Abstract: To solve the magnetic attitude control problem of a bias momentum satellite, a controller design method based on genetic algorithms (GAs) is addressed. The pitch loop of the satellite is controlled by a bias momentum wheel, while the roll/yaw loops are stabilized by two magnetic torquers which are installed along their respective axes. Momentum unloading is achieved through the two magnetic torquers. Sum of squares of three axes attitude errors and angular velocity errors via appropriate weighting serves as objective function that used in GA optimization, and the minimization of objective function is our performance index. Simulation results demonstrate that high pointing and stability accuracy can be obtained using the proposed optimized control scheme.

Keywords: Bias Momentum satellite; attitude control; magnetic control; genetic algorithms

1. INTRODUCTION

Attitude control systems play an important role in the operation of satellite. Since the magnetic attitude control systems are relatively lightweight, require low power and are inexpensive, the use of magnetic torquers for attitude control has become an active research topic (Barry, 2002; Lovera, 2005; Wisniewski, 2000). For bias momentum satellite, union control schemes using bias momentum wheels and magnetic torque rods have been developed through the years (Cavallo, et al., 1993; Jan, 2005; Fichter, 1996). Such system configuration has both the advantage of a minimum number of low cost actuators and the fact that the adopted actuators have no nominally fuel consumption in performing attitude and angular momentum control. Such magnetic attitude control is an attractive alternative for satellites subject to cost and weight constraints.

Satellite attitude control systems design must take into account a number of performance issues, such as stability accuracy, pointing accuracy. Many project designers apply the traditional PD control scheme of which the control parameters are often obtained from method of trial and error, thus the drawbacks of this kind of design obviously cost much time and efforts, and hardly to guarantee the optimal control parameters. Genetic Algorithms (GAs) are search algorithms based on the mechanics of natural selection and natural genetics (Goldberg, 1989). GAs can be adopted in solving these problems by encoding the parameters of the controllers into a chromosome, and defining a fitness measure as a function over the performance demands, thus formulating the design problem as the minimization of an objective function with respect to the controller parameters. This paper deal with the controller design for a three-axis stabilized bias momentum satellite, that is equipped with one momentum wheel and two magnetic torquers. Here a controller design method based on GAs is addressed. The GA optimization process is introduced to turn the control parameters for attitude control to achieve good results.

The paper is organized as follows: in Section 2 we briefly state the dynamics equation of the satellite; the satellite attitude controller design approach is presented in Section 3; the proposed decimal GA
optimization procedure to tune and optimize the performance of the controller in the former section is presented in Section 4; Numerical simulations are carried on and demonstrate the effectiveness of the controller and GA approach in Section 5. We conclude the paper in Section 6.

2. DYNAMICS EQUATIONS OF THE SATELLITE

The attitude dynamics for a satellite with internal angular momentum can be expressed by the Euler’s equations

\[ I \omega + \omega \times (I \omega + h) = -\dot{h} + T_e + T_d \]  \hspace{1cm} (1)

Where \( \omega \in \mathbb{R}^3 \) is the satellite body angular velocity vector, \( I \) is the inertia matrix, \( T_e \) is the control torques vector, \( T_d \) is the external disturbance torques vector, and \( h \) is the wheel angular momentum.

For a pitch momentum bias system with a momentum wheel aligned along the satellite pitch axis, we have

\[ h = \begin{bmatrix} 0 \\ h_p \\ 0 \end{bmatrix} \]  \hspace{1cm} (2)

And we note that

\[ h_p = h_{n} + \Delta h_p \]  \hspace{1cm} (3)

Where \( h_n \) (\( h_n < 0 \)) is the pitch momentum bias, \( \Delta h_p \) is the small variations in wheel momentum for pitch control, and \( |h_n| \gg |\Delta h_p| \).

Under the definition on 3-1-2 rotation, the attitude kinematics are represented by

\[ \omega = \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} \phi \cos \theta - \psi \cos \phi \sin \theta \\ \phi + \psi \sin \theta \\ \psi \cos \theta \cos \phi + \phi \sin \theta \end{bmatrix} \] \hspace{1cm} (4)

\[ -\omega_h = \begin{bmatrix} \cos \phi \sin \psi + \sin \phi \cos \psi \\ \cos \phi \cos \psi \sin \phi - \cos \phi \\cos \psi \sin \phi \] \hspace{1cm} \sin \phi \sin \psi - \cos \phi \\cos \psi \sin \phi \]

Where \( \omega_x \), \( \omega_y \), and \( \omega_z \) are the angular velocity components of the body frame with respect to the inertial frame, decomposed in the body frame axes; \( \omega_h \) is the orbit angular velocity; \( \phi \), \( \theta \), and \( \psi \) are the roll, pitch, and yaw angles, respectively.

Suppose the attitude of the pitch loop has been well controlled, that is \( \dot{h}_p \) fluctuates within a relatively small scope (\( \Delta h_p \) is very small). Hence to the roll-yaw coupling loops, \( \dot{h}_p \) can be treated as a constant, let \( h_p = h_{n} \), and when the satellite vary in a small angle, using equations (2), (3), and (4), the eq.(1) can be written as

\[ I_\phi \dot{\phi} + \omega_\phi h_\phi \varphi - h_\psi y = T_{ex} + T_{ex} \] \hspace{1cm} (5)

As seen from (5), since the linear pitch dynamics is decoupled from the roll/yaw dynamics, the pitch controller and roll/yaw controller are design separately.

3. ANALYSIS AND DESIGN OF ATTITUDE CONTROL

3.1 Pitch control

Based on the simplified model (5), the pitch loop equation controlled only by momentum biased wheel is:

\[ I_\phi \dot{\phi} + \dot{h}_\phi = T_{dp} \]  \hspace{1cm} (6)

\( T_{dp} \) includes the external disturbance torque of the pitch axis.

Let \( \dot{h}_\phi = K_{p\phi} \theta + K_{d\phi} \theta \), (6) can be written as \( I_\phi \dot{\phi} + K_{p\phi} \theta + K_{d\phi} \theta = T_{dp} \). This is a classical second-order control system. We can get control parameters through the genetic algorithm optimization.

Fig. 1. Block diagram of the pitch loop

3.2 Roll/yaw control

Attitude control of roll/yaw loop utilize magnetic actuation, in which the mechanical torque required for attitude control is generated by the magnetic interaction between the geomagnetic field and magnetic torquers. Here Magnetic torquers can be used for attitude and also for wheel momentum unloading.

The control torque generated by magnetic torquers can be expressed as

\[ T_c = T_{mc} + T_{ma} = M \times B = S(B)m \]  \hspace{1cm} (7)

Where \( T_{mc} \) is the control torque generated from the roll/pitch axes’ magnetic torquers, and \( T_{ma} \) is the unloading torque of the bias momentum wheel generated from the roll/yaw axes’ magnetic torquers; \( M = [M_x \ 0 \ M_z]^T \) is magnetic control moment, \( B = [B_x \ B_y \ B_z]^T \) is the vector formed with the components of the Earth’s magnetic field in the body frame of reference, and \( S(B) \) is given by
\[
S(B) = \begin{bmatrix}
0 & B_z & -B_y \\
-B_z & 0 & B_x \\
B_y & -B_x & 0
\end{bmatrix}
\] (8)

\(M\) is the control magnetic moment vector which is split into two parts

\[
M = M_c + M_u = \begin{bmatrix} M_{c_x} \\ 0 \\ M_{c_z} \end{bmatrix} + \begin{bmatrix} M_{u_x} \\ 0 \\ M_{u_z} \end{bmatrix}
\] (9)

Where \(M_c\) is control moment on roll/yaw axis; \(M_u\) is unloading moments for pitch bias angular momentum.

For a momentum bias satellite using two-axis magnetic torquers, we have

\[
\begin{bmatrix}
T_{cx} \\
T_{cy} \\
T_{cz}
\end{bmatrix} = \begin{bmatrix} 0 & B_z & -B_y \\
-B_z & 0 & B_x \\
B_y & -B_x & 0 \end{bmatrix} \begin{bmatrix} M_{c_x} \\ 0 \\ M_{c_z} \end{bmatrix}
\] (10)

For a desired torque \(T = T_c\) to be applied on the satellite, we must generate the magnetic momentum vector \(M\). The matrix in (10) does have an inverse, so

\[
\begin{bmatrix}
M_{c_x} \\
\dot{h}_y \\
M_{c_z}
\end{bmatrix} = \frac{1}{B_y} \begin{bmatrix} 0 & 0 & B_y \\
B_x & B_z & B^2_y \\
-B_y & 0 & 0 \end{bmatrix} \begin{bmatrix} T_{nx} \\ T_{ny} \\ T_{nz} \end{bmatrix}
\] (11)

The control torques in roll/yaw loop can be calculated by the formula

\[
T_{nx} = -(K_{p\phi} \phi + K_{d\phi} \dot{\phi})
\] \(\text{(12)}\)

Substituting (12) into (11), we have the following control law

\[
M_{c_x} = -\frac{1}{B_y}(K_{p\phi} \psi + K_{d\psi} \dot{\psi})
\] \(\text{(13)}\)

\[
M_{c_z} = \frac{1}{B_y}(K_{p\phi} \theta + K_{d\theta} \dot{\theta})
\]

Magnetic torquers generate magnetic moments whose interactions with the earth’s magnetic field produce the torques necessary to remove the excess momentum. The control equation for momentum unloading is as following, see (Sidi, 1997)

\[
M_u = -\frac{k}{|B|}(B \times \Delta h)
\] (14)

Where \(\Delta h = h - h_{nom} = \begin{bmatrix} 0 & \Delta h_y & 0 \end{bmatrix}^T\) is the excess momentum to be removed, \(h\) is the wheel’s momentum vector, \(h_{nom}\) is the nominal wheel momentum vector, and \(k\) is the unloading control gain.

4. CONTROLLER DESIGN VIA GENETIC ALGORITHM

Genetic algorithms are search and optimization techniques based on the mechanics of natural selection and natural genetics. GAs manipulate a population of potential solutions which are called chromosomes or individuals. Chromosomes are encoded representations of all the parameters of the solution. We evaluate chromosomes according to their lever of fitness. To evolve chromosomes that encode better solutions, the GA employs the genetic operators of selection, crossover, and mutation for manipulating the chromosomes in a population. The selection mechanism for parent chromosomes takes the fitness of the parents into account, ensuring that the better solutions have a higher chance to preconrate. Newly generated chromosomes in time replace the existing ones. Through this process the population will converge to a best solution.

The real-coded GA is chosen for this study because the processed parameters are multidimensional ones and vary in large scopes. The real-coded GA can simplify the encoding/decoding procedure and provide high precision results.

The proposed method is described in the following form.

Step 1: Give a reasonable range of control parameters. Select coding method. Set up GA parameters: crossover probability, mutation probability, population sizes, and maximum number of generations.

Step 2: Initialize the population of GA.

Step 3: Calculate the objective function for each chromosome. The best controller parameters for this example obtained by the minimization of the objective function \(f(x)\).

Here we define the objective function \(f(x)\) as

\[
f(x) = \sum_{i=1}^{3} \sum_{i=1}^{n_i} (a_i e_i^T)^2 + \sum_{i=1}^{3} \sum_{i=1}^{n_i} (a_i e_i^T)^2
\]

That is sum of squares of three axes attitude errors and angular velocity errors via appropriate weighting serves as objective function which is used in GA optimization. \(a_i\) and \(a_2\) are set mainly according to order of three axes attitude errors and angular velocity errors.

The objective function is used to provide a measure of how individuals have performed in the problem domain. Another function, the fitness function, is normally used to transform the objective function value into a measure of relative fitness (K.A.De Jong, 1975). Fitness values are derived from objective function values through a scaling or ranking function.
Step 4: Calculate the fitness value for each individual, if the search goal is achieved, or an allowable generation is attained, stop and return; else go to step 5.

Step 5: Selection and reproduction. The individuals with better fitness will be selected into next generation. Apply crossover and mutation.

Step 6: Repeat step 4.

Flow chart of the proposed method is shown in Fig.2.

Fig. 2. Flow chart of the proposed method

5. SIMULATION EXAMPLES

We use an example to demonstrate the validity of the method. The numerical values of satellite parameters are assumed as flowing:

Orbit attitude is 850 km, and inclination is 98.8°.

The moments of inertia are

\[
I = \begin{bmatrix}
106 & 2 & -8 \\
2 & 315 & -13 \\
-8 & -13 & 318
\end{bmatrix} \text{ (kg} \cdot \text{m}^2)\]

Bias angular momentum \( h_x = -60 (N \cdot m \cdot s) \). Max magnetic momentum is 100 (A \cdot m^2). Disturbance torques are

\[
T_y = \begin{bmatrix}
1.5 \cos(\omega_o t) - 2 \\
2 \sin(\omega_o t) + 5 \\
1.5 \sin(\omega_o t) - 3
\end{bmatrix} \times 10^{-4} (N \cdot m)
\]

Where \( \omega_o = 0.00108 \text{rad} / \text{s} \).

The initial attitude angle and initial angular velocity of satellite are

\[
\varphi(0) = \theta(0) = \psi(0) = 1^\circ \\
\dot{\varphi}(0) = \dot{\theta}(0) = \dot{\psi}(0) = 0.02^\circ / \text{s}
\]

For this example the following GA parameters are defined in Table 1.

<table>
<thead>
<tr>
<th>GA Parameter</th>
<th>Value or Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Generation number</td>
<td>30</td>
</tr>
<tr>
<td>Individual number</td>
<td>40</td>
</tr>
<tr>
<td>Parameter number</td>
<td>6</td>
</tr>
<tr>
<td>Coding method</td>
<td>Real-valued coding</td>
</tr>
<tr>
<td>Selection method</td>
<td>Stochastic universal sampling</td>
</tr>
<tr>
<td>Crossover and/or recombination</td>
<td>Discrete recombination</td>
</tr>
<tr>
<td>Probability mutation</td>
<td>0.167</td>
</tr>
<tr>
<td>Fitness assignment</td>
<td>Rank-based fitness assignment</td>
</tr>
<tr>
<td>Generation gap</td>
<td>0.9</td>
</tr>
</tbody>
</table>

Table 2 lists the controller parameters. The significant digit is \(10^{-4}\).

<table>
<thead>
<tr>
<th>(K_{py})</th>
<th>(K_{dy})</th>
<th>(K_{px})</th>
<th>(K_{dx})</th>
<th>(K_{pz})</th>
<th>(K_{dz})</th>
</tr>
</thead>
<tbody>
<tr>
<td>500.0547</td>
<td>6000.2355</td>
<td>-0.0533</td>
<td>-30.0768</td>
<td>-1.0007</td>
<td>-5.1673</td>
</tr>
</tbody>
</table>

Fig. 3. Progress of the objective function
In this paper, magnetic attitude control problem of a three-axis stabilized bias momentum satellite has been considered, and an optimal controller design method based on genetic algorithm is proposed. The creative combination of control methods and GAs can result in a powerful tool that is able to address real engineering control problems.

REFERENCES


