more smoothly by absorbing excessive loops even under a severe mass flow unbalance. Therefore, it is better to keep the looper position close to a desired value during operation to give maximum flexibility for handling sudden changes in strip tension. The looper position can be controlled by the mill motor speed or looper motor angular velocity. There is mutual interaction between the looper angle and the tension which makes it difficult to design a controller, hence resulting in degraded control performance. The looper position and inter-stand strip tension should be controlled simultaneously to effectively reduce the mutual interaction.

Up to now, many control strategies which include robust control (Asada, et al., 2003, Imanary, et al., 1997, Hearns et al., 2000), multivariable control (Asano, et al., 2000), optimal control (Okada, et al., 1998) and so on have been proposed and applied to achieve enhanced performance of the tension and looper control. The main issue of controller design was to reduce mutual interaction and to reject disturbance quickly.

However, there is another specification, that is, the constraints satisfaction, which should be considered in the controller design for ensuring product quality and stable operation in the transient state. In looper and tension control the constraints should be satisfied, for instance the looper angle should not be close to a pass line of the strip and the maximum strip tension should be avoided to satisfy the width and thickness quality requirements. Also, there are requirements on the looper angle and minimum strip tension to avoid strip threading becoming unstable with the consequent trip of the main motors. There are tension disturbances, which might cause constraint violations resulting in severe defects of production quality and instability of operation. Therefore, constraints also should be considered for controller design. A paper (Schuurmans, et al., 2002) suggested a MPC controller design with constraint handling of looper angle. However, it was the design for the SISO case.

This paper investigates a MPC design scheme (Mayne et al, 2000) for the looper and tension control problem, both in handling the mutual interaction and also the constraints. Hence the main contributions of this paper are: (i) a derivation of a mathematical model of the looper and tension components of a hot strip mill; (ii) a critique of an MPC...
control design carried out on this model. The model is derived in section 2, the controller design is discussed in section 3 and conclusions are given in section 4.

2 Process Model

The Dynamic Simulator Model (POSCO, 1997) based on a hot strip mill of POSCO is used as the process. From this a linear model is constructed for controller design. This section gives the underlying dynamics equations and shows how these can be used to construct a useful model. This is then followed by discussion of a linearisation and how to model disturbance effects.

First, define the following variables. \( T \): load torque, \( \sigma \): load torque component by strip tension, \( sT \): load torque component by strip weight, \( wT \): load torque component by looper weight, \( lpr \): looper roll radius, \( \sigma \): strip tension, \( w \): strip width, \( h \): strip thickness, \( g \): acceleration of gravity, \( \rho \): strip density, \( L \): inter-stand strip length, \( lpw \): looper weight.

2.1 Inter-Stand Strip Tension Model

Inter-stand strip tension is defined by eqn. (1) and fig. 2.

\[
\sigma = \frac{E}{L} \left( \frac{dL}{dt} + \frac{(v_{n+1,i} - v_{n,i})}{dt} \right) \tag{1}
\]

\[
= \frac{E}{L} \left( \frac{dL}{dt} + \frac{E}{L} \int (v_{n+1,i} - v_{n,i}) dt \right)
= \frac{E}{L} (L' - L_0) + \frac{E}{L} \int (v_{n+1,i} - v_{n,i}) dt = \frac{E (L' - L)}{L}
\]

where, \( L' = \sqrt{(x^2 + y^2)} + \sqrt{(L_0 - x)^2 + y^2} \) and \( L = L_0 - \int (v_{n+1,i} - v_{n,i}) dt \)

2.2 Load Torque on a Looper Motor

The equation of the looper with inertial \( J_L \) is defined by applying Newton’s second law.

\[
J_L \dot{\theta} = M - T \tag{2}
\]

\[
T = T_\sigma + T_s + T_w \tag{3}
\]

\[
T_\sigma = w \sigma [\sin(\theta + \beta) - \sin(\theta - \alpha)] \tag{4}
\]

\[
\alpha = \sin^{-1} \left( \frac{y_s + l \sin \theta + r_w}{\sqrt{(x_s + l \cos \theta)^2 + (y_s + l \sin \theta + r_w)^2}} \right)
\]

\[
\beta = \sin^{-1} \left( \frac{y_s + l \sin \theta + r_w}{\sqrt{(L_0 - x_s - l \cos \theta)^2 + (y_s + l \sin \theta + r_w)^2}} \right)
\]

\[
T_s = g \rho \pi \ell_0 \cos \theta \tag{5}
\]

\[
T_w = g w_\pi \ell_0 \cos(\theta + \theta_\phi) \tag{6}
\]

2.3 State Space Representation of Looper-tension Model

To design a linear MPC controller the nonlinear models of the Dynamic Simulator are linearised about operating point of 12.9 N/mm² tension and 0.35 rad looper angle. The resultant state space model is constructed as follows.

\[
\dot{x} = A'x + B'u + F'\dot{d}
\]

\[
y = C'x + D'\dot{u}
\]

where, \( x = [w, \sigma, \theta, w_\pi, x_s, x_s, x_s] \)

\[
A' = \begin{bmatrix}
0 & -K_s \frac{J_s}{J_s} & 0 & -\frac{\sigma_\pi}{J_s} & 0 & 0 & 0 \\
EK_s & -EK_s & 0 & -E(1 + f) & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -K_s \frac{K_s}{J_s} & 0 & -K_s \frac{K_s}{J_s} & 0 & 0 & 0 \\
-\frac{\sigma_\pi + K_s \frac{K_s}{J_s}}{L} & 0 & 0 & -R + K_m \frac{1}{L} & K_m \frac{1}{L} & 0 & 0 \\
-\frac{K_m \frac{1}{L}}{L} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 
\end{bmatrix}
\]
\[
E' = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
K_{c0} - K_{p0} & 0 & 0 & 0 & 0
\end{bmatrix}, \quad F' = \begin{bmatrix}
0 \\
E/L \\
0 \\
0 \\
0
\end{bmatrix}
\]

\[
C' = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}, \quad d' = 0
\]

Where, the gain \( K_{c0} \) represents the loop length variation by the looper angle variation. This relationship is shown in fig.4 and linearised by Taylor expansion.

\[
K_{c0} = \frac{dL}{d\theta} \approx \bar{L} + \frac{\partial T(\theta)}{\partial \theta} \quad \text{(8)}
\]

Where, \( \bar{L} \) represents the loop length at equilibrium state.

Gain \( K_{p0} \) represents the relationship between the looper angle and load torque of a looper motor. This relationship is shown in fig.5 and linearised by the following eqn.

\[
K_{p0} = \frac{dT}{d\theta} \approx \bar{T} + \frac{\partial T(w, h, \sigma, \theta)}{\partial \theta} \quad \text{(9)}
\]

Where, \( \bar{T} \) represents the load torque of a looper motor at equilibrium state.

### 2.4 Disturbances

There are several disturbances to the looper and tension control system. One comes from mass flow unbalance due to the set-up mismatch and impact drop phenomena during transient rolling operation. Another disturbance comes from the AGC action. AGC systems are there to get higher gauge quality. AGC systems reject thickness disturbances due to skid marks, roll eccentricity and so on. While the hydraulic screw down system for quick thickness control makes it possible to give quicker response to the AGC, it often creates a disturbance to the tension control system because of the mass flow change caused by quicker roll gap movement. Another large disturbance for downstream loopers occurs at coiling (Imanari, et al. 1997). When the lead end of a strip is coiled, a large tension between the last stand and down coiler is often caused. It also causes tension fluctuation at the finishing mill inter-stand.

All these disturbances severely influence the strip tension and looper angle control performance, thus affecting strip thickness and width.

### 3. MPC Controller Design

A MPC controller is designed to achieve optimal performance of the disturbance rejection and reference tracking in looper and tension control. Integral action is incorporated to allow both offset free tracking and disturbance observation. A major difficult in this application is the constraints on inputs and outputs and also there is a desire for good a priori stability assurances. All these specifications can be included in a by now relatively conventional MPC design such as the LQMP algorithm (Scokaert et al 1998) which here is implemented to looper and tension control using the CLP (Closed Loop paradigm) (Rossiter, 2003).

#### 3.1 LQMP Controller Design

LQMP chooses an optimal state feedback (or control law) \( K \) as that arising from an unconstrained optimal control problem (for conventional MPC this implies infinite horizons). Where this control law is not predicted to cause constraint violations, it is used, and therefore, at least for the nominal case, optimal performance should ensue.

At times the unconstrained control will give predictions that are expected to violate constraints and hence such a law is 'infeasible'. At these times, the first \( n \) predicted control
moves are modified to optimise predicted performance subject to constraint satisfaction. It can be shown (Scokaert et al 1998) that if \( n_c \) is large enough, then this algorithm will find the optimal for the constrained infinite dimensional optimisation. For a wisely chosen \( K \), this value of \( n_c \) may not be too large but such issues are beyond the remit of this paper (Rossiter, 2003) and here we assume \( n_c \) is large enough. A brief summary of the key steps is given next for completeness.

In order to design the MPC controller continuous plant of eqn.(7) is converted to the following equation by discretisation.

\[
x(k+1) = Ax(k) + Bu(k) + Fd(k)
\]
\[
y(k) = Cx(k) + Du(k)
\]

The LQ MPC control law predictions are given by

\[
u_i = -K_{opt} x_i + c_i \quad i < n_c
\]
\[
u_i = -K_{opt} x_i + c_i \quad i \geq n_c
\]

Where, \( K_{opt} \) represents the optimal stabilizing feedback gains without constraints. Vector \( c \) contains control perturbations to ensure constraint satisfaction during transients. Assuming the constraints can be defined, for all \( i \), as

\[
C_u - d \leq 0, \quad C_{max} x_i - d_{max} \leq 0
\]

then, with a conventional 2-norm performance index, the constrained optimisation can be defined as

\[
\min_{x_{i+1},x_{i+2},...} J_i = \sum_{i=0}^{\infty} x_i^T Q x_i + u_i^T R u_i
\]

s.t. constraints (12) \( \quad 1 \)

Substituting in from (11) and using Lyapunov equations to sum the cost to infinity and hence define a suitable matrix \( P \), one can easily show \(^1\) that the cost function \( J \) takes the form

\[
J = c_i^T S c_i + 2 c_i^T S c x_i + c_i = [c_0, \ldots, c_n]
\]

(14)

Where, \( S = H_c^T diag(Q) H_c + H_{\alpha}^T diag(R) H_{\alpha} + H_{\nu}^T PH_{\nu} \)

\( S_{cs} = H_{cs}^T diag(Q) P_{cs} + H_{\alpha}^T diag(R) P_{cs} + H_{\nu}^T P_{cs} \)

\[
H_c = \begin{bmatrix} B & 0 & 0 & \ldots \ \\
\Phi B & B & 0 & \ldots \\
\Phi_2 B & \Phi B & B & \ldots \end{bmatrix}, \quad P_c = \begin{bmatrix} \Phi \\
\Phi^2 \\
\vdots \\
\Phi^n \end{bmatrix}
\]

\[
H_{\alpha} = \begin{bmatrix} I & 0 & 0 & \ldots \ \\
-KB & I & 0 & \ldots \\
-K\Phi B & -KB & I & \ldots \end{bmatrix}, \quad P_{cs} = \begin{bmatrix} -K \\
-K\Phi \\
-K\Phi^2 \\
\vdots \end{bmatrix}
\]

and \( \Phi = A-BK \).

\(^1\) Typically \( S_{cs} = 0 \).

The required on-line optimisation is a quadratic program which in most cases is considered tractable.

### 3.2 Linearised model and tuning

\( \begin{align*}
L = 5505 [mm], \quad J_\alpha = 2.1 [kg \cdot m^2], \quad f = 0.0703, \quad R_c = 0.09141 [\Omega], \\
L_n = 0.00122 [H], \quad \zeta_R = 20.0 [Nm/\cdot A], \quad R_n = 0.0610 [\Omega], \\
J = 0.00122 [H], \quad J_t = 1.91 [kg \cdot m^2], \quad \zeta_\phi = 3.326 [Nm/\cdot A], \quad g_i = 10.
\end{align*} \)

The weights in (13) were taken as \( Q = C^T C \), \( R = I \).

Control law (11-13) was implemented with \( n_c = 5 \) under the following scenario:

The simulation was performed for the no.6 and no.7 stand in rolling mills with step disturbance of strip speed 15 [mm/sec] at 1 [sec] which is generated from 0.3 [%] changes in thickness. In the simulation, it is assumed that the strip is 1240 [mm] in width, 2.53 [mm] in target thickness, 12.9 [mm] in tension reference and 20 [degree] in looper angle reference.

### 3.3 Simulation Results

With the MPC controller designed above, simulation was performed for a two inputs and two outputs looper and tension control process. Fig. 5 and Fig.6 show the performance of the tension and looper angle control.

**Fig. 5 Tension control**

**Fig.6. Looper angle control**
These simulation results show that MPC can achieve offset free tracking under the constant disturbance owing to the integral action. Moreover, it represents better control performance compared with the conventional PI control scheme.

4 Conclusions

This paper makes two main contributions. First it derives a simple model for the looper and tension control problem in a hot strip mill which can be used for investigations of different control strategies.

Secondly a critique is given of an application of an MPC scheme. It is demonstrated that the designed MPC can achieve offset free tracking under the constant disturbance. Moreover, it has better control performance than the conventional PI control due to its systematic handling of interaction. Therefore, It has been done the MPC control scheme can be a useful design strategy for looper and tension control problem as handling constraints as well as mutual interaction systematically.

References


