Local Instabilities due to Inverter and Rectifier Dead Times in Open-loop Induction Motor Drives

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Abstract

Undesired low-frequency self-sustained speed oscillations are encountered in fan, compressor and pump drives utilizing open-loop frequency-controlled induction motor drives. Discontinuous rectifier current at light loads and the dead-time of the inverter switches, which are inherent features of the drive are the main sources of such oscillations. This paper proposes a concise analytical method to predict the risk of self-sustained oscillations due to these two phenomena, as a function of drive parameters such as load level, values of dc-link filtering elements and motor parameters. Good accuracy of instability prediction is verified by dynamic simulation and by extensive experimentation.

1 Introduction

Robust, reliable and cheap, induction motor drives appear as a workhorse for industrial applications such as fans, pumps and compressors, where they are operated in so-called constant voltage/frequency (V/f) mode. With the introduction of solid-state inverters and PWM voltage control, V/f control became popular for low-performance low-cost induction motor drives. However, practical applications at low speeds (5-30% of the rated) and light loads often encounter local stability problems.

Induction motor may become unstable at low speeds even when supplied by pure sinusoidal voltages [1]. For PWM inverter-supplied induction motors, stability analyses become more complicated. A stability study of rectifier-inverter induction motor drive is usually performed by neglecting the harmonic content of the stator voltages and applying root locus [1] or Nyquist stability criterion [2] to the small perturbation model, which is obtained by linearisation around an operating point. Drive instability may occur over a wide range of low speeds if the drive parameters are improperly selected. These early studies [1,2] neglect the effects of inverter dead time and also consider the rectifier and L-C filter in a continuous operating mode.

Dead time between switching from positive to negative side or vice versa is needed in order to protect inverter elements from short-circuit situations. It guarantees safe operation but adversely affects the inverter performance, as it produces a momentary loss of voltage control. Since this is repeated in every inverter switching cycle, detrimental effect may become significant when the drive is operated at light load in low speed region with high switching frequency. Extensive experimental investigation of unstable regions and oscillating frequencies as a result of dead time was presented in [3].

Switching-type inverters also include a regenerative interval, which is also a source of instability [3]. Fluctuations of this interval can result in amplification of self-sustained oscillations. Ever-faster switching devices do not improve the local stability since their utilization results in a high switching frequency; hence the cumulative effect of time delay remains essentially the same [4]. It has been shown in [4] that dead time effects are related to the load current amplitude and the load power factor. These effects can be balanced by modifying the PWM control circuit with a compensator based on the current feedback; two compensation methods are proposed in [4]. An analysis of the voltage distortion due to combined effect of dead time and non-ideal characteristics of power devices is presented in [5].

Papers [3-5] consider rectifier and L-C filter in a continuous operating mode, which is incorrect at light load operation. This paper addresses the issue by presenting a concise but thorough method for predicting stability of PWM inverter-fed induction motor drive by root locus criterion. The method does include both effects – inverter dead time and rectifier discontinuous operating mode. Regions of local instability are established by root locus analyses of linearised models for four induction motors and a parameter sensitivity study is performed. The findings are validated by dynamic simulation of non-linear model and by experiments.

2 Drive Structure and Modelling

A standard three-phase PWM inverter drive is shown in Fig.1, consisting of rectifier, filter, inverter and motor. Low-cost PWM inverters need to be small and light, hence the filter inductance \( L_f \) will normally be low. In such a case, current \( i_R \) through the inductance will have discontinuous mode as shown in Fig.2 if the average \( i_R \) is small enough (at light mechanical load). Overall mathematical model consists of six or seven differential equations, depending on the rectifier mode.
2.1 Modelling of the L-C circuit

During continuous operation ($i_R > 0$) the filter model is:

$$\frac{dV_d}{dt} = \frac{V_R - V_d}{L_f}$$

(1)

$$\frac{di}{dt} = \frac{i_R - i_i}{C_f}$$

(2)

where $V_R$ and $V_d$ are rectifier and DC-link voltages, $R_f$ is filter resistance and $i_i$ is inverter current. During discontinuous mode there is no current $i_R$, hence the model becomes:

$$\frac{dV_d}{dt} = -\frac{V_R - V_d}{RC_f}$$

(3)

The stability study of the whole system will be performed in the d-q synchronously rotating reference frame [6], hence eqs. (1)-(3) must be rewritten. By choosing a suitable initial angle of the rotating reference frame, the following is obtained:

$$i_i = -\frac{3m_a}{4} i_q; \quad V_d = -\frac{2}{m_a} V_q.$$  \hspace{1cm}(4)

where $m_a$ is the amplitude modulation index.

2.2 Induction motor modelling

The general d-q model of an induction motor [6] is highly non-linear hence linearisation is needed. It yields:

$$\frac{d\Delta y}{dt} = A_{IM} \Delta y + \Delta \omega t b_2 + \Delta v + \Delta T_d b_L;$$

(5)

where the induction motor matrix $A_{IM}$ is in the Appendix B, while the state vector $\Delta y$, input voltage vector $\Delta v$, load disturbance vector $b_L$ and supply-disturbance vector $b_2$ are:

$$\Delta y^T = \begin{bmatrix} \Delta \psi_d & \Delta \psi_q & \Delta \psi_D & \Delta \psi_Q & \Delta \omega \end{bmatrix}$$

(6)

$$\Delta v^T = \begin{bmatrix} \Delta v_d & \Delta v_q & 0 & 0 & 0 \end{bmatrix}$$

(7)

$$b_L = \begin{bmatrix} 0 & 0 & 0 & -p/J \end{bmatrix}$$

(8)

$$b_2^T = \begin{bmatrix} \psi_{q0} & -\psi_{d0} & \psi_{Q0} & -\psi_{D0} \end{bmatrix}$$

(9)

2.3 Modelling of inverter dead time

During dead time interval $t_d$, both switches in one inverter leg (say T1 and T4) are switched off. Polarity of inverter output voltage at point A is determined by the direction of the phase A current, which flows through either D1 or D4, until either T1 or T4 is switched on. Amplitude of that output voltage is practically equal to $V_d$ and it has a six-step nature over the sine-wave cycle [7]. Hence the first harmonic of the loss $\Delta v_i$ in the inverter output voltage (due to the dead time) is:

$$(\Delta v_i)_1 = \frac{4}{\pi} V_d \frac{t_d}{T_s}.$$  \hspace{1cm}(10)

3 Stability Analyses

Modelling presented in section 2 enables formulation of system matrix, $A$ which can be used to determine stability of the system. There are two different system matrices: $A^{CONT}$ (Appendix C) for continuous and $A^{DISC}$ (Appendix D) for discontinuous rectifier operation. Both are a sum of matrices describing induction motor and the LC filter and matrices describing the influence of dead time:

$$A^{CONT} = A^{IM+LC} + A^{DEADTIME}$$

(11)

$$A^{DISC} = A^{DISC+IM+LC} + A^{DEADTIME}$$

(12)
3.1 Root locus stability

Root loci of the linearised system described by (11) or (12), without any damping available from the load, are shown in Figures 3 and 4, for motor No.1 (data in Appendix A). The switching frequency is set to 4 kHz, the fundamental supply frequency is varied from 5 to 50 Hz, while the dead time is firstly set to $t_D = 7$ ms (Fig.3) and then neglected (Fig.4). It is evident that the inclusion of the inverter dead time brings some roots into the right-hand side of the plane (positive real part), which indicates that the system becomes unstable. When the dead time is neglected, the system is stable, albeit with very low damping at low/medium supply frequencies.

![Image of root loci with dead time] Figure 3: Root loci with $t_D = 7$ ms, (1 p.u. = 100π rad/s)

![Image of root loci without dead time] Figure 4: Root loci with dead time neglected ($t_D = 0$)

3.2 Computer simulation

Dynamic drive stability is initially verified by a computer simulation of the full non-linear model of the drive. This approach requires the simulation time step to be much smaller than the dead time, resulting in a very slow simulation. Simulation of small-speed transients could be executed much faster by using a linearised drive model.

Speed vs. time responses are shown in Figures 5 and 6. Three simulated voltage inverter frequencies are 30 Hz, 25 Hz and 20 Hz, while the switching frequency is set to 4 kHz. The drive initially operates unloaded, then a load of 8% is applied at $t = 3$ s and removed at $t = 4$ s.

Simulation with idealised inverter switching (Fig.5) shows no serious problems, although the damping is low at 20 Hz operation. Inclusion of inverter dead time gives a realistic representation of the physical drive. Simulations with dead time of $t_D = 7$ ms indicate self-sustained speed oscillations of limited amplitude – limited load cycling, as shown in Fig.6. Therefore the simulation confirms the root-locus analyses and indicates that the system may become unstable at light loads and low speeds. Inherent system non-linearity ‘saturates’ the oscillations to a limited amplitude and thus prevents fully unstable operation.

![Image of speed responses with dead time] Figure 5: Speed responses to load changes with $t_D = 0$ ms

![Image of speed responses without dead time] Figure 6: Speed responses to load changes with $t_D = 7$ ms

3.3 Parameter sensitivity

Various parameters will influence the exact region of local instability, with the supply frequency and the load level being the two most influential factors. The reason for this is that higher levels of load will reduce the discontinuity in rectifier current (current will rise) and also reduce the regenerative...
interval in inverter (power factor will increase). Other parameters of some importance are dead time duration, switching frequency, motor size and motor parameters related to the magnetic energy stored in the motor core. Unstable regions are predicted by analysing linearised mathematical models and confirmed by dynamic simulation of full models.

Changes of instability region in the frequency-torque plane for motor No.1 are shown in Figure 7 (due to different dead time duration) and in Figure 8 (due to inverter switching frequency, for a fixed inverter dead time). It has been found that the system is stable if the switching frequency is below 2.25 kHz.

Three more induction motors (data in Appendix A), all of them larger than the motor No.1, have been studied. An increase in motor power shifts the unstable region to a lower frequency and higher torque region, as shown in Figure 9. The value of the filter capacitor $C_f$ has also been investigated and it was found that it has a negligible impact on the unstable area, as indicated in experimental study [3].

![Figure 7: Unstable area vs. dead time (4 kHz, motor 1).](image)

![Figure 8: Unstable area vs. switching frequency ($t_d = 10 \mu s$).](image)

4 Experiments

Motor 1 was subjected to extensive experimentation that confirmed the local instability of the drive at light loads. In the absence of a speed sensor, traces of inverter current $i$ and phase A current $i_a$ were used to detect sustained oscillations. The inverter current should be relatively constant while the phase current should be sinusoidal. The illustration of instability is shown in Figure 10, where time traces of $i$ are shown for fundamental supply frequencies of 11Hz, 13Hz, 17Hz and 19Hz, respectively for traces 1, 2, 3, 4. Offsets of 0.1A, 0A, −0.1A and −0.2A (respectively) were applied to these traces in order to show all of them in a single figure. Self-sustained oscillations are obvious for 13Hz and 17Hz supply frequencies.

![Figure 10: Experimentally recorded inverter current $i$ for four fundamental supply frequencies (4.2 kHz, 4 $\mu s$, motor 1).](image)

Comparison of time traces of the phase current $i_a$ at stable (Fig. 11) and locally unstable (Fig. 12) operation indicates that the expected sinusoidal nature of the current is distorted when the local instability occurs.
5 Conclusions

The paper presents a linearised mathematical model of the induction motor drive. The model incorporates discontinuous mode of rectifier/filter current, as well as the inverter dead time. The linearised model is used to predict local unstable regions of operation in the frequency-torque plane. Changes of unstable area due to several parameters (duration of inverter dead time, switching frequency, motor parameters, filter capacitance) have been investigated.

Predictions of local instability and self-sustained oscillations are verified by dynamic simulation of a full non-linear mathematical model and by a number of experiments. It is shown that the simple linearised model is very accurate for prediction of local instability.

References


Appendices

Appendix A: Motor Parameters

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<th>Motor 2</th>
<th>Motor 3</th>
<th>Motor 4</th>
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<td>ZK 160 M4</td>
<td>ZK 180 L4</td>
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<tr>
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<td>$L_m [H]$</td>
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<td>$J [kgm^2]$</td>
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<td>0.055</td>
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Appendix B: Linearised Induction Motor Matrix

$$A_{IM} = \begin{bmatrix} -\frac{1}{T_s} \omega_0 & k_r & 0 & 0 \\ \frac{k_s}{T_r} & -\omega_0 & 0 & 0 \\ 0 & \frac{1}{T_r} & 0 & 0 \\ -c \psi_{d0} & c \psi_{d0} & c \psi_{q0} & -c \psi_{q0} \frac{k_r}{J} \end{bmatrix}$$

where: $k_s = \frac{L_m}{L_s}$; $k_r = \frac{L_m}{L_r}$; $L_s = L_s \sigma$; $L_r = L_r \sigma$;

$$\sigma = 1 - \frac{L_m^2}{L_s L_r}$$

while $\omega_0, \omega_0, \psi_{d0}, \psi_{d0}, \psi_{q0}, \psi_{q0}$ are the values of supply angular frequency, mechanical speed and d-q flux components (respectively) at the nominal operating point around which linearisation is performed.
Appendix C: Linearised System Matrix for Continuous Mode

\[
A^{\text{CONT}} = \begin{bmatrix}
-\frac{R_f + \frac{3\omega_m L}{\pi}}{L_f} & \frac{2}{m_a} & 0 & 0 & 0 & 0 & 0 \\
-\frac{2}{m_a L_f C_f} & 0 & 0 & -\frac{3m_a^2}{8L_s C_f} & 0 & \frac{3m_a^2 k_r}{8L_s C_f} & 0 \\
0 & Bi_{d0} & -\frac{1}{T_s} + Ai_{d0}^2 - C & \omega_{0} + Ai_{d0}i_{q0} & k_r \left(\frac{1}{T_s} - Ai_{d0}^2 + C\right) & -k_r Ai_{d0}i_{q0} & 0 \\
0 & 1 + Bi_{q0} & -\omega_{0} + Ai_{d0}i_{q0} & -\frac{1}{T_s} + Ai_{q0}^2 - C & -k_r Ai_{d0}i_{q0} & k_r \left(\frac{1}{T_s} - Ai_{q0}^2 + C\right) & 0 \\
0 & 0 & \frac{k_s}{T_r} & 0 & -\frac{1}{T_r} & \omega_{0} - \omega_{0} & -\psi_{Q0} \\
0 & 0 & 0 & \frac{k_s}{T_r} & -\left(\omega_{0} - \omega_{0}\right) & \frac{1}{T_r} & \psi_{D0} \\
0 & 0 & -c\psi_{Q0} & c\psi_{D0} & c\psi_{q0} & -c\psi_{d0} & -\frac{k_r}{J}
\end{bmatrix}
\]

where:
\[A = \frac{4}{\pi} \frac{V_d}{I_s} \frac{t_{\Delta}}{I_s} \quad B = \frac{8}{\pi} \frac{V_d}{I_s} \frac{t_{\Delta}}{I_s} \quad C = \frac{8}{\pi} \frac{V_d}{I_s} \frac{t_{\Delta}}{I_s}\]

\[i_{d0}, i_{q0}\] are the values of stator current d-q components at the nominal operating point around which linearisation is done.

Appendix D: Linearised System Matrix for Discontinuous Mode

\[
A^{\text{DISC}} = \begin{bmatrix}
-\frac{1}{RC_f} & 0 & -\frac{3m_a^2}{8L_s C_f} & 0 & \frac{3m_a^2 k_r}{8L_s C_f} & 0 \\
Bi_{d0} & -\frac{1}{T_s} + Ai_{d0}^2 - C & \omega_{0} + Ai_{d0}i_{q0} & k_r \left(\frac{1}{T_s} - Ai_{d0}^2 + C\right) & -k_r Ai_{d0}i_{q0} & 0 \\
1 + Bi_{q0} & -\omega_{0} + Ai_{d0}i_{q0} & -\frac{1}{T_s} + Ai_{q0}^2 - C & -k_r Ai_{d0}i_{q0} & k_r \left(\frac{1}{T_s} - Ai_{q0}^2 + C\right) & 0 \\
0 & \frac{k_s}{T_r} & 0 & -\frac{1}{T_r} & \omega_{0} - \omega_{0} & -\psi_{Q0} \\
0 & 0 & \frac{k_s}{T_r} & -\left(\omega_{0} - \omega_{0}\right) & \frac{1}{T_r} & \psi_{D0} \\
0 & -c\psi_{Q0} & c\psi_{D0} & c\psi_{q0} & -c\psi_{d0} & -\frac{k_r}{J}
\end{bmatrix}
\]