I. INTRODUCTION
Decentralized Control methodology provides effective economic and practical solutions to the control of complex process plants with multi inputs and multi outputs. Simple tuning procedure for single input single output controllers and implementation consideration are among the many reasons for its wide use (Skogestad and Postlethwaite, 1996; Bristol, 1966; Wittenmark and Salgado, 2002). However, a successful decentralized design would require an appropriate input-output section a priori (, 1966). Since the seminal work of (Bristol, 1966) and presentation of the RGA concept for input-output pairing, there have been many extensions and modification to the method (Niderlinski, 1971; Conley and Salgado, 2000; Khaki-Sedigh and Shahmansoorian, 1996; Wittenmark and Salgado, 2002).

In spite of the extensive research of the previous decades, the input-output pairing problem of nonlinear multivariable plants or linear multivariable plants with uncertain or time varying parameters is an open problem (Khaki-Sedigh and Moaveni, 2003a, b; Glad, 1999). In this paper, a new on-line estimation for RGA matrix using neural network for nonlinear or uncertain linear multivariable plants is proposed.

II. NEURAL NETWORKS APPROACH TO RGA
Consider the general nonlinear multivariable plants with \( m \) inputs and \( m \) outputs. To define RGA for the general plant an identification structure as in fig 1, with two neural network identifiers in the forward and backward directions are proposed.
Structures of the two neural identifiers, NN-I and NN-II, are the same, and shown in Fig.2, each has 2 linear layers in the input and the output layer and has no hidden layer. Each layer has \( m \) neurons that are equal to the number of system inputs-outputs.

Training the neural networks to identify the plants dynamics using the back propagation methods gives:

\[
\mathbf{Y} = \mathbf{WU}
\]  

Where, \( \mathbf{Y} \) is the output vector, \( \mathbf{U} \) is the output vector and \( \mathbf{W} \) is the brain neural network gain. Also, an inverse dynamics of the multivariable plants is identified using the inverse identifier (NN-II), given by:

\[
\mathbf{U} = \mathbf{W}_r \mathbf{Y}
\]  

Where, \( \mathbf{W} \) and \( \mathbf{W}_r \) are the weight matrices of the neural network identifiers. Now by the classical definition of RGA (Bristol, 1966) and using equations (1) and (2) the new definition for RGA is proposed as:

\[
\Gamma_n = \mathbf{W} \otimes \mathbf{W}_r^T
\]  

The weight matrices are computed online hence the \( \Gamma_n \) matrix are updated in each iteration, so in (3) we have a new definition of interaction measure, computable for linear and nonlinear and linear uncertain or time varying multivariable plants.

III. SIMULATION RESULTS

Examples are used to illustrate the effectiveness of the proposed RGA definition.

Example 1
Consider a system with the discrete transfer matrix (Wittenmark and Salgado, 2002)

\[
\mathbf{G}(z) = \begin{bmatrix}
0.1 & 0.08 \\
(z - 0.8)(z - 0.5) & (z - 0.8)(z - 0.1)
\end{bmatrix}
\]

The corresponding classical RGA is:

\[
\Gamma_1 = \begin{bmatrix}
0.8242 & 0.1758 \\
0.1758 & 0.8242
\end{bmatrix}
\]

While, the neural RGA is given by

\[
\Gamma_n = \begin{bmatrix}
0.7898 & 0.1994 \\
0.1647 & 0.7832
\end{bmatrix}
\]

Comparing equations (5) and (6), shows that \( \Gamma_n \) is very similar to \( \Gamma_1 \) and therefore \( (u_1 - y_1), (u_2 - y_2) \) are appropriate pairs. Fig. 3 shows the element convergence of the neural RGA.

Example 2
Consider the Quadruple-tank nonlinear system shown in fig 4 (Johansson, 2000). The nonlinear state space model is given by:

\[
\begin{align*}
\dot{x}_1 &= \alpha_1 \sqrt{2g h_1} + \alpha_2 \sqrt{2g h_2} + \frac{r_1}{A_1} u_1 \\
\dot{x}_2 &= \alpha_3 \sqrt{2g h_3} + \alpha_4 \sqrt{2g h_4} + \frac{r_2}{A_2} u_2 \\
\dot{x}_3 &= \alpha_5 \sqrt{2g h_5} + \frac{1 - \gamma_1}{A_3} u_3 \\
\dot{x}_4 &= \alpha_6 \sqrt{2g h_6} + \frac{1 - \gamma_2}{A_4} u_4
\end{align*}
\]

(7)
Linearization of (7) gives the transfer function matrix:

\[
G(s) = \begin{bmatrix}
\frac{\gamma G_1 k_1}{1+sT_1} & \frac{(1-\gamma_2)k_2}{(1+sT_1)(1+sT_2)} \\
\frac{(1-\gamma_1)k_2}{(1+sT_1)(1+sT_2)} & \frac{\gamma G_2 k_2}{1+sT_2}
\end{bmatrix}
\] (8)

Where,

\[c_i = \frac{T_ik_ik_c}{A_i}, \quad c_4 = \frac{T_4k_ik_c}{A_2}, \quad T_i = \frac{A_i}{a_i} \sqrt{\frac{2k_0}{g}}\]

Therefore the conventional RGA for the linear model of the quadruple-tank is

\[
RGA(G) = \begin{bmatrix}
\lambda & 1-\lambda \\
1-\lambda & \lambda
\end{bmatrix}
\] (9)

Where

\[\lambda = \frac{\gamma_1\gamma_2}{\gamma_1 + \gamma_2 - 1}\]

If \(0 < \gamma_1 + \gamma_2 \leq 1\), the appropriate pairs are \((u_1 - y_2), (u_2 - y_1)\) and if \(1 < \gamma_1 + \gamma_2 \leq 2\) the appropriate pairs are \((u_1 - y_1), (u_2 - y_2)\).

Assume that the quadruple-tank has approximately the following physical constants:

\[
\begin{aligned}
A_1 &= A_2 = 28 \text{ (cm}^2) \\
A_2 &= A_3 = 32 \text{ (cm}^2) \\
a_1 &= a_2 = 0.071 \text{ (cm}^2) \\
a_2 &= a_3 = 0.057 \text{ (cm}^2) \\
k_2 &= 0.50 \text{ (V/cm)} \\
g &= 981 \text{ (cm/s}^2) \\
k_i &= 2.9
\end{aligned}
\] (10)

In the 1st case we select \(\gamma_1 = \gamma_2 = 0.7\), from (9) we know that the conventional RGA matrix is (11) and so appropriate pairs are \((u_1 - y_2), (u_2 - y_2)\). Now, the neural RGA is obtained. Fig 5 shows the convergence of the elements of the neural RGA.

\[
\Gamma_{n} = \begin{bmatrix}
1.025 & -0.025 \\
-0.025 & 1.025
\end{bmatrix}
\] (11)

In the 2nd case, at 200 second \(\gamma_1, \gamma_2\) change to 0.3, therefore appropriate pairs will change to \((u_2 - y_1), (u_1 - y_2)\). Elements convergences of the neural RGA definition before and after 200th second are shown in fig 6.

\[
\Gamma_{n} = \begin{bmatrix}
1.0594 & -0.2191 \\
-0.2481 & 1.0852
\end{bmatrix}
\]

In the 2nd case, at 200 second \(\gamma_1, \gamma_2\) change to 0.3, therefore appropriate pairs will change to \((u_2 - y_1), (u_1 - y_2)\). Elements convergences of the neural RGA definition before and after 200th second are shown in fig 6.

IV. CONCLUSION

In this paper, a neural RGA is proposed for general nonlinear multivariable plants. This method is based upon the Neural Network identification. It is shown that it can be applied to linear and nonlinear MIMO systems and can be computed on-line. The new definition of RGA has been compared with classical RGA index. The new RGA definition has similar properties as the classical RGA index.

REFERENCES

- Khaki-Sedigh A. and B. Moaveni (2003), Relative Gain Array analysis for uncertain multivariable
plants, Proc. of 7th European Control Conference (ECC03), Cambridge, London, UK.


