SIMPLE ANALYTICAL RULES FOR PI-PD CONTROLLERS TO TUNE INTEGRATING AND UNSTABLE PROCESSES

Ibrahim Kaya[*] and Derek P. Atherton[+]

[*]*Inonu University,*
Engineering Fac.,
Dept. of Electrical & Electronics Eng.
44280, Malatya, TURKEY
Email: ikaya@inonu.edu.tr

[+]*University of Sussex,*
School of Engineering and Information Technology,
BN1 9QT, Brighton, UK
Email: dpatherton@sussex.ac.uk

Abstract: Proportional-Integral-Derivative (PID) controllers are still extensively used in industrial systems. In the literature, many publications can be found considering PID controller design for processes with integrators and unstable transfer functions. However, due to structural limitations of PID controllers, generally, good closed loop performance cannot be achieved with a PID for controlling aforementioned processes and usually a step response with a high overshoot and oscillation is obtained. On the other hand, PI-PD controllers are proved to give very satisfactory closed loop performances for these processes. Hence, the paper introduces a simple approach to obtain parameters of a PI-PD controller for the control of integrating and unstable processes. Simulation examples are given to illustrate the value of the approach proposed.

Keywords: PID, PI-PD, performance index, unstable processes, integrating processes.

1. INTRODUCTION

Proportional-Integral-Derivative (PID) controllers are still widely used in industrial systems. The controller has three parameters to be adjusted. Appropriate values for these parameters can be found using many theoretical approaches if a plant transfer function is given or by one of several tuning rules, which can be found in the literature, based on typical process models. Often the PID controller is taken to have the error as it's input to the closed loop system which produces an undesirable 'derivative kick' at it's output for a step input to the feedback loop even when the D term has a filter. Also, it is well known that it is difficult to get good closed loop step responses with a PID controller for processes with plant transfer functions having integrators and unstable poles.

Some recent publications addressing the control of unstable processes from different points of view can be found in Poulin and Pomerleau (1996), Park et al. (1998), Ho and Xu (1998), and for integrating processes can be found in Poulin and Pomerleau (1996), Wang and Cluett (1997), Kwak et al. (1997).

Optimization has always been a powerful design method to determine tuning parameters of a PID controller. Tuning rules for stable processes based on a first order plus dead time (FOPDT) model and minimised for integral performance criteria were given by Zhuang and Atherton (1993). Recently, Visioli (2001) has presented tuning formulas for the minimisation of integral performance criteria for both integrating and unstable processes. However, large overshoots and long settling times resulted as a PD or

PID was used for the control of integrating and unstable processes, respectively.

The PI-PD controller has been shown to give improved performance in controlling unstable processes (Atherton and Boz, 1998; Kaya, 1999; Atherton and Majhi, 1999), and integrating processes (Kaya, 1999; Atherton and Majhi, 1999), for both set point tracking and disturbance rejection. Therefore, the purpose of this paper is to introduce a simple procedure to obtain tuning formulas for the PI-PD controller to minimise an integral performance criterion such as Integral of Squared Error (ISE) or Integral of Squared Time weighted Error (ISTE).

2. PI-PD STRUCTURE

In the conventional PID control algorithm, the proportional, integral and derivative parts are implemented in the forward loop, thus acting on the error between the set point and closed loop response. This PID controller implementation may lead to an undesirable phenomenon, namely the derivative kick. Also, by moving the PD part into an inner feedback loop, an unstable or integrating process can be stabilised and then can be controlled more effectively by the PI controller in the forward path. Therefore, the control structure shown in Fig. 1, which is known as a PI-PD control structure, can be used for performance improvement.

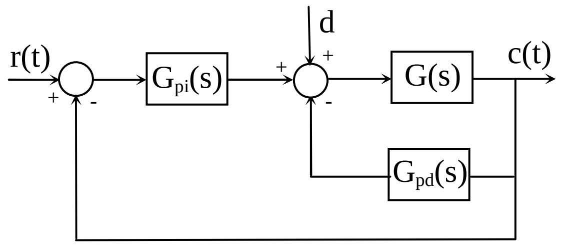

Fig. 1: The PI-PD control structure

In this structure, $G(s)$ is the plant transfer function and $G_{pi}(s)$ and $G_{pd}(s)$ are the PI and PD controller transfer functions, respectively, which have the following ideal forms:

$$G_{pi}(s) = K_p(1 + \frac{1}{T_i s}) \qquad (1)$$

$$G_{pd}(s) = K_f(1 + T_f s) \qquad (2)$$

This structure, which uses an inner feedback loop, is not totally a new concept. Benouarets (1993) was the first to mention the PI-PD controller structure. Unfortunately, its true potential was not recognized there as it was used to control plants with simple stable real pole transfer functions where its advantages are relatively minor. Later, Kwak et al. (1997) and Park et al. (1998) used a PID-P control structure for controlling integrating and unstable processes, respectively. However, as they still use the derivative term, D, in the forward path the structure may result in a derivative kick. Also, they use a gain only controller to alter the open loop unstable or integrating processes to open loop stable processes and then use the PID controller for an effective control of the overall system. It is better to use an inner feedback loop with a PD controller rather than a P only controller, as this not only converts the open loop unstable or integrating processes to open loop stable processes but also guarantees more suitable pole locations. To clarify this better, consider the PD controller of the form given by eqn. (2) and a general plant transfer function of

$$G(s) = \frac{b_m s^m + b_{m-1}s^{m-1} + \ldots + b_1 s + b_0}{a_n s^n + a_{n-1}s^{n-1} + \ldots + a_1 s + a_0} \qquad (3)$$

The closed loop transfer function for the inner loop, with $G(s)$ given by eqn. (3), is

$$G_{il}(s) = \frac{b_m s^m + b_{m-1}s^{m-1} + \ldots + b_1 s + b_0}{a_n s^n + a_{n-1}s^{n-1} + \ldots + (a_1 + K_f b_1 + K_f T_d b_0)s + (a_0 + K_f b_0)} \qquad (4)$$

provided that $n > m + 2$. The modification in the last two terms of the denominator of eqn. (4), due to insertion of PD controller used in the feedback loop, is clear. Let us assume that the coefficients a_0 and a_1 take suitable values to make the plant transfer function given in eqn. (3) an unstable or integrating plant transfer function. The PD controller used in the inner feedback loop can be used to convert it to an open loop stable plant transfer function for the PI controller used in the forward loop, which can then be used for a more satisfactory closed loop performance. Another point, which should be pointed out, is that the use of PI-PD controller gives more flexibility than a PID-P controller to locate the poles of open loop plant transfer function $G_{il}(s)$ in more desired locations with simultaneous use of K_f and T_f rather than a gain only parameter K_f.

3. TUNING RULES FOR PI-PD CONTROLLERS

In this section tuning rules for control of integrating and unstable processes are presented for the PI-PD controller, based on minimisation of integral performance criteria. The following performance index is used.

$$J_n(\theta) = \int_0^\infty \left\{t^n e(\theta, t)\right\}^2 dt \qquad (5)$$

where θ denotes the variable parameters which can be chosen to minimise $J_n(\theta)$. When $n = 0$, the index is known as the ISE and $n = 1$ as the ISTE and $n = 2$ as the IST^2E. The minimisation for the ISTE criterion usually gives responses with small overshoots and short settling times.

3.1 PI-PD Tuning Rules for Integrating Processes

It is assumed that the model

$$G(s) = \frac{K}{s} e^{-Ls} \tag{6}$$

can approximate integrating processes.

Determining four parameters of the PI-PD controller simultaneously is difficult using the performance index given by eqn. (5). The main aim of the inner PD controller is to stabilise and place closed loop poles of the inner loop to more appropriate positions so that the forward PI controller can control the resulting open loop stable system more effectively. Therefore, the inner PD controller parameters are identified as follows. The closed loop transfer function for the inner loop is

$$G_{il} = \frac{G(s)}{1 + G(s)G_{pd}(s)} \tag{7}$$

Substituting for $G(s)$ and $G_{pd}(s)$, using 1/1 Páde approximation for the time delay in the denominator and choosing $T_f = L/2$ gives

$$G_{il} = \frac{Ke^{-Ls}}{(1 - KK_f L/2)s + KK_f} \tag{8}$$

For a stable system

$$0 < K_f < \frac{2}{KL} \tag{9}$$

must be satisfied. Rearranging eqn. (8) gives

$$G_{il} = \frac{K' e^{-Ls}}{T' s + 1} \tag{10}$$

where

$$K' = 1/K_f \tag{11}$$

and

$$T' = \frac{1 - KK_f L/2}{KK_f} \tag{12}$$

Now, the integral performance index can be performed using the stable FOPDT model given by eqn. (10) to determine PI controller parameters. Tuning rules for such a model and PI controller were found by Zhuang and Atherton (1993), which are repeated below

$$K_p = \frac{a_1}{K'} \left(\frac{L}{T'}\right)^{b_1} \tag{13}$$

$$T_i = \frac{T'}{a_2 + b_2(L/T')} \tag{14}$$

The coefficients in eqns. (13) and (14) for the ISE and ISTE minimisations are given in Table 1.

Table 1: PI tuning formulae

L/T'	0.1-1.0		1.1-2.0	
Criterion	ISE	ISTE	ISE	ISTE
a_1	0.980	0.712	1.072	0.786
b_1	-0.892	-0.921	-0.560	-0.559
a_2	0.690	0.968	0.648	0.883
b_2	-0.155	-0.247	-0.114	-0.158

Therefore, tuning parameters for a PI-PD controller to control integrating processes are given by eqns. (9), (13) and (14) for K_f, K_p and T_i, respectively. T_f is given by $L/2$. Choosing larger K_f values, which satisfies the inequality given by eqn. (9), results in smaller overshoots and shorter settling times.

Example 1: Consider $G(s) = 0.0506e^{-6s}/s$ as an illustrative example regarding integrating processes, which was considered by Visioli (2001). The inner loop PD controller has $T_f = L/2 = 3$. $K_f = 0.5$, $K_f = 1.0$ and $K_f = 2.0$ are selected, in order to show the effect of K_f on the system performance. The stabilized inner loop plant transfer function will have the gain and time constants of 2.0, 1.0 and 0.5 for K', and 36.526, 16.763 and 6.881 for T', respectively. The forward PI controller minimised for the ISTE will have $K_p = 1.882$, $T_i = 39.381$ for $K_f = 0.5$, $K_p = 1.834$, $T_i = 19.058$ for $K_f = 1.0$ and $K_p = 1.616$, $T_i = 9.143$ for $K_f = 2.0$. Responses to a unity step input change and disturbance with magnitude of -1.0 introduced at $t = 100$ s are given in Fig. 2. Almost similar responses are obtained for different K_f values. However, faster disturbance rejections are achieved with larger K_f values. Fig. 3 compares the results of the PI-PD controller with the design proposed by Visioli (2001), which has $K_p = 2.960$, $T_d = 2.70$ for set-point tracking and $K_p = 4.410$, $T_i = 10.980$ and $T_d = 2.940$ for disturbance rejection. The controller parameters are obtained for the ISTE minimisation as for the PI-PD controller. The method proposed by Visioli (2001) gives steady state errors for disturbances as the controller tuned for set-point tracking does not involve an integrator. When the controller tuned for disturbance rejection is used, the disturbance is rejected but a very large overshoot for a set-point change is resulted. Fig. 4 illustrates the resultant control variables.

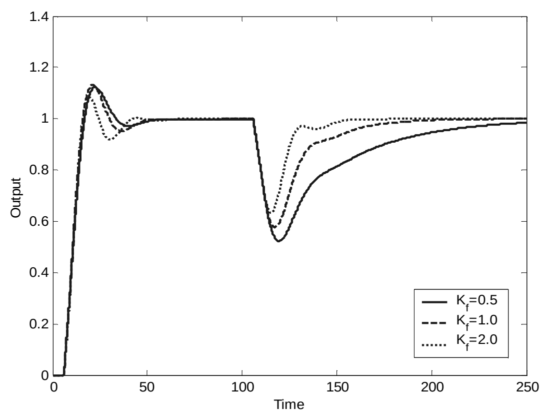

Fig. 2: Set-point and disturbance responses for PI-PD control with different K_f values for example 1

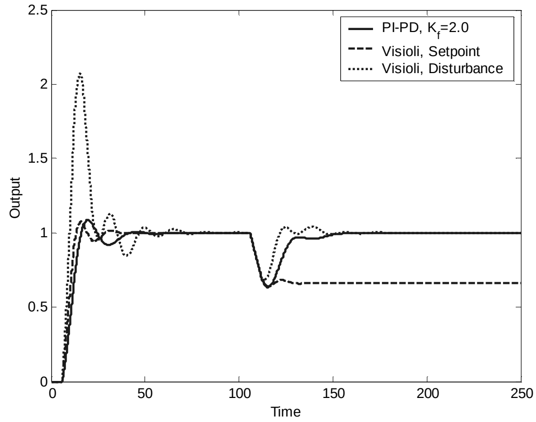

Fig. 3: Set-point and disturbance responses for example 1

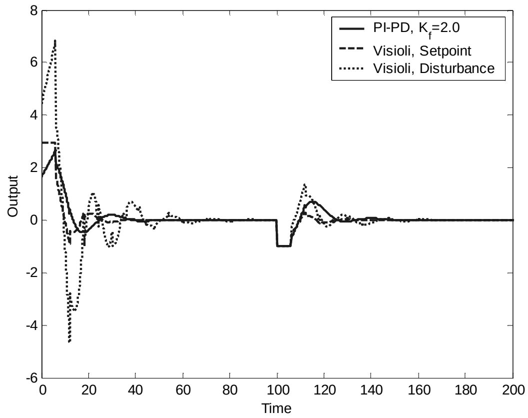

Fig. 4: Control variables for set-point and disturbance responses of example 1

3.2 *PI-PD Tuning Rules for Unstable Processes*

The plant transfer function is assumed to be modelled by

$$G(s) = \frac{K}{Ts - 1} e^{-Ls} \qquad (15)$$

Substituting for $G(s)$ and $G_{pd}(s)$ in eqn. (7), using 1/1 Páde approximation for the time delay in the denominator and choosing $T_f = L/2$, as for the integrating case, results in

$$G_{il} = \frac{Ke^{-Ls}}{(T - KK_f L/2)s + KK_f - 1} \qquad (16)$$

For a stable system

$$\frac{1}{K} < K_f < \frac{2T}{KL} \qquad (17)$$

must be satisfied. Rearranging eqn. (16) gives

$$G_{il} = \frac{K' e^{-Ls}}{T' s + 1} \qquad (18)$$

where

$$K' = \frac{K}{KK_f - 1} \qquad (19)$$

$$T' = \frac{T - KK_f L/2}{KK_f - 1} \qquad (20)$$

Choosing K_f satisfying the inequality given by eqn. (17) and $T_f = L/2$, the overall inner closed loop is a stable FOPDT. Hence, eqns. (13) and (14), with K' and T' given by eqns. (19) and (20), can be used to obtain the tuning parameters for the forward PI controller from Table 1.

Example 2: $G(s) = e^{-0.2s}/(s-1)$, which was studied by Visioli (2001), has been selected as the illustrative example for unstable processes. The PD tuning parameters are $K_f = 4$, which satisfies the condition given by eqn. (17), and $T_f = 0.2/2 = 0.1$. Hence, $K' = 0.333$ and $T' = 0.200$. Using these values in eqns. (13) and (14) for the ISTE criterion gives $K_p = 2.138$ and $T_i = 0.277$. The PID controller parameters proposed by Visioli (2001) for the ISTE minimisation are $K_p = 6.230$, $T_i = 0.73$ and $T_d = 0.09$. Fig. 5 illustrates responses to a unity step input change and disturbance with magnitude of -0.5 introduced at $t = 5$ s. The outstanding performance of the proposed PI-PD over the optimal PID method proposed by Visioli (2001) is clear. Results for control variables are given in Fig. 6.

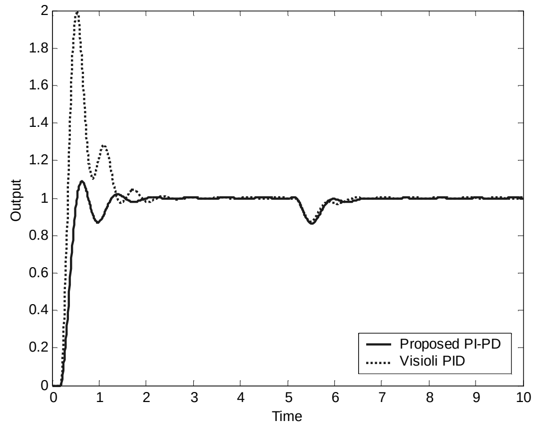

Fig. 5: Set-point and disturbance responses for example 2

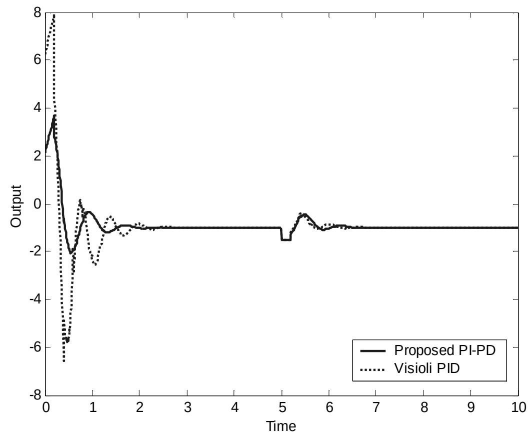

Fig. 6: Control variables for set-point and disturbance responses of example 2

Example 3: A higher order plant transfer function of $G(s) = e^{-0.3s}/(2s-1)(s+1)$ is considered. The model of $G(s) = e^{-1.07s}/(3.341s-1)$ was obtained using a single relay feedback test (Kaya, 1999). The inner loop PD controller parameters were selected as $K_f = 3.00$ and T_f as the half of the time delay, that is, T_f =0.535. These controller parameters convert the inner feedback loop to a stable FOPDT with $K^{'} = 0.500$ and $T^{'} = 0.868$. The PI controller parameters can be found from eqns. (13) and (14), which are calculated to be $K_p = 1.398$ and $T_i = 1.261$ for the ISTE minimisation. The parameters of PID parameters suggested by Visioli (2001) for the ISTE minimisation is $K_p = 6.080$, $T_i = 0.816$ and $T_d = 0.0963$. Responses for both designs to a unity step input change and a disturbance with magnitude of -0.5 introduced at t =35 s are given in Fig. 7. The PI-PD gives far superior performance than the PID proposed by Visioli (2001) for set-point tracking. Corresponding control variables are given in Fig. 8.

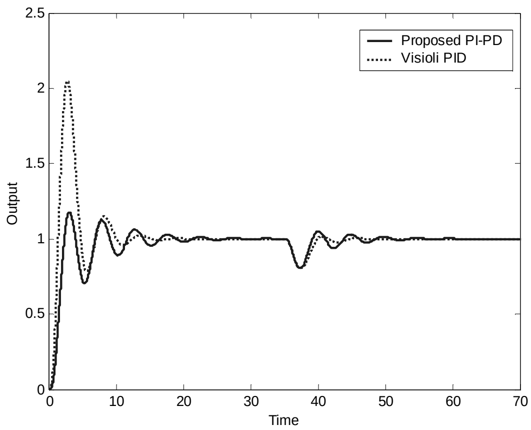

Fig. 7: Fig. x: Set-point and disturbance responses for example 3

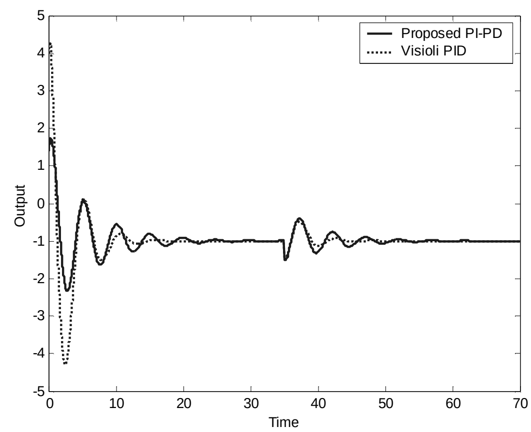

Fig. 8: Control variables for set-point and disturbance responses of example 3

4. CONCLUSIONS

PID controllers are still widely used in industrial systems. The effectiveness of minimisation of integral performance criteria has been proven to find good tuning parameters of PID controllers. However, due to structural limitations of PID controllers, good performances cannot be achieved with PID controllers for the control of integrating and open loop unstable systems, even when the integral performance criteria are used. Hence, a simple procedure to determine tuning rules for PI-PD controllers, which is efficient has been presented for these systems. The PD part is used to convert the integrating or unstable processes to a stable one. Then the ISE or ISTE minimisation is used to find tuning parameters for the forward PI controller to control the stabilized inner loop system. The value of the proposed approach has been shown by examples.

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