WORST-CASE STATE-SPACE $\mu$-ANALYSIS FOR SYSTEMS SUBJECT TO REAL PARAMETRIC UNCERTAINTY

M. Halton*, M. J. Hayes†*, P. Iordanov‡

*Dept. of Electronic and Computer Engineering, University of Limerick, Ireland
†Dept. of Electronic and Computer Engineering, NUI Maynooth, Ireland
‡E-mail: martin.j.hayes@ul.ie Phone: +353-61-202577 Fax: +353-61-202572

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Abstract

This paper considers the application of the structured singular value and the skewed structured singular value to the robust stability of systems subject to strictly real parametric uncertainties. The focus is on the calculation of improved lower bounds for this type of uncertainty using frequency independent techniques necessary to counteract the discontinuous nature of the analysis in this instance. Three state-space formulations that transform the original structured singular value problem into a frequency independent skewed structured singular value problem are detailed. The formulations are critically assessed by performing a robust stability analysis on a safety-critical experimental steer-by-wire vehicle.

1 Introduction

For many physical systems, it is appropriate to consider the effect of parameter based uncertainty on system stability and performance. The structured singular value, $\mu$, provides a rigorous means of analysing the robustness of such systems [4]. Although $\mu$ analysis provides a general framework for robust analysis, in practice skew $\mu$ (or $\mu^s$) problems commonly occur where not all the uncertain parameters are allowed to vary freely. One example of skew $\mu$ is a stability analysis where frequency is considered as an uncertain variable. By reformulation of the problem incorporating frequency as a perturbation parameter, the gridded $\mu$ problem becomes a skew $\mu$ problem where no frequencies are missed in the search. Although these skew $\mu$ problems can be reformulated as iterative $\mu$ problems, direct methods for obtaining bounds on skew $\mu$ are desirable for computational reasons. Recent work on the development of direct methods to determine both upper and lower bounds on skew $\mu$ for suitably mixed uncertainty is detailed in [7, 6].

The focus here is on the special case where the parametric uncertainty is constrained to be real. It is known that this so-called “real $\mu$” problem is a discontinuous function of the problem data [2]. The $\mu$-Tools lower bound algorithm generally fails to converge for this class of problem, in which case the lower bound on $\mu$ is taken as zero. The discontinuous nature of real $\mu$ dictates that frequency sweeps are not guaranteed to return the peak value of $\mu$ or the worst-case perturbation unwrapped from a lower bound due to the finite nature of any frequency grid. As an alternative approach, three frequency independent state-space formulations are detailed here. Commonly, lower bound algorithms for skew $\mu$ based around Matlab $\mu$-Tools algorithms exhibit convergence difficulties for this class of problem [6]. In this work, a new skew $\mu$ optimisation-based lower bound algorithm is presented. It is suggested that this new method coupled with the aforementioned state-space formulations return better lower bound results than existing frequency sweeping techniques. As an intuitively appealing and practical example, a robust stability analysis for a safety-critical steer-by-wire system taken from [9] is considered where the parameter uncertainty is both strictly real and repeated.

This paper is outlined as follows, section 2 introduces the nomenclature used and details the formal definitions of both the structured singular value and the skew structured singular value. Section 3 details new and existing computational techniques based on frequency independent state-space formulations. Section 4 focuses on the special case of strictly real parametric uncertainty introducing the optimisation formulation for a lower bound on skew $\mu$. Section 5 gives a descriptive overview of the steer-by-wire system. A robust stability analysis is carried out using standard $\mu$-Tools techniques and the methodologies developed in this paper.

2 Robustness Analysis Techniques

![Figure 1: Canonical $\mu$ analysis framework.](image-url)

The $\mu$ approach for systems analysis is based on the observation that problems involving interconnections of linear time invariant (LTI) systems with uncertain parameters and unmod-
Note that block structure allows for repeated real scalars \( \delta \) and complex scalars \( \tilde{\delta} \). The structured singular value, introduced.

If two block structures are defined as

\[ \mathbf{K} = (k_1, \ldots, k_{m_r}, k_{m_r+1}, \ldots, k_{m_r+m_c}, \ldots, k_{m_r+m_c+1}, \ldots, k_m) \]

with \( m = m_r + m_c + m_c \). This \( m \)-tuple specifies the dimensions of the perturbation blocks, which determines the set of allowable perturbations, namely define

\[ \mathbf{X}_\mathbf{K} = \left\{ \Delta = \text{block diag}(\delta_1^{(i)} I_{k_1}, \ldots, \delta_m^{(i)} I_{k_m}, \ldots, \delta_1^{(m_r)} I_{k_{m_r}}, \ldots, \delta_m^{(m_r)} I_{k_{m_m}}, \ldots, \Delta_{m_c}^{(C)}, \ldots, \Delta_{m_c}^{(C)} \) \right\} \]

with \( \delta_i^{(j)} \in \mathbb{R}, \delta_c^{(j)} \in \mathbb{C}, \Delta_{m_c}^{(C)} \in \mathbb{C}^{k_{m_r}+k_{m_c}+m_c+i} \times k_{m_r}+k_{m_c}+m_c+i \).

Noting \( \mathbf{X}_\mathbf{K} \subset \mathbb{C}^{n \times n} \) (where \( n = \sum_{i=1}^{m} k_i \)) and that this block structure allows for repeated real scalars (\( \delta_i^{(j)} I \)), repeated complex scalars (\( \tilde{\delta}_c^{(j)} I \)), and full complex blocks (\( \Delta_{m_c}^{(C)} \)). Noting this block structure, the following definition, taken from [4], is introduced.

**Definition 1** The structured singular value, \( \mu_{\mathbf{K}}(\mathbf{M}) \), of a matrix \( \mathbf{M} \in \mathbb{C}^{n \times n} \) with respect to a block structure \( \mathbf{K}(m_r, m_c, m_c) \) is defined as

\[ \mu_{\mathbf{K}}(\mathbf{M}) = \frac{1}{\min_{\Delta \in \mathbf{X}_\mathbf{K}} \{ \gamma(\Delta) : \det(I_n - \Delta \mathbf{M}) = 0 \} \}
\]

with \( \mu_{\mathbf{K}}(\mathbf{M}) = 0 \) if no \( \Delta \in \mathbf{X}_\mathbf{K} \) solves \( \det(I_n - \Delta \mathbf{M}) = 0 \).

Linear Fractional Transformations (LFTs) are used to reorganise a perturbed problem with uncertainty into the feedback interconnection in Figure 1. In particular, if \( \mathbf{M} \in \mathbb{C}^{n \times n} \) is partitioned as

\[ \mathbf{M} = \begin{bmatrix} \mathbf{M}_{11} & \mathbf{M}_{12} \\ \mathbf{M}_{21} & \mathbf{M}_{22} \end{bmatrix} \]

with \( \mathbf{M}_{11} \in \mathbb{C}^{n_1 \times n_1}, \mathbf{M}_{12} \in \mathbb{C}^{n_1 \times n_2} \) and \( n = n_1 + n_2 \), then an upper LFT will be described as

\[ \Delta \ast \mathbf{M} = \mathbf{M}_{22} + \mathbf{M}_{21} \Delta (I_{n_1} - \Delta_1 I_{n_1})^{-1} \mathbf{M}_{12} \]

If two block structures are defined as \( \mathbf{X}_{\mathbf{K}_1} \subset \mathbb{C}^{n_1 \times n_1}, \mathbf{X}_{\mathbf{K}_2} \subset \mathbb{C}^{n_2 \times n_2} \), then the augmented block structure \( \mathbf{X}_{\mathbf{K}} \subset \mathbb{C}^{n \times n} \) is defined as

\[ \mathbf{X}_{\mathbf{K}} = \{ \Delta = \text{block diag}(\Delta_f, \Delta_u) : \\Delta_f \in \mathbf{X}_{\mathbf{K}_1}, \Delta_u \in \mathbf{X}_{\mathbf{K}_2} \} \]

where \( \mathbf{X}_{\mathbf{K}_1} = \{ \Delta_f \in \mathbf{X}_{\mathbf{K}_1} : \gamma(\Delta_f) \leq 1 \} \).

The skewed structured singular value is the smallest structured singular value of a subset of perturbations that destabilises the system \( \mathbf{M} \) with the remainder of the perturbations contained within a fixed range. Formally stating this

**Definition 2** The skewed structure singular value, \( \mu_{\mathbf{K}}^*(\mathbf{M}) \), of a matrix \( \mathbf{M} \in \mathbb{C}^{n \times n} \) with respect to a block structure \( \mathbf{K}(m_r, m_c, m_c) \) is defined as

\[ \mu_{\mathbf{K}}^*(\mathbf{M}) = \frac{1}{\min_{\Delta \in \mathbf{X}_{\mathbf{K}}} \{ \gamma(\Delta) : \det(I_n - \Delta \mathbf{M}) = 0 \} \]

with \( \mu_{\mathbf{K}}^*(\mathbf{M}) = 0 \) if no \( \Delta \in \mathbf{X}_{\mathbf{K}} \) solves \( \det(I_n - \Delta \mathbf{M}) = 0 \).

3 **State-space Approaches using skew \( \mu \)**

In general, robustness analysis problems correspond to a question of checking the value for

\[ \mu_{\mathbf{K}}(\mathbf{M}(s)) \]

over the closed right-half-plane (where \( \mathbf{M}(s) \) is a stable system). This approach can be computationally intensive and an appropriate frequency range and the fineness of the grid must be decided. Even still with such an approach, there still remains the possibility of missing important points especially as real \( \mu \) may be discontinuous. Instead three alternative frequency independent state-space approaches are presented to counteract these issues.

3.1 **State-space \( \mu \) - formulation 1**

The development of the first two tests is based on the fact that a transfer function can be expressed as a LFT of a constant matrix on the frequency variable. Given a transfer function \( \mathbf{M}(s) \) its differential equation representation is considered and expanded using the usual state-space formula

\[ \mathbf{M}(s) = \mathbf{C}(s \mathbf{I}_p - \mathbf{A})^{-1} \mathbf{B} + \frac{1}{s} \mathbf{I}_p \ast \mathbf{M} \]

where \( \mathbf{M} \) is the constant matrix

\[ \mathbf{M} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} \]

and \( p \) is the dimension of the state-space. The idea is to replace \( \mu_{\mathbf{K}}(\mathbf{M}(s)) \) with this expression and include \( \frac{1}{s} \mathbf{I}_p \ast \mathbf{M} \) as one of the uncertainties. Instead of testing \( \frac{1}{s} \mathbf{I}_p \) over the right-half-plane, a better solution is to test within a unit circle. This is achieved by employing a bilinear transform, where the transformation

\[ T = \begin{bmatrix} \mathbf{I}_p & \sqrt{2} \mathbf{I}_p \\ \sqrt{2} \mathbf{I}_p & \mathbf{I}_p \end{bmatrix} \]

is used to generate \( \frac{1}{s} \mathbf{I}_p \) in the right-half-plane from \( \delta \omega \mathbf{I}_p \ast T \) where \( \delta \omega \mathbf{I}_p \) lies within the unit disk. The test now follows by applying the main loop theorem and evaluating \( T \ast \mathbf{M} \) (Figure 2).

**Theorem 1** (10) (state-space \( \mu \))

Suppose that \( \mathbf{M}(s) \) has all of its poles in the open left-half-plane (i.e. nominal stability) and let \( \beta > 0 \). Let a minimal state-space representation for \( \mathbf{M}(s) \) be given as

\[ \mathbf{M}(s) = \mathbf{C}(s \mathbf{I}_p - \mathbf{A})^{-1} \mathbf{B} + \mathbf{D} \]
Then for all $\Delta \in \mathcal{M}(X_\Delta)$ with $\|\Delta\|_\infty \leq \beta$, the perturbed closed-loop system in Figure 2 is uniformly stable if and only if

$$
\mu_{K}(T \ast \hat{M}) < 1
$$

where

$$
T \ast \hat{M} = \begin{bmatrix}
A_1^{-1}(I_p + A) & \sqrt{\beta}A_1^{-1}B \\
\sqrt{\beta}C A_1^{-1} & \frac{1}{\beta}(C A_1^{-1}B + D)
\end{bmatrix}
$$

(6)

3.3 State-space $\mu$ - formulation 3

The basis for the third approach differs from the first two but remains a frequency independent approach with frequency represented as an uncertain (real) parameter. Unlike the previous two approaches, instead of checking for frequency points over the right-half-plane corresponding to $\omega \in [0, \infty]$, this approach allows a frequency interval to be selected a priori where $\omega \in [\alpha, \infty]$. Using the transformation

$$
T = \begin{bmatrix}
0 & I_p \\
\frac{1}{2} I_p & \frac{1}{2} I_p
\end{bmatrix}
$$

and introducing the parameters

$$
\omega_0 = \frac{1}{2}(\omega + \omega) \quad \alpha_\omega = \frac{1}{2}(\omega - \omega)
$$

The following theorem extends results first presented in [8] where the test again follows by applying the main loop theorem and evaluating $T \ast \hat{M}$.

Theorem 3 (state-space $\mu$ 3)

Suppose that $M(s)$ has all of its poles in the open left-half-plane and let $\beta > 0$. Given a minimal state-space representation of $M(s)$ and the uncertainty structure $X_\Delta$ defined in theorem 1 and theorem 2 respectively, then for all $\Delta \in \mathcal{M}(X_\Delta)$ with $\|\Delta\|_\infty \leq \beta$, the perturbed closed-loop system is uniformly stable if and only if

$$
\mu_{K}(T \ast \hat{M}) < 1
$$

where

$$
T \ast \hat{M} = \begin{bmatrix}
\frac{j}{\beta}\alpha_\omega A_3^{-1} & \sqrt{\frac{\beta}{\pi}}A_3^{-1}B \\
-j\sqrt{\beta}A_3^{-1} & \frac{1}{\beta}(C A_3^{-1}B - D)
\end{bmatrix}
$$

(9)

with

$$
A_3^{-1} = (A - j\omega_0 I_p)^{-1}
$$
It is with abuse of notation that $T * \hat{M}$ is used for all three cases, the reasons for which will become apparent. A full description of the third approach is presented in [5]. As they exist, all three state-space formulations from theorems 1, 2 and 3 give a one shot constant matrix $\mu$ test for the general robust stability problem with a yes/no answer. This can be improved upon by reformulating as a skew $\mu$ problem. This is quantified in the following proposition.

**Proposition 1 (state-space skew $\mu$)**

Formulation of frequency as an uncertainty parameter is a skew $\mu$ problem. Since the frequency parameter is skewed (fixed in range), (6), (8) and (9) can be recast as a skew $\mu$ formulation.

$$\mu^s_K(T * \hat{M}) < 1$$

(10)

with $\delta_\omega I = \Delta_f$ and $\Delta = \Delta_v$.

It is now possible to obtain the worst-case perturbation from each test and the value of $\delta_\omega$ containing the worst-case frequency information. Furthermore, this information may be determined or unwrapped using the following expressions for each of the three approaches

$$s = \frac{1 - \delta_\omega}{1 + \delta_\omega}$$

$$s = -\frac{1}{1 - \delta_\omega}$$

$$s = \frac{j}{1} (\omega_0 + \alpha_\omega \delta_\omega)$$

4 Optimisation skew $\mu$ lower bound

This optimisation-based approach is outlined in [5] for strictly real-valued parameter uncertainty. It has been extended here to determine a lower bound on skew $\mu$ where the uncertainty can consist of (possibly repeated) real and complex scalars.

**Theorem 4 (Optimised skew $\mu$ lower bound - Robust Stability)**

Let $0 \leq \theta_d \in \mathbb{R}$. For $M \in \mathbb{C}^{m \times n}$ and any compatible block structure $\hat{K}(m_{e}, m_{c}, 0, m_{v}, m_{r}, 0)$, a lower bound on $\mu^s_K(M)$ can be determined from

$$\mu^s_K(M) = \frac{1}{\min_{\Delta \in \mathcal{X}_K} \{\|\Delta_v\| : \det(I_n - \Delta M) \leq \theta_d\}}$$

(11)

where the solution is (the inverse of) a constrained minimisation problem. If each real uncertain scalar is represented by one optimisation variable $\delta^r_l$ and each complex scalar is represented by two optimisation variables

$$\delta^c_l = \delta^{rc}_l + \delta^{im}_l j$$

then the vectors of optimisation variables $x_f$ and $x_v$ associated with the perturbation sets $\Delta_f$ and $\Delta_v$, respectively may be obtained from the mappings

$$\Delta_f \mapsto x_f \in \mathbb{C}^{m_e + 2m_r}$$

$$\Delta_v \mapsto x_v \in \mathbb{C}^{m_v + 2m_e}$$

where

$$x_f = (\delta^r_l, \ldots, \delta^{rc}_{m_r}, \delta^{im}_{m_r}, \ldots, \delta^{rc}_{m_e}, \delta^{im}_{m_e})$$

$$x_v = (\delta^r_{m_r}, \ldots, \delta^{rc}_{m_r}, \delta^{im}_{m_r}, \ldots, \delta^{rc}_{m_e}, \delta^{im}_{m_e})$$

(12)

The objective function can then be quantified as

$$f(x_v) = \|\Delta_v\|$$

subject to the nonlinear and simple bound constraints

$$\left(I_n - \Delta M\right) \leq \theta_d$$

$$1 \leq x_f \leq 1$$

Note that “$\theta_d$” is the digital implementation of zero and is generally of magnitude $10^{-8}$ or less. As part of this work, several different formulations of the nonlinear non-convex constraint (13) have been considered. The uncertainty block structure is implemented where one optimisation variable represents a real-valued uncertainty parameter, two optimisation variables are used to represent a complex-valued uncertain parameter corresponding to the it’s real and complex parts. This lower bound algorithm has been developed in Matlab using the Optimization Toolbox [3] and may be used with the state-space formulation to provide a certifiably-safe robustness metric for systems subject to real parametric uncertainty. This is illustrated on the following steer-by-wire system.

5 Steer-by-Wire Application

In this section, an experimental steer-by-wire vehicle is considered (Figure 3). The open-loop dynamics of the vehicle can be described using the single track or bicycle model with states (outputs) of sideslip angle $\beta$ at the centre of gravity (c.g.) and yaw rate $r$ (Figure 4). $\theta$ represents the actual steer angle at the pinion. The vehicle uses a combination of Global Positioning System (GPS) technology and Inertial Navigation Sensor (INS) measurements to accurately estimate the sideslip angle and yaw rate in real-time. With steer-by-wire technology, an active full state feedback controller can be developed where the steer angle is a linear combination of states and the driver command angle $\theta_d$.

$$\theta = K_r r + K_\beta \beta + K_d \theta_d$$

A physically intuitive way to modify the vehicle’s handling characteristics is to define a target front cornering stiffness as

$$\hat{C}_f = C_f (1 + \eta)$$

and the state feedback gains as

$$K_\beta = -\eta, \quad K_r = -\frac{\alpha}{V} \eta, \quad K_d = 1 + \eta$$

where $\eta$ is the desired fractional change in the original front cornering stiffness $C_f$. Substituting this feedback law, the
closed-loop state-space matrices can be derived with

\[
A = \begin{bmatrix}
-\hat{C}_f - C_r & -1 + \frac{C_r b - \hat{C}_f a}{mV} \\
\frac{C_r b - \hat{C}_f a}{I_z} & -\frac{\hat{C}_f a^2 - C_r b^2}{I_z V}
\end{bmatrix}
\]

and

\[
B = \begin{bmatrix}
\frac{\hat{C}_f}{mV} \\
\frac{\hat{C}_f a}{I_z}
\end{bmatrix}^T
\]

Since a vehicle’s handling characteristics are heavily influenced by tyre cornering stiffness, the effect of this modification is to make the vehicle either more oversteering or understeering depending on the sign of \(\eta\). Full details can be found in [9].

For a robust stability analysis, levels of uncertainty are incorporated into the system for \(\eta = -0.5\) effectively making the vehicle more understeering (Table 1). The structured perturbation block for this problem consists only of strictly real repeated and nonrepeated uncertain parameters.

\[
\Delta = \text{diag}(\delta_{\omega}, \delta_{I_z}, \delta_{C_f} I_6, \delta_{C_r} I_4, \delta_{a} I_5, \delta_{b} I_4, \delta_{V} I_3)
\]

Figure 5 shows the bounds on \(\mu\) using the Matlab \(\mu\)-Tools Toolbox [1] for a frequency sweep of 300 points for the frequency range \([10^{-3}, 10^3]\). Although not obvious from the plot, the peak value of \(\mu\) for this sweep is 0.3021 at 0.001 rad/s. For problems of this type, unexpected peaks can occur at \(\omega = 0\) rad/s. Indeed a local peak does occur at this frequency where the upper bound on \(\mu\) is 0.2484. Since the overall peak value of \(\mu\) is less than unity for the normalised perturbation set, this signifies that the system is robustly stable. Shifting the focus, the \(\mu\)-Tools mu function, with the exception of a few grid points, fails to return quality lower bound solutions. Even for the exceptions evident from Figure 5, the accuracy of \(\det(I - M_{11}\Delta)\) is only of magnitude of \(10^{-4}\). Consequently, no candidate destabilising perturbation set \(\Delta_d\) from this algorithm is considered. For a more rigorous approach, the original system matrix \(M\) is transformed into each of the three frequency independent state-space formulations ((6), (8) and (9)). In each case, the resulting augmented perturbation block now includes frequency expressed as a repeated uncertain parameter.

\[
\Delta := \text{diag}(\delta_{\omega}, \delta_{I_z}, \delta_{C_f} I_6, \delta_{C_r} I_4, \delta_{a} I_5, \delta_{b} I_4, \delta_{V} I_3)
\]

It is worth emphasising that this new perturbation block (15) increases by the number of states of the system. A more fundamental point is that the type of uncertain parameter \(\delta_{\omega}\) is not specified, this is deliberate omission because it may be real or complex depending on which state-space transformation is used. Using the optimisation-based skew \(\mu\) algorithm, the qual-

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**Table 1: Parameter values for vehicle.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Nominal Value</th>
<th>Variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>(m)</td>
<td>kg</td>
<td>1640</td>
<td>± 10%</td>
</tr>
<tr>
<td>(I_z)</td>
<td>N/m²</td>
<td>3500</td>
<td>± 10%</td>
</tr>
<tr>
<td>(C_f)</td>
<td>N/rad</td>
<td>100000</td>
<td>± 5%</td>
</tr>
<tr>
<td>(C_r)</td>
<td>N/rad</td>
<td>160000</td>
<td>± 5%</td>
</tr>
<tr>
<td>(a)</td>
<td>m</td>
<td>1.3</td>
<td>± 10%</td>
</tr>
<tr>
<td>(b)</td>
<td>m</td>
<td>1.5</td>
<td>± 10%</td>
</tr>
<tr>
<td>(V)</td>
<td>m/s</td>
<td>13.4</td>
<td>± 10%</td>
</tr>
</tbody>
</table>

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**Table 2: Summary of results.**

<table>
<thead>
<tr>
<th>Description</th>
<th>Interval</th>
<th>Max (\mu) Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>State-space (\mu) 1</td>
<td>([0, \infty])</td>
<td>0.1033</td>
</tr>
<tr>
<td>State-space (\mu) 2</td>
<td>([0, \infty])</td>
<td>0.1831</td>
</tr>
<tr>
<td>State-space (\mu) 3</td>
<td>([0, 1])</td>
<td>0.2389</td>
</tr>
</tbody>
</table>
ity of the lower bound improved for all three state-space approaches (Table 2). Due to the nature of the third approach, the frequency interval may be refined to $[0, 1]$ rad/s with no further improvement by subdividing this interval. This is very beneficial because it is possible to refine the frequency interval where the peak value of $\mu$ occurs. In this instance it occurred at 0.2389. The other major benefit of this lower bound algorithm is that a candidate worst-case destabilising perturbation set $\Delta_d$ is returned and the worst-case perturbation set $\Delta_{wc}$ can be determined using the value of $\mu = \beta_l$, i.e. $\Delta_{wc} = \beta_l \Delta_d$. Figure 6 shows the nominal and perturbed time-domain responses for the sideslip angle subject to a sinusoidal input. This perturbation set corresponds to the peak lower bound $\mu$ value of 0.2389. It can be quickly concluded that the effect of the uncertainty is counteracting the effect of the active controller, with the worst-case closed loop response tending towards that of the open loop.

6 Conclusions

In this paper, different methodologies for assessing the robust stability of systems subject to strictly real parametric uncertainty have been detailed. Three frequency independent state-space formulations have been shown to address the discontinuity issue associated with “real $\mu$”. The advantage of the third formulation is that a frequency interval may be selected a priori. Improved lower bounds solutions were obtained for an experimental steer-by-wire vehicle using an optimisation-based skew $\mu$ algorithm, with candidate worst-case perturbation sets returned. A current research objective is to facilitate full complex blocks so that the $H_\infty$ robust performance question can also be assessed, possibly obtaining improved lower bound solutions for $\mu$ and skew $\mu$ for this class of problem.

References


