MODELLING, SIMULATION AND CONTROL OF A HYDROELECTRIC PUMPED STORAGE POWER STATION

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Abstract

In this paper, a Simulink model of the Dinorwig pumped-storage hydroelectric power station is controlled using Model Predictive Control (MPC). The response of the plant with MPC is compared with that of a classic proportional and integral controller (PI), as currently implemented on the system. It is shown that constrained multivariable MPC can achieve good control over the operating envelope of the plant. This control seems to be robust; it maintains its performance in the SISO and MIMO cases without changing its tuning parameters. The advantage of using a constrained MPC with prefilter to reduce the cross-coupling interaction of this system is also shown.

1 Introduction

The main idea in this work is to design an algorithm for predictive control of the Dinorwig hydroelectric plant that could improve its response to demand changes in power. It is hoped that faster and more accurate control can be achieved with predictive control than with classic control such as Proportional and Integral (PI).

Dinorwig is a large pumped storage hydroelectric scheme located in North Wales, which is operated by the First Hydro Company. The station has six 300 MW rated turbines, driving synchronous generators which feed power into the national grid. Dinorwig provides rapid response frequency control when peak demands occur. This hydroelectric station has a single tunnel drawing water from an upper reservoir (lake Marchlyn) into a manifold, which splits the main flow into six penstocks (figure 1). Each penstock feeds a turbine to generate power using a guide vane to regulate the flow. The electrical power generated is controlled by individual feedback loops on each unit. The reference input to the power loop is the grid frequency deviation from its 50 Hz set point, thus forming an outer frequency control loop.

The model of the hydroelectric power plant is based on the work in [7], where a multivariable nonlinear simulation model of the plant is derived. This has provided an improved understanding of its characteristics. Its main features are non-minimum-phase (NMP) dynamics, poorly damped poles (associated with water-hammer in the supply tunnel and electrical synchronisation) and a nonlinear relationship between flow and power. It is also known that there is significant hydraulic coupling between the turbines because of the common supply (cross-coupling interaction). This makes the plant a good candidate for the application of predictive control. Although it is known that there are several examples of predictive control being applied to conventional power plant [10,12], its application to hydroelectric plant has only appeared recently [9,11,13]. This paper begins with a discussion of the multivariable linear model used in this work, followed by a description of multivariable Generalized Predictive Control (GPC), and then the technique utilized in this work to reduce the cross-coupling interaction is described. Finally some results and conclusions are drawn.

2 Model description

The hydroelectric plant model can be separated into three subsystems: guide vane, hydraulics and turbine/generator (figure 2). In this work a linearised model [14] of the hydraulic subsystem was used (figure 3). The transfer function of the guide vanes used to control the water flow is given in equation (1).

\[
G = \frac{1}{0.19s + 1} \frac{1}{0.4s + 1} \text{(set_position)}
\]  

Figure 1: Schematic of the hydraulic power station layout.

Figure 2: The three subsystems of the hydroelectric plant.

Figure 3 shows the hydraulic subsystem. In this model \( G \) is the per unit (p.u.) gate opening, \( G_o \) is the operating
point, $P_{\text{mech}}$ is the mechanical power produced by a single turbine, $T_{\text{mt}}$ is the water starting time of the main tunnel, $T_w$ is the water starting time of any single penstock and $T_{\text{wt}}$ is the water starting time of the main tunnel and a single penstock ($T_{\text{wt}}=T_{\text{mt}}+T_w$). The values of $T_{\text{mt}}$, $T_w$ and $T_{\text{wt}}$ depend directly on the constructional dimensions of the main tunnel and the penstocks. $A_s$ is the turbine gain, whose value depends directly on the turbine MW and 50 Hz.

This model was implemented in Simulink [8]. It was designed to be scalable, allowing different behaviours to be selected according to the objective of the study. For the purposes of this work a multivariable linear model was chosen.

## 3 MIMO GPC

### 3.1 Unconstrained multivariable GPC

This section contains a summary of the GPC method. GPC finds the future control signals by looking for the minimum of a quadratic cost function, equation (2), over one horizon of prediction [2].

$$J(N_1, N_2, N_s) = \sum_{j=1}^{\infty} \left[ (r(t+j) - w(t+j)) \cdot R + \sum_{j=1}^{N} \| u(t+j-1) \|_2^2 \right] \quad (2)$$

In this equation $\hat{y}(t+j|t)$ is the optimum predicted output of the system, $A$ is (1-$q^{-1}$) operator, $N_1$ and $N_2$ are the minimum and the maximum of the prediction horizon (N), $N_s$ is the control horizon, $R$ and $Q$ are positive definite weighting matrices and $w(t+j)$ is the future reference trajectory.

The plant is modelled using the Model Controller Auto-Regressive Moving-Average with integrator (CARIMA) form, equation (3).

$$A(q^{-1})y(t) = q^{-d}B(q^{-1})u(t-1) + C(q^{-1})e(t)$$

where $A(q^{-1})$ and $C(q^{-1})$ are monic polynomial matrices of order $n \times n$, and $B(q^{-1})$ is a polynomial matrix of order $n \times m$. The model can be represented by:

$$\hat{y}(t+j) = G_j(q^{-1})\Delta u(t+j-1) + F_j(q^{-1})w(t+j)$$

where $G_j$ is a lower triangular matrix of $N \times N$ with elements $g_i$ being points on the step response of the plant. $f$ is the free response, whose elements can be calculated recursively by:

$$f_{j+1} = q^{-d}\Delta u(t+j)$$

with $f_0 = y(t)$, $\Delta (q^{-1}) = \Delta A(q^{-1})$ and $\Delta u(t+j) = 0$ for $j \geq 0$

If the control signal is considered constant after $N_u$ steps of control, the set of predictions

$$y_{N_1,2} = \left[ \hat{y}(t + N_2), \hat{y}(t + N_1 + 1), \ldots, \hat{y}(t + N_u) \right]$$

affecting the cost function, equation (2), can be expressed as:

$$y_{N_1,2} = G_{N_1,2} \Delta u + f_{N_1,2}$$

where:

$$\Delta u = \left[ \Delta u, \ldots, \Delta u, \Delta u + N_u - 1 \right]$$

and $G_{N_1,2}$ is a submatrix of $G$, with $G_i=0$ for $i<0$.

$$G_{N_1,2} = \begin{bmatrix} G_{N_1} & G_{N_1-1} & \cdots & G_{N_1-N_u} \\ G_{N_1-1} & G_{N_1-2} & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ G_{N_2} & G_{N_2-1} & \cdots & G_{N_2-N_u} \end{bmatrix}$$
Then equation (2) can be rewritten as:

\[ J = (G_{n2}u_N + f_{n2} - w)^T \overline{R}(G_{n2}u_N + f_{n2} - w) + u_N^T \overline{U}u_N \]  

(5)

where \( \overline{R} = \text{diag}(R, \ldots, R) \) and \( \overline{U} = \text{diag}(\lambda, \ldots, \lambda) \).

The optimum control signal can be calculated by:

\[ u = (G_{n2}^T \overline{R}G_{n2} + \overline{U})^{-1} G_{n2}^T \overline{R}(w - f_{n2}) \]  

(6)

3.2 Constrained multivariable GPC

It is common to formulate an equation of control assuming that all signals possess an unlimited range. However, this is not realistic because in the real world all plants are subject to constraints, [3,12], usually for constructional and safety reasons. Limited range of actuators and limited slew rate are some examples of constructional constraints. Levels in tanks and pressures in vessels are examples of safety constraints.

In Predictive control, three kinds of constraints are normally considered, limits on control signals (saturation), rate limits on control signals and limits on output signals. These constraints can be represented by the following equations:

\[ u \leq u(t) \leq u_{\text{max}} \quad \forall t \]

\[ \Delta u \leq u(t) - u(t-1) \leq \Delta u \quad \forall t \]

\[ y \leq y(t) \leq y_{\text{max}} \quad \forall t \]

If equation (2) is expressed in the following form:

\[ J(u) = \frac{1}{2} u^T H u + b^T u + f_0 \]

(7)

where \( f_0 = (f_{n2} - w)^T (f_{n2} - w) \), \( b^T = 2(f_{n2} - w)^T G_{n2} \), and \( H = 2(G_{n2}^T G_{n2} + \lambda I) \) the predictive control formulation can be expressed as a Quadratic Programming problem, with

\[ J(u) = \frac{1}{2} u^T H u + b^T u \]

(8)

as the function to optimise under the following constraints: \( A_{gg}u \leq b_c \).

Note that \( f_0 \) is not used because at every stage of optimisation, it is a constant whose value does not depend on \( u \), and every value of \( J(u) \) and \( u \) are non-negative.

\( A_{gg} \) and \( b_c \) are the following matrix and vector of order \( n_N \times m \) and \( n_N \) respectively

\[
A_{gg} = \begin{bmatrix}
I \\
-I \\
-I \\
G_{N23} \\
-G_{N23}
\end{bmatrix}, \quad b_c = \begin{bmatrix}
11 - u \\
11 + u \\
1\Delta u_{\text{max}} \\
1\Delta u_{\text{max}} \\
1\overline{\sigma} - f
\end{bmatrix}
\]

\( \overline{I} \) where

A Matlab program for the multivariable GPC algorithm was developed for this study. The program accepts data for the CARIMA model and other parameters such as the control horizon (\( N_u \)), the prediction horizon (\( N \)), the weight sequence of control effort (\( \lambda \)) and the weight sequence of the predictive trajectory (\( \alpha \)). This program is integrated with the power plant model in Simulink.

3.3 Model for control

GPC does not need the full complexity of the plant model, [1]. Instead, a much simpler discrete time model, obtained by the reaction curve technique can be used. The guide vanes subsystem has a substantial influence on the overall system dynamics. For this reason, the predictive model includes this subsystem. The model is symmetric because the units are assumed to be identical (which is almost the case in practice).

A MIMO Simulink model with 6 units at an operating point fixed at 0.95 p.u. was used to obtain, by the reaction curve method, a first order open loop transfer function, equation (9).

\[ G(s) = \frac{1.12e^{-1.4318s}}{1.25s + 1} \]

(9)

The cross-coupling interaction was modelled as the simple transfer function of equation (10) which has one zero and one pole. The equations were transformed to discrete time with a sample time of 0.25 seconds.

\[ X_i(s) = -0.6s + 1 \]

(10)

The model was then converted to CARIMA form:

\[
\begin{bmatrix}
A_{k1} & 0 & 0 & 0 & 0 & y_1(t) \\
0 & A_{k2} & 0 & 0 & 0 & y_2(t) \\
0 & 0 & A_{k3} & 0 & 0 & y_3(t) \\
0 & 0 & 0 & A_{k4} & 0 & y_4(t) \\
0 & 0 & 0 & 0 & A_{k5} & y_5(t) \\
0 & 0 & 0 & 0 & 0 & A_{k6} u(t)
\end{bmatrix}
= \begin{bmatrix}
b_0 & b_0 & b_0 & b_0 & b_0 & b_0 & w_1(t) \\
b_0 & b_0 & b_0 & b_0 & b_0 & b_0 & w_2(t) \\
b_0 & b_0 & b_0 & b_0 & b_0 & b_0 & w_3(t) \\
b_0 & b_0 & b_0 & b_0 & b_0 & b_0 & w_4(t) \\
b_0 & b_0 & b_0 & b_0 & b_0 & b_0 & w_5(t) \\
b_0 & b_0 & b_0 & b_0 & b_0 & b_0 & w_6(t)
\end{bmatrix}
\]

(11)

The model for cross-coupling is:

\[ A_{k1} = 1 - 1.7924z^{-1} + 0.7972z^{-2} \]

\[ B_k = 0.203z^{-1} - 0.1977z^{-2} \]

\[ b_y = -0.04 + 0.073z^{-1} - 0.033z^{-2} \]

Delay = 1.43 s.

4 Cross-Coupling interaction

An alternative approach was used in order to increase the accuracy of the transfer function for the cross-coupling interaction. In this case a full linear hydraulic model plus the dynamics of the guide vane were included, giving the discrete time model of equation (12).

\[
X_k(z) = \frac{-0.04757z^{-2} - 0.02751z^{-2} + 0.06817z^{-2} + 0.006913}{z^4 - 1.800z^{-2} + 1.093z^{-3} - 0.2616z^{-4} + 0.02113}
\]

(12)

The model of the plant using (9) and (12) in CARIMA form is:
As discussed in section 3, the central idea of GPC is to find a set of control signals \( u(t), u(t+1), \ldots, u(t+N) \), where \( N \) is the horizon of prediction, which optimises a quadratic function. In this method of control the reference of the system can be changed to produce different responses. This work takes advantages of this characteristic, in an attempt to decouple the interaction produced when some units are regulating a fixed power output and others are tracking a power reference derived from the frequency error. This approach can be seen as applying a prefilter, \([2,4]\) which deliberately modifies the references of the fixed-power units to compensate for the changes in reference of the frequency-control units. This is illustrated in figure 6. The prefilter attempts to synthesise a signal that cancels the coupled perturbation by producing a sharp, transient change in the reference input of the fixed-power unit. An effective decoupling signal is produced by the transfer function (13).

\[
p_f(z^{-1}) = \frac{g(1-z^{-1})}{1+dz^{-1}} \tag{13}
\]

In (13), the gain \( g \) fixes the value of the overshoot and \( d \) controls the duration of the signal and its decay rate back to the original reference. The best values of \( g \) and \( d \) depend on the number of units whose references are changing and the operating point. It was found that \( g=0.8 \) and \( d=0.1 \) gave good decoupling over a range of operating conditions.

![Figure 6: Block diagram of GPC showing the modification of references.](image)

## 5 Results

### 5.1 Direct response

Simulations were carried out to verify the advantages of using multivariable constrained GPC in this application. A multivariable GPC with constraints and a PI with anti wind up mode were used in all simulations. A rate limit of \( \pm0.2 \) p.u. was imposed on the control for both PI and GPC. A saturation limit of \( P_d/A_d \), that depends on the reference, was imposed on the GPC. The parameters of the PI were fixed to the current values used at Dinorwig \((K=0.1 \text{ and } T_i=0.12)\). The GPC parameters used in these simulations were \( N_u=3 \), \( N=3 \) and \( \lambda=70 \) for the unconstrained case. For the constrained case the GPC parameters were \( N_u=10 \), \( N=10 \) and \( \lambda=120 \).

Initially, the operating point is established at 0.86, and then a 0.04 step is applied at \( t=100 \) to unit 1 while the references to units 2-6 remain fixed. Figure 7 shows the step response of the system under PI and GPC controls. As is seen both GPC responses are faster than obtained using the PI. The GPC with constraint is the fastest of these two. The PI response also has a slightly bigger NMP undershoot.

Figure 8 shows the response when a 0.04 step is applied to all 6 units simultaneously at \( t=200 \). Constrained GPC produces a faster response but has a bigger NMP undershoot. The PI response has a very high overshoot, which is eliminated by using GPC.

![Figure 7: Step response of the plant using GPC and PI controls, one unit changing and the other five with fixed references.](image)

![Figure 8: Step response of the plant using GPC and PI controls, all units changing.](image)

### 5.2 Cross coupling interaction

To evaluate the cross coupling interaction a 0.04 step was applied simultaneously at \( t=350 \) to units 2-6 and the perturbation of unit 1 was observed. Figure 9 shows the effect of the prefilter when using unconstrained GPC. Depending on the value of the filter gain there is a trade-off in the response. If this gain is increased the undershoot is reduced, but the overshoot is increased and vice versa. However, increasing the filter gain
reduces the integral of the power output. In order to obtain a balanced cross coupling response a 0.8 gain prefilter was selected.

The simulation was repeated with constrained GPC and PI. Figure 10 shows the new cross coupling interaction of the system. The GPC with constraints was simulated with and without the prefilter. The PI response has a longer settling time and a higher overshoot that the GPC. The GPC response without the prefilter has a bigger undershoot, but almost no overshoot. When the prefilter is included the undershoot is reduced, but the overshoot increases.

6 Conclusions

The results have shown how multivariable constrained GPC can be applied to a hydroelectric pumped-storage station to improve the dynamic response. The employment of input constraints in the GPC appears to be a good approach to increasing the robustness of the system. The inclusion of the prefilter seems to reduce the effects of the hydraulic cross coupling interaction. In general these are promising results for using multivariable GPC with constraints in this application.

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