Abstract: This paper presents a modelling technique and an optimal operation scheme for water distribution systems under full range of operating scenarios with/without leakage. Predictive control strategies are formulated based on piecewise affine control model of the system and the control problem is then cast into hybrid predictive control category. Optimal system operation is fulfilled by softly switching between different predictive control strategies each of which fits for a different operating scenario. Feasibility of the proposed modelling and optimal operational scheme is verified by an example network implemented using Matlab-Epanet computer simulation.

Keywords: water supply system, predictive control, leakage, optimal operation, switching

1. INTRODUCTION

In urban areas, the increased consumer demand versus the scarcity of water resources has presented a challenging task for water industry. Effective control techniques for preventing water leakage, minimizing water waste, and saving operational cost are always needed. Model predictive control (MPC), which is based on the mathematical model of the system and a short-term demand prediction, has been a commonly used technique to operate a water supply/distribution system (Brdys and Ulanicki, 1994). The system can be operated within a hierarchical optimizing control structure (Brdys and Tatjewski, 2005). The control objectives and corresponding control strategies are determined on-line by an upper supervisory control level (SuCL) in order to best respond to current operation state and operating conditions of the plant. In this paper, we propose an optimal operation scheme by softly switching between different control strategies each of which fits for a specific control objective together with a specific operating scenario with/without leakage. Three typical control strategies are considered: the cost control strategy, the leakage prevention control strategy, and the least leakage control strategy.

In realization of a predictive control strategy, the highly nonlinear head-flow relationship in water network components, e.g. pipes, valves and pumps, often makes the corresponding optimization problem very complex. Several mathematical programming techniques have been used by researchers e.g. linear programming (Crawley and Dandy, 1993), and nonlinear programming (Ormsbee and Reddy, 1995). For optimal pump scheduling control problem, due to the existence of pump on-off states, integer or logical variables are often involved, which implicitly casts the control problem into hybrid predictive control category. In this paper, we report a piecewise affine (PWA) modelling method and formulate the control model and corresponding optimization problems into mixed integer linear programs, for which efficient solvers exist, e.g. CPLEX (ILOG, 2003).

2. PIECEWISE AFFINE FORMULATION OF CONTROL MODEL

2.1 Physical Modelling of Distribution Systems

A water distribution system forms a network, the arcs of which are pipes/pumps and the nodes of which are
reservoirs, consumption nodes and pipe junctions. Pressure and flows in the network are controlled by valves and pumps. The network equations can be obtained from the continuity of flow at each node and balance of pressure between adjacent nodes:

\[
\sum_{j \in M_i} q_j - \sum_{j \in M_i} q_j - d_i = \begin{cases} 
0 & \text{for } i \in M \\
 l_{q_i} & \text{for } i \in M_i 
\end{cases} 
\]

\[h_{i+} - h_{i-} = \Delta h_j (q_j, u_j)\]  

where:
- \(q_j\) is the flow at arc \(j\) (liter/sec)
- \(h_i\) is the head at node \(i\) (m)
- \(d_i\) is the demand flow at node \(i\) (liter/sec)
- \(l_{q_i}\) is the leakage flow at node \(i\) (liter/sec)
- \(u_j\) is the control variable representing the state of valve or pump at arc \(j\)
- \(\Delta h_j\) is the head-flow characteristic function at arc \(j\)
- \(M, N\) are the set of non-leaky (leaky) nodes
- \(J_i, (N_j)\) the set of arcs whose start (end) node are \(i\)

The leakage flow \(l_{q_i}\) at node \(i\) can be modelled using an orifice function of the form (Rossman, 2000)

\[l_{q_i} = C_k p_i^\alpha,\]

where \(C_k\) is the emitter coefficient, and \(p_i\) is the pressure at leaky node \(i\). \(\alpha\) is the emitter exponent typically assigned a value of 0.5 to reflect leakage from bursts (May, 1994).

For pipes, the head-flow characteristic function \(\Delta h_j\) is expressed by Hazen-Williams formula

\[\Delta h_j (q_j) = R_i q_j^{1.852},\]

where \(R_i\) is the pipe resistance.

When arc \(j\) represents a pump, \(q_j\) can be described by a quadratic function when the pump is switched on:

\[\Delta h_j (q_j) = A_j q_j^2 + B_j q_j s_j + C_j s_j^2,\]

where \(0 < s_j \leq 1\) is the speed factor and takes a value of 1 for fixed speed pumps. \(A_j < 0\) is the resistance coefficient, and \(B_j\) often takes the value of \(B_j \leq 0\) in order to ensure a single stable operating flow point, and \(C_j\) is the shut-off head.

For a group of \(L\) fixed speed pumps having the same hydraulic characteristics in parallel, the head is the same for each individual pump but the overall flows are summed together. Hence, (5) is written as

\[\Delta h_j (q_j) = \begin{cases} 
A_j \left(\frac{q_j}{u_j}\right)^2 + B_j \left(\frac{q_j}{u_j}\right) + C_j & \text{if } u_j \neq 0 \\
0 & \text{if } u_j = 0 
\end{cases} \]

where \(0 \leq u_j \leq L\) is the number of pumps switched on.

2.2 PWA Formulations

The nonlinear head-flow relationship (4) for a pipe is approximated by \((n-1)\) piecewise linear segments as illustrated in Fig.1.

The piecewise linear relationship can be formulated as the following linear equalities and inequalities:

\[\Delta h = H_i + \sum_{i=1}^{n} \lambda_i (H_{i+1} - H_i)\]

\[q = Q_i + \sum_{i=1}^{n} \lambda_i (Q_{i+1} - Q_i)\]

\[1 \geq \lambda_1 \geq \delta_1 \geq \cdots \geq \lambda_{n-1} \geq \delta_{n-1}\]

where \(\delta \in [0, 1]\) and \(\lambda \in R\) for \(i = 1, \ldots, n-1\) are introduced integer variables and continuous variables respectively.

The leakage-pressure relationship (3) can also be formulated into the form of (7) using the same piecewise linearization method. However, for a pump group, whose characteristic function is as (6), there exist \(L\) hydraulic curves but only one curve can be chosen for each time step, which is shown in Fig. 2.


\[
\Delta h = \sum_{k=1}^{L} \delta_k \times \left( H_{k,i} + \sum_{j=1}^{m} \lambda_{i,j} (H_{k,i,j} - H_{k,i,j+1}) \right)
\]

\[
q = \sum_{k=1}^{L} \delta_k \times \left( Q_{k,i} + \sum_{j=1}^{m} \lambda_{i,j} (Q_{k,i,j} - Q_{k,i,j+1}) \right)
\]

\[
\sum_{i=1}^{N} \delta_i \leq 1, \quad \delta_i \in \{0,1\}, \quad \forall k = 1,\ldots,L
\]

\[
1 \geq \lambda_{i,j} \geq \delta_{i,j} \geq \cdots \geq \lambda_{i,m+1} \geq \delta_{i,m+1}, \quad \forall k = 1,\ldots,L
\]

where \( \delta_i \in \{0,1\} \), \( \lambda_i \in R \) \( \forall i = 1,\ldots,n-1 \).

According to the techniques described by Williams (1993), propositional logic can be equivalently transformed into mixed-integer linear inequalities. Thus, it can be easily verified that for binary integer variable \( \vartheta \) and continuous variable \( \lambda \) :

\[
z = \vartheta \lambda \quad \text{is equivalent to}
\]

\[
m \leq \lambda \leq M \\
 m \vartheta \leq z \leq M \vartheta \\
 z \geq \lambda - M(1-\vartheta)
\]

Hence, the product terms by integer and continuous variables in (9) are eliminated and transformed into linear inequalities by introducing auxiliary variables \( z_{kj} = \delta_k \lambda_{i,j} \), for \( k = 1,\ldots,L \) and \( i = 1,\ldots,n-1 \).

Power consumption \( p \) for a pump group composed of a number of pumps connected in parallel can be written as (Brdys and Ulanicki, 1994):

\[
p(q, \Delta h, \eta) = \sum_{i=1}^{U} \xi_i \frac{\Delta h}{\eta_i}
\]

where \( \xi \) is the unit conversion coefficient relating hydraulic quantities to electrical energy, and \( \eta_i \) is the \( i^{th} \) pump efficiency, \( i = 1,\ldots,U \). Consider the fact that \( \Delta h \) and \( \eta \) are both functions of flow \( q \) and pump combination, then for each known pump combination, the pumping power \( p \) can be expressed as a function of pumped flow \( q \) only. Hence, there are altogether \( L \) power-flow curves available but still only one such curve can be chosen for each time step.

3. CONTROL SCHEME

In daily operation of water distribution systems, a period of water demand prediction ahead of current time is usually needed to be the basis for generating optimal pump actions so as to achieve certain control objective, e.g. least pumping cost. Since too long or infinitive time horizon demand prediction is not accurate or unavailable, a relatively short prediction horizon is more realistic, and this is applied in a receding horizon manner, which forms the key idea of MPC technique. The corresponding optimization problem that reflects the control strategy is solved repetitively at every new time step and only the first element of the computed control sequence is applied.

3.1 Pumping Cost Minimisation Control Strategy

In operational control of water distribution systems, it is a very common control objective to obtain the least pumping cost, while satisfying consumer water demand and pressure constraints at certain nodes. Moreover, in order to keep a sustainable operation day after day, it is expected that reservoir/tank levels can come back to their original states after a certain period of operation. For instance, if the network is daily operated and the prediction horizon \( H_p = 24h \), we hope that the tank levels at the beginning of the next day could have the same level as the beginning of today. Hence, the overall objective function for this control strategy is written as:

\[
J_c = \sum_{s=1}^{T} \gamma(t) \sum_{j=1}^{G} \sum_{i=1}^{S} \frac{\xi_{s,j,i} \Delta h_{s,j,i}(t)}{\eta_{s,j,i}(t)} + \rho \sum_{s=1}^{S} \left[ v_s(t + H_p) - v_s(t) \right]
\]

where \( H_p \) is the prediction horizon. \( \rho \) is a weighting value. \( \gamma(t) \) is a power unit charge in £/kWh for the \( (t+1) \) time stage. \( v_s \) is the level of the \( s^{th} \) tank, \( s = 1,\ldots,S \). \( \eta_i \) is the pump efficiency of the \( i^{th} \) pump in the \( j^{th} \) pump group, \( i = 1,\ldots,U \), \( j = 1,\ldots,G \).

The optimization constraints are composed of:

- nodal flow continuity equations (1) with \( M_i \in \{0\} \)
- water elements head-flow equations (2), which are formulated into mix-integer linear inequalities
- volume balance equations of tanks/reservoirs
- pressure limits for certain nodes, \( U \leq h \leq \bar{U} \)
3.2 Excessive Pressure Minimisation Control Strategy

Since very high pressure may cause pipe fracture and water leakage, the control strategy of minimizing the overall network pressure can be applied for leakage prevention purposes. Following the same arguments for the levels of reservoir/tank, the objective function for this control strategy is written as:

$$J_p = \sum_{i=1}^{k_{th}} \sum_{t=1}^{t_{s1}+1} \left[ \Delta t \sum_{i=1}^{\text{NLN}} h_i(t) + \rho \sum_{i=1}^{\text{NLN}} \left| v_i(t+1) - v_i(t) \right| \right]$$

where $\text{NLN}$ is the number of nodes in the network and $h_i(t)$ denotes the head of $i$th node at time $t$.

The optimization constraints for this strategy have the same form as those of the least pumping cost strategy, since in both cases no leakage is involved.

3.3 Leakage Minimisation Control Strategy

When the network suffers leakage, minimizing the water loss becomes important. The objective function of leakage control strategy can be expressed as:

$$J_l = \sum_{i=1}^{k_{th}} \sum_{t=1}^{t_{s1}+1} \left[ \Delta t \sum_{i=1}^{\text{NLN}} l_i(t) + \rho \sum_{i=1}^{\text{NLN}} \left| v_i(t+1) - v_i(t) \right| \right]$$

where $\text{NLN}$ is the number of leaky nodes in the network and $l_i(t)$ denotes the leak flow rate from the $i$th leaky node at time $t$.

Due to the existence of leaky nodes, the set $M_l$ is not empty. The optimization constraints for the leakage control strategy incorporates the leakage term $l_i$ as shown in (3), which is an apparent difference from the constraints of the least pumping cost strategy.

3.4 Softly Switched Predictive Control Scheme

In operational control of water distribution systems, switching from one control strategy to another is sometimes inevitable due to the change of operating conditions. Since switching control strategies in a hard way may introduce some unwanted impulsive phenomena, e.g. sudden change of nodal pressure, an effective soft switching method is needed to achieve smooth departing from current control strategy into a new control strategy (Wang et al., 2005).

Supposing the current predictive control strategy is based on optimization problem (16):

$$\min_{u(t)} \sum_{i=1}^{\text{NLN}} f_i(x(t), u(t), \hat{t}) + \rho \left[ v_i(t+H_p) - v_i(\hat{t}) \right]$$

subject to: $g_i(x(t), u(t), \hat{t}) \leq 0$

and a new predictive control strategy is reflected by optimization problem (17):

$$\min_{u(t)} \sum_{i=1}^{\text{NLN}} f_i(x(t), u(t), \hat{t}) + \rho \left[ v_i(t+H_p) - v_i(\hat{t}) \right]$$

subject to: $g_i(x(t), u(t), \hat{t}) \leq 0$

where the state vector $x(t)$ refers to the pressure and flow in the network, $v(t)$ denotes level of tanks, and the input vector $u(t)$ is the pump actions.

In order to smoothly switch from the current control strategy to the new strategy, intermediate combined control strategies are introduced, which are reflected by optimization problems of the following form:

$$\min_{u(t)} \sum_{i=1}^{\text{NLN}} \left[ w_1(t, \hat{t}) f_i(x(t), u(t), \hat{t}) + w_2(t, \hat{t}) f_i(x(t), u(t), \hat{t}) \right]$$

subject to: $g_i(x(t), u(t), \hat{t}) \leq 0$

$$x_{\min} \leq x(t) \leq x_{\max}$$

where $w_1$ and $w_2$ soft switching weighting vectors the values of which are not only different within the prediction horizon, but also varying with every new time steps. Let the switching starts at $T_s$, $w_1$ and $w_2$ are generated by a simple algorithm below:

Algorithm 1.

- If $t < k_p + H_p$, $w_1(t, \hat{t}) = (\lambda_1)^{t-k_p}$, $\forall \hat{t} \in [\hat{t}_1,...,\hat{t} + T_s - 1]$
- If $t \geq k_p + H_p$, $w_1(t, \hat{t}) = 0$, $\forall \hat{t} \in [\hat{t}_1,...,\hat{t} + T_s - 1]$

where $T_s$ denotes the switching duration time.

It is worth to point out that each of the above control strategies is hybrid due to the existence of integer variables. Hence, as opposed to (Wang et al., 2005), the softly switched control strategy is also hybrid. A distribution system under varying operating scenarios can be operated by separately applying these control strategies and switching between them softly. The decision of when to switch control strategy depends on current operating conditions, and such decision is often made by SuCL. For simulation study purposes, we assume the decision on switching time is given.

4. SIMULATION CASE STUDY

The distribution network, which is depicted in Fig. 4 and Tables 1-4, includes 2 reservoir sources and 1 storage tank which acts as a supplementary water source. Water is pumped from the reservoir sources by two pump stations to the consumption nodes.

![Fig. 4 Dual-loop water supply/distribution network](image-url)
Computer implementation is based on Matlab-Epanet simulation environment. The piecewise affine control model and control strategies are realized in Matlab. Optimization problems are solved by CPLEX Mixed Integer Optimizer (ILOG, 2003), which can be called through Matlab MEX-DLL interface. Epanet is used as the water network simulator generating “real-life” data, which are fed back to update the initial state of the predictive controllers for each time step.

Table 1 Nodal data for the pipe network

<table>
<thead>
<tr>
<th>Node ID</th>
<th>Elevation (m)</th>
<th>Minimum head (m)</th>
<th>Maximum head (m)</th>
<th>Demand (l/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>18.0</td>
<td>21.0</td>
<td>35.0</td>
<td>5.0</td>
</tr>
<tr>
<td>2</td>
<td>16.0</td>
<td>18.0</td>
<td>32.0</td>
<td>15.0</td>
</tr>
<tr>
<td>3</td>
<td>14.0</td>
<td>16.0</td>
<td>30.0</td>
<td>15.0</td>
</tr>
<tr>
<td>4</td>
<td>10.0</td>
<td>12.0</td>
<td>28.0</td>
<td>15.0</td>
</tr>
<tr>
<td>5</td>
<td>12.0</td>
<td>14.0</td>
<td>30.0</td>
<td>15.0</td>
</tr>
<tr>
<td>6</td>
<td>9.0</td>
<td>10.0</td>
<td>28.0</td>
<td>20.0</td>
</tr>
<tr>
<td>7</td>
<td>18.0</td>
<td>20.0</td>
<td>32.0</td>
<td>5.0</td>
</tr>
<tr>
<td>8</td>
<td>18.0</td>
<td>21.0</td>
<td>35.0</td>
<td>5.0</td>
</tr>
</tbody>
</table>

Table 2 Tank/Reservoir data for the pipe network

<table>
<thead>
<tr>
<th>Node ID</th>
<th>Elevation (m)</th>
<th>Minimum level above bottom (m)</th>
<th>Maximum level above bottom (m)</th>
<th>Tank diameter (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>28.0</td>
<td>3.0</td>
<td>10.0</td>
<td>25.0</td>
</tr>
<tr>
<td>10</td>
<td>15.0</td>
<td>— Reservoir nodes —</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>11</td>
<td>15.0</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>

Table 3 Pipe data for the pipe network

<table>
<thead>
<tr>
<th>Pipe ID</th>
<th>Start node</th>
<th>End node</th>
<th>Length (m)</th>
<th>Diameter (mm)</th>
<th>Coefficient C Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>102</td>
<td>1</td>
<td>2</td>
<td>1014.0</td>
<td>450.0</td>
<td>115</td>
</tr>
<tr>
<td>203</td>
<td>2</td>
<td>3</td>
<td>1767.0</td>
<td>360.0</td>
<td>115</td>
</tr>
<tr>
<td>703</td>
<td>7</td>
<td>3</td>
<td>1334.0</td>
<td>300.0</td>
<td>115</td>
</tr>
<tr>
<td>402</td>
<td>4</td>
<td>2</td>
<td>1767.0</td>
<td>240.0</td>
<td>115</td>
</tr>
<tr>
<td>406</td>
<td>4</td>
<td>6</td>
<td>1767.0</td>
<td>180.0</td>
<td>115</td>
</tr>
<tr>
<td>503</td>
<td>5</td>
<td>3</td>
<td>1767.0</td>
<td>240.0</td>
<td>115</td>
</tr>
<tr>
<td>504</td>
<td>5</td>
<td>4</td>
<td>1767.0</td>
<td>240.0</td>
<td>115</td>
</tr>
<tr>
<td>506</td>
<td>5</td>
<td>6</td>
<td>2334.0</td>
<td>180.0</td>
<td>115</td>
</tr>
<tr>
<td>805</td>
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<td>5</td>
<td>1767.0</td>
<td>360.0</td>
<td>115</td>
</tr>
<tr>
<td>907</td>
<td>9</td>
<td>7</td>
<td>1014.0</td>
<td>300.0</td>
<td>115</td>
</tr>
</tbody>
</table>

Table 4 Pump data for the pipe network

<table>
<thead>
<tr>
<th>Pump ID</th>
<th>Head node</th>
<th>Tail node</th>
<th>Coeff. A (10^{-3})</th>
<th>Coeff. B</th>
<th>Shut-off head (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>111/112</td>
<td>10</td>
<td>1</td>
<td>-0.6297</td>
<td>0</td>
<td>22.67</td>
</tr>
<tr>
<td>181/182</td>
<td>11</td>
<td>8</td>
<td>-0.4354</td>
<td>0</td>
<td>21.33</td>
</tr>
</tbody>
</table>

Fig. 5 Daily demand profile

Optimization problems are formulated with a moving horizon \(H_F = 24\) hours ahead of present time and the sampling period is fixed to 1 hour. Electricity tariff difference is taken into account: low tariff is charged at 4.51p/kWh for the night time between 10:00pm–6:00am and high tariff is 9.72p/kWh for the daytime between 10:00pm–6:00am.

4.1 Non-leaky Network

In Fig. 6-8, the least pumping cost strategy considers different electricity tariff and the pumping actions are then more active in low tariff period. Whereas, the least excessive pressure control strategy does not use tariff information and the pumping actions are mainly affected by the network daily demand profile Fig. 5.

4.2 Leaky Network

When the network suffers leakage, e.g. across nodes 4, 5 and 6 with the emitter coefficient \(C_E = 0.15\) LPS/m\(^{0.5}\) for each leak, then the leakage minimisation control strategy should be applied to reduce water loss. Although technically it is possible to apply least pumping cost strategy or least pressure strategy, yet the leakage amount reduced by these two strategies is less than directly applying the leakage minimisation strategy. Simulation results Fig. 9-11 show that using the least leakage strategy may lose 19.312Ml water in a month, whereas the other two control strategies will lose 21.087Ml and 20.268Ml, respectively.
4.3 Softly Switched Operational Control

If the network is operated under varying operating scenarios, an optimal operation can be achieved by softly switching between these control strategies to meet different control objectives as shown in Fig. 12.

Simulation results for the softly switched operational control scheme under varying operating scenarios are shown in Figs. 13-14.

5. CONCLUSIONS

An obvious merit of the proposed PWA model is that the predictive control strategies can be conveniently implemented as mixed integer linear programs and the solution to corresponding optimization problems is guaranteed global optima. In the considered three predictive control strategies, long-term sustainable operation of the distribution system is guaranteed by periodically control the tank level back to its original status. When water distribution systems are operated under full range of operating scenarios with/without leakage, it has also been reported that an optimal operation of the system can be achieved by softly switching between control strategies.

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