Abstract: This paper deals with the vibration control problem of the light rail vehicle’s (LRV’s) pantograph. The concepts of force cancellation, skyhook damper and track-following spring are jointly applied to form an active vibration control system. A systematic and effective optimization process for design of the active suspension parameters of pantograph using the constrained multiobjective evolutionary algorithm is presented. Simulation results are presented to illustrate the proposed design.

Keywords: Light rail vehicle, pantograph, optimization, force cancellation, skyhook damper

1. INTRODUCTION

Light rail vehicles (LRVs) are equipped with one pantograph that collects current from the contact wire. When the LRVs operate, the pantograph pan-head is in contact with the contact wire and electricity is transferred to the train. For the proper operation of LRVs, the catenary system must be supplied with stable electrical power through solid contact with the pantograph. The complex interactions between the pantograph and contact wire can deteriorate the quality of the electricity transfer and are difficult to surmount.

Recently, some researchers have tried to design active controllers for the pantograph systems of high speed trains (Yuki et al., 2003; Wu and Brennan, 1998; Corriga et al., 1994). The central issue is to reduce variations of the contact force between the pantograph and overhead contact wire. However, the pantograph system of those papers was modelled with a constant stiffness to represent the catenary. In (Wu and Brennan, 1998), the authors have discussed optimal control strategy to achieve a relatively constant contact force between an active pantograph and the overhead wire, which was shown to be effective against external disturbances. However, finding the optimal control gains while simultaneously satisfying all conflicting objectives and hard constraints is far beyond the scope of the paper.

Here we introduce an optimal active suspension design for the pantograph system which is related to a specific objective function for reducing the time-varying stiffness fluctuation between the pan-head and the contact wire. The ideas of force cancellation and skyhook damper commonly adopted in the wheel’s suspension design are introduced here. To tackle the highly complicated design task we develop an improved parameter optimization scheme by applying the evolutionary algorithms (EAs) as an computational tool. Compared with the traditional approaches, efficiency of the solution search is enhanced by incorporating the Pareto ranking strategy. We show that it is appropriate to deal with the complicated task considered herein.

2. PANTOGRAPH CONFIGURATION AND CATENARY CHARACTERISTICS

We consider the pantograph system of LRVs shown as in Fig. 1. The 3 DOF dynamic model of the pantograph-catenary system is represented by a kinematical chain with elastic joints, whose end-effectors act on the environment with unknown time-varying stiffness and the whole system dynamics can be represented by the spring-damp-mass system shown in Fig. 2.

In Fig. 2, \(P(t)\) indicates the time-varying stiffness between the pantograph shoe and catenary; the suspension between the frame masses in the model represents the elbow transmission. The stiffness
fluctuation between the droppers is neglected. It was shown in (Wu and Brennan, 1997) that the time-varying stiffness \( P(t) \) can be expressed as
\[
P(t) = \left[ k_0 (1 + \alpha \cos \frac{2 \pi V}{L}) \right] y_1 = \left( k_0 + \Delta k_0(t) \right) y_1, \quad (1)
\]
where \( k_0 \) is the average stiffness, \( \Delta k_0(t) \) denotes the uncertain stiffness and \( \alpha \) is the stiffness variation coefficient in a span, \( k_{\text{max}} \) and \( k_{\text{min}} \) represent the maximum and minimum stiffness of the overhead wire in a span, respectively; the time-varying stiffness of the catenary is related to the vehicle travelling speed \( V \); \( L \) is the length of one span.

Equations of motion of the pantograph model can be lumped into a second-order matrix equation:
\[
M \ddot{q} + C \dot{q} + Kq = u \quad (2)
\]
where the generalized coordinates vector is \( q = [y_1 \quad y_2 \quad y_1]^T \), the inertia matrix, the damping matrix, the stiffness matrix, and the control vector are, respectively, defined as
\[
M = \text{diag}(m_1, m_2, m_3) \quad , \quad C = \begin{bmatrix} c_1 & -c_2 & 0 \\ -c_3 & c_1 + c_2 & -c_3 \\ 0 & -c_2 & c_1 + c_2 \end{bmatrix}
\]
\[
K = \begin{bmatrix} k_0 + (k_0 + \Delta k_0) & -k_0 & 0 \\ -k_0 & k_2 + k_0 & -k_2 \\ 0 & -k_2 & k_2 + k_0 \end{bmatrix} , \quad u = [F_1 \quad F_2 \quad F_3]^T
\]
where the upper mass \( (m_3) \) represents the pan-head, the middle and lower \( (m_1, m_2) \) masses represent the plunger and frame arms, respectively; \( F_1(t) \) and \( F_2(t) \) are lift forces by aero force at high speeds; \( F_3(t) \) is the static uplift force applied to the pantograph; \( k_1, k_2, k_3 \) represent the stiffness of pan-head suspension, the plunger suspension, and the frame suspension, respectively; \( c_1, c_2, c_3 \) represent the viscous damping of the pan-head suspension, the plunger suspension, and the frame suspension, respectively. Aiming at achieving the minimum vibration, the parameters \( k_1, c_1, k_2, c_2, k_3 \) and \( c_3 \) closely connected to the objective have to be finely tuned.

3. DESIGN OF ACTIVE PANTOGRAPH SYSTEM

To achieve best contact effect, it is necessary to cancel the forces caused by the passive springs and dampers, which act on the pan-head resulting in heave accelerations. For the active control design, the active forces are generated with equal magnitude but in opposite direction to the passive forces so that the pan-head is completely isolated from the plunger.

According to the force cancellation, the pantograph is separated into two subsystems: pan-head and plunger. By observing the dynamic equations, after force cancellation, the pan-head act like a double integrator without damping and hence is inherently unstable. To stabilize the vehicle motion, it is necessary to measure velocities of the pan-head with respect to the inertia frame. This is named as the skyhook damper. For the optimization of suspension working space, the distance between pan-head and track should also be fed back to maintain the reasonable ranges among pan-head, plunger, and track, and to make the vehicle’s pantograph follow the tendency of the track profile. This is the track-following spring. The desired control configuration is illustrated in Fig. 3. The active control law can be summarized as follows
\[
F_i = F_{a1} + F_{a2} = k_i(y_i - y_2) + c_i(y_i - y_2) - k_i(y_i - y_1) - c_i y_1 \quad (3)
\]

A. Performance objectives

Traditional pantograph suspension system designs can only deal with the static deformations, they fail to obtain the dynamic deformation between of the pantograph and contact wire. In practice, the pantograph system is severely influenced by the irregular rail surface which may cause vibrations and deteriorate quality of transmission of electricity to vehicles. The individual objective functions considered are given as follows
\[
J_i = \frac{1}{3N} \sum_{j=1}^{N} \left[ y_j^2 (jT) + y_j^2 (jT) + y_j^2 (jT) \right] \\
J_1 = \max_{i \in [2,3]} \left| y_j (jT) - y_j (jT) \right| \\
J_2 = \max_{i \in [2,3]} \left| y_j (jT) - y_j (jT) \right| \\
J_3 = \max_{i \in [2,3]} \left| y_j (jT) \right|
\]
where \( N \) is the number of sampling cycles for measuring the related transient signals, \( \tau \) is the sampling period. To achieve the better power transmission, the suspension length should be optimally tuned to absorb the fluctuations caused by the contact wire irregularity and the stiffness fluctuation of the catenary.

Since the suspensions can only move within the available travelling ranges, therefore, the following constraints should be imposed:
\[
g_i (= J_i) \leq h_i , \quad i = 2,3,4 \quad (5)
\]

B. Optimal robust control design

Robust stability analysis considers the effect of time-varying stiffness resulting from the uneven overhead wire. The perturbed system model is given as follows
\[
\dot{x}(t) = \left( \tilde{A} + \Delta \tilde{A}(t) \right) x(t) + Bu \\
u = -\tilde{G} x
\]
where \( \tilde{G} \) is the feedback control gain matrix and
\[
\tilde{A} = \begin{bmatrix} 0_{3 \times 3} & I_{3 \times 3} \\ -M \cdot ^{-1} K & -M \cdot ^{-1} C \end{bmatrix} , \quad \Delta \tilde{A}(x,t) = \begin{bmatrix} 0_{3 \times 3} & 0 \\ -M \cdot ^{-1} \Delta K(t) & 0 \end{bmatrix}
\]
in which \( \Delta \tilde{A}(x,t) \) is the uncertain part of \( \tilde{A} \), which corresponds to changes of uneven stiffness of the overhead wire.

The plant uncertainty \( \Delta \tilde{A}(t) \) can be further decomposed as
\[
\Delta \tilde{A}(t) = B \Delta(t) \quad (7)
\]
where the uncertain matrix $\Delta(t)=[N_0,0]$ with
$N=\text{diag}(\Delta_k(t),0,0)$.

It is easy to show that $(A,B)$ is controllable. The closed-loop system is now given by
\[
\dot{x}(t)=(\hat{A}-BG)x(t)+\Delta\hat{A}(t)x(t)
\]
(8)

It can be shown that the problem for determination of the robust control law is equivalent to considering minimization of the following cost functional:
\[
V=\int_{t_0}^t \{x'Qx+u'Ru\}dt
\]
(9)
where $R=R^T>0$ and $Q=F+I_0$ with $F$ defined by
\[
F=\min \left\{ F| F \geq \Delta'(t)\Delta(t), \forall t \right\}
\]
(10)

For the current problem, $F=\text{diag}(0.25(k_{\text{max}}-k_{\text{min}})^2,0,0)$. It has been shown that the solution minimizing the LQR problem (9) is also a solution to the robust control problem (Lin, 1997). Based on this fact we can derive a robust control law given by
\[
u(t)=-R^{-1}B^TPx(t)
\]
(11)
where $P$ is the unique positive definite solution to the following algebraic Riccati equation:
\[
A'P+PA+Q-PBR^{-1}BP=0
\]
(12)

Since the weighting matrix $\Gamma$ directly affects the stabilizing control gain $G=R^{-1}B^TP$, it can be used as a tuning factor to alter system performance. We therefore set
\[
R=\text{diag}(r_1,r_2,r_3), \quad r_i>0, \forall i
\]
where $r_1,r_2,r_3$ are treated as the tuning parameters.

C. Solving for Multiobjective Optimization Problem

To apply the EGA, we first define the $j$-th individual of the active pantograph system by
\[
\hat{S}_j=[k_j, c_j, \ldots, r_j, r_j, \ldots]
\]
(13)

With reference to the formulation given in (Ehrgott and Galperin, 2002), we can represent the multiobjective problem in the following form:
\[
\min \left\{ F_j(\hat{S}_j) \right\}
\]
(14)
where $f_j$ is a weighting factor corresponding to the $q$-objective function $J_q$.

Next, a Pareto optimal set is applied to deal with the multiple objective optimization problems. The ranking arrangement is used to identify the near Pareto optimal set within the population of EGA. It performs the step with the best population of individuals that dominate the $j$-th individual $[\hat{S}_j]^T$.

To proceed, define
\[
\rho^* = \frac{w}{\text{rank}_j(\hat{S}_j,g)}, \quad \gamma_j = \frac{\max(p_j J_j(\hat{S}_j)/F_q)}{\max(J_j(\hat{S}_j)/F_q)} + \chi
\]
(15)

where the constant $\chi>\ln4$, $w$ is the number of total individuals at the current population $g$ and the multiobjective rank of $[\hat{S}_j]^T$ in the current population is defined by
\[
\text{rank}_j(\hat{S}_j,g) = 1+p^*_q
\]
(16)

where $p^*_q$ is the number of individuals that dominate the $j$-th individual (in the sense of Pareto domination (Goldberg, 1989) for the minimization of multiple objective functions) with respect to $J_q$. Furthermore, to cover the general situations that $A(\cdot)$ may not be smooth enough with respect to the variables, (13) is approximated as a scalar optimization problem described by
\[
A_j(\hat{S}) = \frac{1}{\gamma_j} \ln \left( \sum_{i=1}^{\gamma_j} \rho_i e^{\gamma_i}(\gamma_i) \right)
\]
(17)

Through the substitution we can adopt the following generalized objective formulation
\[
\min \left\{ A_j(\hat{S}) \right\}
\]
(18)

The generalized, constrained multiobjective fitness model can now be defined as
\[
\max f(\hat{S})
\]
subject to $g_j(\hat{S}) \leq h, i=2,3,4$
(19)
where $h,i=2,3,4$ denote the permissible travelling ranges of suspensions.

4. MIXED EVOLUTIONARY/GRADIENT SEARCHING ALGORITHM

An evolutionary-gradient search algorithm (EGA) taking into account the gradient directions of the objective function to find out the robust solution while speeding up the parameter searching is proposed. The EGA procedure starts with a group of randomly selected individuals
\[
\hat{x}_1=[\hat{x}_1, \hat{x}_1, \ldots, \hat{x}_1, \hat{x}_1]^T, j=1,\ldots,w
\]
During evolution, $\lambda=hw$ mutants are generated at each generation via
\[
\hat{x}_i(\hat{k},\hat{e})=\hat{x}_i(\hat{k},\hat{e})+N(0,\sigma^2), j=1,\ldots,w, i=1,\ldots,\lambda
\]
(20)

Note that each mutant is subject to the nominal stability requirement:
\[
\Re \lambda(A-BG)<0, \forall i
\]
(21)

Selection of the offsprings is based on the observation of the distribution of robust solutions for solving the optimization problem. Usually, the preferable robust solutions should lie in the regions with moderate gradients nearby so that the solutions won’t be too sensitive to the plant changes while achieving the optimality.
The process is summarized in the follows.

**Step 1** (generate mutants): Randomly generate the mutants \( \tilde{x}_i^p, i = 1, \ldots, \lambda \) by using (20) and subject to (21).

**Step 2** (preliminary screening): Calculate the adaptable average fitness level from the parent individuals

\[
\Psi^g = \left[1 + \ln \frac{N^g}{N^g_p} \right] \frac{1}{w} \sum_{i=1}^w f(\tilde{x}_i^p)
\]

where we have defined \( \Omega^g(p) \) as the set of superior individuals passing the preliminary screening:  
\[
\Omega^g(p) = \left\{ \tilde{x}_i^g \mid f(\tilde{x}_i^g) \geq \Psi^g, i = 1, \ldots, \lambda \right\}
\]
with \( N^g_p = \text{Num}(\Omega^g(p)) \) being the number of elements in \( \Omega^g \) and \( p \) denoting the qualified mutants.

**Step 3** (gradient search): First, determine the gradient \( \nabla f(i) \) for the individuals belonging to \( \Omega^g(p) \):

\[
\nabla f(i) = \left[ \frac{f(\tilde{x}_i^g) - f(\tilde{x}_1^g)}{\tilde{x}_1^g - \tilde{x}_i^g}, \frac{f(\tilde{x}_i^g) - f(\tilde{x}_2^g)}{\tilde{x}_2^g - \tilde{x}_i^g}, \ldots, \frac{f(\tilde{x}_i^g) - f(\tilde{x}_w^g)}{\tilde{x}_w^g - \tilde{x}_i^g} \right]^T
\]

Next, pick out the mutants with their corresponding gradient ascents lying between \( S_i \) and \( S_j \):

\[
\Theta^g(p) = \left\{ \tilde{x}_i^g \mid \nabla f(i) \leq \Psi^g \leq \nabla f(i) \right\}
\]
with \( || \) being some vector norm. The number of elements in \( \Theta^g(p) \) is denoted by \( M^g_p = \text{Num}(\Theta^g(p)) \) with \( p \) denoting the qualified mutants.

**Step 4** (elitism): The \( M^g_p \) mutants of \( \tilde{x}_i^g \) and the \( w \) parent individuals of \( \tilde{x}_i^g \) are sorted according to the magnitude of their fitness values defined. The \( w \) bests of the \( M^g_p + w \) members, denoted by \( \tilde{x}_i^{g+1} \), are sorted out and kept as the parents for the next generation.

Return to Step 1, selection of the qualified mutants is performed until all members are feasible. When the stopping condition is reached, an individual with the highest fitness in the converged population is picked up. A graphical illustration of the EGA is displayed in Fig. 4.

5. RESULTS AND DISCUSSIONS

Vibration analyses of the pantograph-catenary system are performed in the time domain. The catenary is treated as an uncertain time-varying system due to the time-varying stiffness. We suppose that the distance between each supporter is one unit of span. Accelerations at the pan-head \( m_1 \), plunger \( m_2 \) and frame arm \( m_3 \) are measured. Duration of the computer simulations is 10 s. For the EGA, the population size of individuals is taken to be 100. Two hundred generations are executed to validate convergence of the fitness function.

-Case 0 (nominal parameter design)

Simulation has been carried out on the basis of the pantograph catenary model with the parameters given in (Corriga et al., 1994). The overall acceleration of the pantograph system is 1.9259 m/s² (r.m.s.). In the following cases, the active control strategy will be adopted to reduce the acceleration of pantograph-catenary system and compromise the deflection of suspensions.

-Case 1 (passive pantograph design)

Consider the passive pantograph parameters which are restricted to the range from 80% to 120% of their nominal values in Case 0 to examine vibration of the overhead wire and check whether the natural frequency of pantograph mass still meets the specification. It is found that the overall acceleration in the passive pantograph system reduces to 1.3745 m/s² (r.m.s.). However, this only shows a slight performance improvement.

-Case 2 (active pantograph design)

We combine the EGA with the optimal robust controller. The overall acceleration of pantograph system has been further reduced to 0.4804 m/s² (r.m.s.). Consequently, the overall vibration is significantly improved by 75.56%. Obviously, all deflections of the suspensions are enlarged; however, they are still within the allowable ranges.

Figure 5 illustrates convergence of the fitness functions for the passive and active vibration control designs using the proposed optimization scheme. It is seen that the active pantograph gets the final fitness value of 2.5007 whereas the passive design gives only 1.5757. This demonstrates distinctive performance in solving the complicated constrained multiobjective optimization problem with our proposed approach. Figure 6 shows the resulting pantograph vertical accelerations for different suspension designs.

6. CONCLUSIONS

We show that force cancellation, skyhook damper and track-following spring can be jointly applied to construct an active vibration control system for the vehicle’s pantograph. Determination of the suspension parameters is assisted by a systematic and effective optimization scheme where the evolutionary-gradient algorithm serves as an efficient solution search tool. Simulation results are presented to illustrate the proposed design.

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REFERENCES


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**Fig. 1 Mechanism of pantograph**

**Fig. 2 Dynamic model of the LRV’s pantograph**

**Fig. 3 Pantograph with skyhook damper**

**Fig. 4 (w, λ) Mixed evolutionary/gradient search strategy**

**Fig. 5 Convergence of fitness values for passive and active vibration control of the pantograph**

**Fig. 6 Comparison of performance for the nominal case, active suspension design by EGA and passive suspension design by EA**