Abstract:
This paper compares low order MIMO controller techniques suitable for an engine idle speed problem. One method of mapping the maximum singular values into parameter planes to create an interactive technique for meeting the nominal performance $H_\infty$ norm constraint is discussed. Two other controller techniques are applied to the same multivariable idle speed disturbance rejection problem. The relative merits of each design are discussed and simulation results presented.

Keywords: Fixed order, Frequency domain, Idle speed, MIMO, Parameter space, PID.

1. INTRODUCTION
The idle speed problem is a compromise between low engine speed to economise on fuel and the ability to satisfactorily reject disturbances (Hrovat and Sun, 2002). Torque disturbances due to electrical demand on the alternator, power-steering pump or air conditioning units can quickly load the engine causing a reduction in speed which can cause undesirable noise and vibration characteristics for the driver, or possibly engine stall. The problem of regulating speed has typically required careful control of the air entering the engine by either an electronic throttle or air-bypass valve. For faster response to disturbances, though at the cost of engine efficiency, the spark channel may be used where this is retarded from ‘maximum best torque’ (MBT) around the idle speed to allow additional control action as necessary. During disturbances the fuel channel is typically used to maintain the air to fuel ratio ($AFR$) as close to stoichiometric as possible. (Ganagopadhyay and Meckl, 2001) suggest a different approach to the idle problem for a natural gas automotive engine, where air and fuel are inputs to control the speed and AFR. This formulation of the idle speed problem is interesting since it allows the spark to be kept at MBT and the fuel is then used to aid in disturbance rejection. Including the fuel is such away also avoids having sequential SISO loops which can often give less performance or stability. The model in (Ganagopadhyay and Meckl, 2001) is difficult to control due to the large amount of interaction, making it poorly suited to SISO loop-by-loop control.

One major constraint when designing controllers for industrial applications is the order of the controller. Fixed, low order controllers are widely implemented in engineering systems due to their well understood characteristics, low computational demands and ease of tuning. In automotive applications, moreover, validated commercial engine management software code (‘strategies’) generally require controllers to be implemented in fixed-order look-up table format. $H_\infty$ theory provides a powerful method for designing controllers, these are generally of high order and so may not be
implemented directly and then require ‘order reduction’.

Parameter space techniques on the other hand offer a way of generating low order controllers directly. For these methods the controller order may be fixed and so can be made independent of the model order. However for systems with very complicated dynamics it may become difficult or impossible to meet several specifications simultaneously with a controller of a particular order. In these cases either the fixed order should be increased or the specifications should be relaxed.

Frequency domain parameter space methods (Shafiei and Shenton, 1997), (Besson and Shenton, 1999), and (Besson and Shenton, 2000) offer significant advantages over other controller design techniques, since the frequency response of the plant found from non-parametric identification can be used directly and as such irrational systems including systems with multiple pure time delays can be handled directly, without the need for rational approximations. This is in distinction to Riccati $H_\infty$ methods where rational representations are required. Furthermore, the frequency response approach is treated uniformly for continuous or discrete systems. A further important merit of the these methods over Riccati based techniques is that weighting functions do not need to be proper or indeed rational.

2. IDLE SPEED ENGINE MODEL

A linearised mean value discrete model of a natural gas engine around the idle operating point is taken from (Ganagopadhyay and Meckl, 2001) where also a novel eigenvalue PI controller design techniques is suggested. The control inputs to the plant are throttle, $\delta \alpha$ ($\%_{\text{max}}$) and fuel, $\delta \dot{m}_{fi}$ (lbm/h) the disturbance input is torque, $T$ (Nm) and the outputs are engine speed, $\delta n$ (rpm) and AFR ($\delta L$). The problem is presented as one of disturbance rejection and therefore the objective is to design a MIMO controller that minimises speed undershoot when a torque load such as air conditioning or power steering is applied suddenly to the engine. The plant has significant channel interaction and therefore a MIMO controller is suggested. In (Ganagopadhyay and Meckl, 2001) it is demonstrated that using a PI controller adequate performance cannot be achieved using two SISO Zeigler-Nichols controllers, nor with a MIMO method suggested in (Peltomaa and Koivo, 1983). The lean-burn natural gas engine plant model was given as

$$G_{11}(z) = \frac{\delta n}{\delta \alpha} = \frac{2.56 z^{-1} + 2.0 z^{-2}}{1 - 1.486 z^{-1} + 0.529 z^{-2}}$$

$$G_{12}(z) = \frac{\delta n}{\delta \dot{m}_{fi}} = \frac{-6.38 z^{-1} - 3.04 z^{-2} + 1.28 z^{-3}}{1 - 1.486 z^{-1} + 0.504 z^{-2}}$$

$$G_{21}(z) = \frac{\delta L}{\delta \alpha} = \frac{0.64}{z - 0.545}$$

$$G_{22}(z) = \frac{\delta L}{\delta \dot{m}_{fi}} = \frac{-2.03}{z - 0.537}$$

The transfer function between the external torque disturbance and the speed is given by:

$$G_d(z) = \frac{\delta n}{\delta T} = \frac{0.679 z^{-1} - 0.332 z^{-2}}{1 - 1.486 z^{-1} + 0.529 z^{-2}}$$

where the maximum step disturbance of 3.7 ft lbf (5 Nm) is assumed, corresponding to the load of a power-steering pump. The torque disturbance was found to have negligible effect on the AFR. Figure 1 shows the open loop block diagram for the engine model and controller $K$. Where the torque disturbance is modelled by adding only to the speed channel.

The aim of this problem is to minimise the speed undershoot when a step torque is applied, without excessive control effort. The fuel channel is fast acting and therefore control efforts can be large during transients, in contrast the electronic throttle in this example is limited to 4% of maximum throttle per s.

3. LOW ORDER CONTROLLER DESIGN

In this section three controller design techniques are compared for performance and suitability for an engine management system.
3.1 Parameter Space design

Recently, important MIMO parameter space results, suitable for developing computer algebra systems, have been presented by (Muhler, 2002). Parameter space boundaries are given in the form of equations in the determinant of the Hamiltonian matrices corresponding to the continuous $H_\infty$ and $H_2$ Riccati equations, and should thereby lead to symbolic parameter space methods for rational continuous systems.

An alternative parameter space method of mapping the singular values for a range of discrete frequencies is presented in (Dickinson and Shenton, 2006). Since the following method relies only on the frequency response of the system, both discrete and continuous systems can be handled uniformly, and irrational systems with multiple pure time delays handled without additional difficulty. In application to a square plant (possibly irrational) frequency response matrix, $G$ with $n$ inputs and outputs, let the controller to be designed be $n \times n$ rational transfer function matrix

$$K = \begin{bmatrix}
K_{1,1} & \ldots & K_{1,n} \\
\vdots & \ddots & \vdots \\
K_{n,1} & \ldots & K_{n,n}
\end{bmatrix}$$

where for a continuous controller each element has the structure

$$K_{ij} = \frac{b_{2ij}s^2 + b_{1ij}s + b_{0ij}}{a_{2ij}s^2 + a_{1ij}s + a_{0ij}} \quad (1)$$

The nominal performance requirement for a multivariable system is dependent on the singular values of the primary sensitivity function $S$

$$S = \frac{I}{I + G(j\omega)K(j\omega)}$$

which is shaped by a weighting function $W_S$ such that

$$||W_S(\omega)S(j\omega)||_\infty \leq 1, \ \forall \omega \in [0; +\infty)$$

The parameter space technique graphically determines the controller boundaries for the regions that meet the weighted primary sensitivity function requirement on the maximum singular values

$$\sigma[W_S(j\omega)S(j\omega)] < 1$$

by finding the solution of

$$\sigma[S(j\omega)^{-1}W_S(\omega)^{-1}] = 1$$

By choosing two controller parameters at a time regions satisfying the above requirement can be mapped for a given frequency. Superimposing a range of discrete frequencies of interest allows a designer to graphically select a pair of controller gains which meet the specification across all frequencies. This process is iterative between different planes and controller elements until the specifications are met for all frequencies. A scaling factor in the weighting functions $\gamma$ is gradually increased in the iterations.

For this example it was found that a large improvement could be made over the MIMO controller suggested in (Ganagopadhyay and Meckl, 2001) by using the weighted primary sensitivity function parameter space method, provided the control effort of the throttle was monitored with regards to the $\gamma$ weighting function scaling factor. Using the guidelines suggested in (Skogestad and Postlethwaite, 1996) a suitable primary sensitivity weighting function was selected. For good tracking response when subject to disturbances high gains at low frequencies are desirable. To limit the number of iterations required a low frequency breakpoint is also conveniently included in the weighting function. The selection for the shape of the weighting was accordingly chosen as

$$W_S = \gamma \frac{0.2s + 1}{s + 0.005} I_2$$

For proper comparison with the Eigenvalue based technique of (Ganagopadhyay and Meckl, 2001) only a pure MIMO PI parameter space controller is designed here. Therefore, each of the elemental controllers of eqn 1 has coefficients $b_{2ij} = 0, a_{2ij} = 0, a_{0ij} = 0$ and $a_{1ij} = 1$ with $b_{1ij}$ and $b_{0ij}$ ($K_F$ and $K_I$) determined in the parameter space. The parameter space design must begin with an initial stable start controller. In this case this could be $b_1 = b_0 = 0$ for all elements. However the parameter space method is also an excellent tuning method for existing controllers. For example the parameter space method could be used to re-tune coefficients of a Riccati controller after order reduction. Therefore to demonstrate this merit the initial controller used here will be the Zeigler-Nichols SISO controller which was shown to be inadequate for this problem.

$$K_{ZN}(s) = \begin{bmatrix}
0.1s + 0.2 & 0 \\
0 & -0.04s - 0.5
\end{bmatrix}$$

Since the problem is concerned with a physically discrete event system (four-stroke engine cycle), only frequencies up to the Nyquist frequencies need to be considered. A total of 250 discrete frequencies logarithmically spaced from 0.01 to 31.4 rad/s were used for the design. Starting with $\gamma = 0.2$ four iterations to $\gamma = 0.7$ were necessary
before the required performance was achieved without exceeding the control effort constraint on the throttle. Further increasing $\gamma$ was found to lead to better performance but excessive throttle control effort. An example plane for two of the parameters is shown in Figure 2.

The resulting parameter space controller was found to be

$$K_{PS}(s) = \begin{bmatrix} 0.1623s + 0.812 & -0.1649s - 1.748 \\ -0.0716s^2 + 0.2296 & -2.998s - 1.148 \end{bmatrix}$$

### 3.2 Eigenvalue MIMO PI controller design

In the multivariable PI controller design method suggested by the authors of the idle speed model developed, the proportional gains are selected to ensure good disturbance rejection by inspection of the eigenvalues. The integral gains are chosen to decouple the plant at steady state and ensure good tracking by looking at the closed loop eigenvalues. The suggested discrete designed controller for this method has a continuous equivalent

$$K_{EIG}(s) = \begin{bmatrix} 0.037s + 0.119 & 0.02s - 1.98 \\ -0.106s - 0.198 & -0.064s - 1.235 \end{bmatrix}$$

### 3.3 Riccati $H_{\infty}$ controller design

A higher order controller is also designed with the two algebraic Riccati Equation (ARE) method using the Matlab Robust Control toolbox to investigate the possible performance benefits of various order controllers. When designing such a Riccati $H_{\infty}$ controller for this problem it was found that the primary sensitivity function alone was not sufficient to achieve acceptable performance whilst meeting the control effort constraint on the throttle, and consequently the control effort sensitivity was also used. For acceptable performance, a second order weighting function, equation (2), was required to get both good speed and settling times. The control effort, equation (3), was bounded at all frequencies for the throttle and less severely on the fuel by the weighting functions:

$$W_S = \gamma \frac{s^2 + 50s + 500}{100s^2 + 30s + 0.1} I_2$$

and

$$W_U = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.005 \end{bmatrix}$$

These weightings result in an 11th order controller. The order can be reduced without significant loss in performance using a balanced model truncation on the normalized coprime factors to an 8th order controller. Further reduction is possible but the performance rapidly decreases. Figure 3 compares the full order controller with a number of reduced order controllers. It can be seen that there is significant loss in performance below 8th order where the 5th order controller is very oscillatory. 4th order and below were found to be unstable, this trend was true for all the common reduction methods found in the Matlab Robust Control toolbox (Balas and et al, 2005).

### 4. RESULTS

In this section the three controller design techniques are discussed. Each controller was converted to its discrete form using a Tustin bilinear transformation and used to compare time response results from a simulation. Time response results comparing the parameter space and Riccati designed MIMO controllers to the MIMO controller suggested by (Ganagopadhyay and Meckl, 2001) are presented in Figures 4, 5, 6...
and 7, 8, 9 and 10. Figures 4, 5, 6 and 7 show the results of the three controllers for a step torque disturbance. A small decrease in the maximum overshoot, significantly better settling times and a much less oscillatory response were found from the higher order algebraic Riccati and low order parameter space controllers. The improvements in performance can also be seen in figures 8 and 9 which show the response of the two outputs to a step change in the speed demand.

The reduced 8th order ARE controller was found to have the most robustness, indicated on the plots of primary sensitivity function in Figure 10. The parameter space method was found to give similar robustness to the PI design based on the eigenvalues, although with a lower absolute maximum value. However, none of the controllers were designed specifically for robustness. Without suffering less robustness, the performance improvement of the parameter space design over the suggested eigenvalue-designed PI controller is substantial for the same order controller. The ARE controller has very similar levels of performance to the parameter space design, with the benefit
of additional robustness, however it was demonstrated that its order could not be reduced below 5th.

5. CONCLUSIONS

The techniques presented in this paper are for achieving nominal performance for multivariable problems. For MIMO problems there are only a limited number of techniques that can produce fixed low order controllers, which are essential for use in industrial applications such as engine management systems.

A parameter space technique defining controller regions satisfying the $H_\infty$ norm was used successfully on a MIMO problem and the merits of the method were shown in simulation. The method is well adapted to both continuous and discrete systems for producing low order controllers. The idle speed disturbance rejection problem showed that mapping just the primary sensitivity function gives large improvements over existing low order controller design methods, with performance close to full order algebraic Riccati solution.

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REFERENCES


