Abstract: In this paper, the conditions for the equivalence between the supervisory level and regulatory level control strategies based on the same objective function are established. Both controllers use the same general objective function with constraints. It is shown that the supervisory controller modifies the control action of the fixed regulatory controller in such a way that the solution of general objective function with constraints is generated at the regulatory level. In order to demonstrate the theoretical results, the proposed controllers are applied to a power plant boiler simulator.

Keywords: Supervisory control, predictive control, optimisation, boiler simulator.

1. INTRODUCTION

For the industrial process, the plant optimisation is an important issue that could be solved using supervisory control strategies. However, the supervisory controllers usually are based on the steady state of the costs, which provide the optimal static set points [1].

In the last years, there are some papers that deal with dynamic models. For example, de Prada [2] proposes a predictive control strategy based on the optimisation of an economic index. This strategy is applied to a chemical reactor. Katebi [3] describes a decentralised control strategy, based on the optimisation of a GPC objective function. The objective function has only regulatory objectives. The control strategy is applied to a thermal power plant simulator. Also, there are some industrial applications, for example a DMC supervisory controller for a petrochemical process (YHP) [4] that gives good economic results based on linear dynamic models. On the other hand, Bemporad [5] and Angeli [6] propose a reference governor at the supervisory level. The objective function is given by the minimization of the reference trajectory error. The main goal is to satisfy certain constraints. The algorithms are developed using a state-space representation. A different approach for a reference governor with the same objective is proposed by Gilbert [7]. In this case, the reference governor is given by a non-linear pre-filter. Tadeo et. al [8] proposes a constrained predictive supervisory controller that can deal with the retuning of the PID controllers at regulatory level. The typical MBPC objective function is considered. Recently, Uduedi & Ordys [9] presented the equivalence between supervisory control strategy based on GPC objective function and a regulatory GPC controller. The state space formulation is considered for multivariable systems. In this work, based on Lagrange theory with Kuhn Tucker conditions, we derive the equivalence between a supervisory control strategy and a regulatory controller, both with constraints, based on the same general objective function. This objective function may represent not just regulatory criterion (GPC), but also economic criterion or others.

The control design is first described, including the supervisory optimal controller and the regulatory optimal controller derivations. The equivalence of the two strategies is demonstrated. The proposed supervisory and regulatory controllers are assessed using a boiler simulator. Finally, the conclusions are summarised.
2. OPTIMAL CONTROL STRATEGY DESIGN

2.1 Problem statement.

The control strategy proposed here is based on the optimisation of a general objective function subject to equality and inequality constraints. This problem can be solved by numerical algorithms and the predictive control theory [10]. Next, the main components of the optimal control design procedure are described.

Process modelling

A Controlled Auto-regressive and Moving-average (CARIMA) model will be used [11][12]:

\[ A(q^{-1})y(t) = B(q^{-1})u(t) + C(q^{-1})e(t) \]  

with \( \Delta = 1 - q^{-1} \Delta A(q^{-1}) = I_{\text{nnx}} + a_1 q^{-1} + \cdots + a_{na} q^{-na}, \)

\( \Delta B(q^{-1}) = b_1 q^{-1} + \cdots + b_{nb} q^{-nb-2} \)

\( C(q^{-1}) = 1 + c_1 q^{-1} + \cdots + c_{nc} q^{-nc}. \)

The variables \( y(t), u(t) \) and \( e(t) \) are \( nx1 \) output vector, \( mx1 \) input vector and \( nx1 \) noise vector at time \( t \), respectively.

Modelling of the regulatory level

Optimisation of the plant operation can be solved by adding a supervisory optimising level to the control hierarchy. Thus, a fixed controller at the regulatory level is considered and represented by:

\[ A_r(q^{-1})u(t) = B_r(q^{-1})r(t) + B_y(q^{-1})y(t) \]  

with \( r(t) \) is \( nx1 \) set-point vector and \( A_r(q^{-1}) = 1 + a_c q^{-1} + \cdots + a_{cna} q^{-cna}, \)

\( B_r(q^{-1}) = b_{ro} + b_{ri} q^{-1} + \cdots + b_{rnb} q^{-rnb}, \)

\( B_y(q^{-1}) = b_{yo} + b_{yi} q^{-1} + \cdots + b_{ynb} q^{-ynb}. \)

General objective function and constraints

The objective function used at the supervisory level is considered and represented by:

\[ J = \sum_{j=1}^{Ny} \hat{y}(t+j)\Psi_{j}\hat{y}(t+j) + \sum_{i=1}^{Nu} u_i(t+i-1)\Psi_{ui}u(t+i-1) \]

\[ + \sum_{i=1}^{Nu} \Delta u_i(t+i-1)\Psi_{ui}\Delta u(t+i-1) \]

\[ + \sum_{j=1}^{Ny} \sum_{i=1}^{N_j} \hat{y}_i(t+j)\Psi_{ji}u_i(t+i-1) + \sum_{j=1}^{N_j} \sum_{i=1}^{N_j} \hat{y}_i(t+j) \]

\[ + \sum_{j=1}^{Ny} \sum_{i=1}^{N_j} \hat{y}_i(t+j)\xi_{ji}u_i(t+i-1) + \sum_{j=1}^{N_j} \xi_{ji}\Delta u_i(t+i-1) \]

where \( \hat{y}(t+j) \) are the j-step ahead predictions of the controlled variables based on data up to time \( t \), \( u(t+i-1) \) are the manipulated variables, \( \Delta u(t+i-1) \) are the increments of the manipulated variables; \( \Psi \) and \( \xi \) are weighting sequences matrices/vectors; \( N_u \) and \( N_y \) are prediction horizons. This objective function may represent different optimisation goals at the supervisory level. For instance the operational costs, the profit of a plant, or the process energy consumption. Also, regulatory level criteria can be included.

Normally, the following constraints are considered:

\[ u_{\text{min}} \leq u(t+i-1) \leq u_{\text{max}} \]

\[ \Delta u_{\text{min}} \leq \Delta u(t+i-1) \leq \Delta u_{\text{max}} \]

\[ y_{\text{min}} \leq \hat{y}(t+j) \leq y_{\text{max}} \]

with \( i = 1, ..., Nu, j = 1, ..., Ny. \)

Next, two solution algorithms are proposed in order to solve the optimisation problem.

2.2 Supervisory control strategy

In the proposed supervisory level control strategy shown in Figure 1, all the regulatory controllers are considered. Thus, the supervisory optimiser gives the optimal set-points \( r \) for the regulatory level, resulting from the optimisation of the objective function \( J \) [10]. In this diagram, the regulatory level is fixed. Other variables are: the trajectory of an external reference \( w \), the controlled variables \( y \), the manipulated variables \( u \) and non-measured disturbances \( e \). In the general case, the system may be multivariable, therefore \( r, w, y, u \) and \( e \) are vectors.

![Fig. 1. A supervisory control strategy diagram](image-url)

Optimisation problem 1

Optimisation of the objective function defined in equation (3) subject to the constraints defined in equations (4) to (6) is considered. The optimal solution is obtained using the Kuhn –Tucker conditions for constrained optimisation problems. In this case, the process and the regulatory level models are considered as constraints. The equation (1) can be reformulated as:

\[ \Delta A(q^{-1})y(t) = \Delta B(q^{-1})u(t) + C\Delta(t) \]  

where \( \Delta A(q^{-1}) = A_1(q^{-1}) + A_2(q^{-1})q^{-1}, \)

\[ \Delta B(q^{-1}) = B_1(q^{-1}) + B_2(q^{-1})q^{-1} \]

\[ C(q^{-1}) = C_1(q^{-1}) + C_2(q^{-1})q^{-1}. \]

Then, the j-step predictions of controlled variables, by using Diophantine equations, are:

\[ A_r(q^{-1})\hat{y}(t+j) + A_s(q^{-1})y(t) \]

\[ = B_r(q^{-1})u(t+j) + B_y(q^{-1})u(t) + C(q^{-1})e(t) \]

with \( E[C(q^{-1})e(t+j)]=0 \) and \( C_2(q^{-1})e(t) \) can be obtained from the measurements of \( y(t) \) and \( u(t) \). On the other hand, the constraints on the manipulated variables at future instants, using the regulatory level model (equation (2)) are:

\[ A_s(q^{-1})u(t+i-1) + A_s(q^{-1})u(t-1) = B_r(q^{-1})r(t+i-1) \]

\[ + B_y(q^{-1})u(t-1) + B_y(q^{-1})y(t+i-1) + B_e(q^{-1})e(t) \]
The optimal solution is obtained using the Kuhn–Tucker conditions. In this method, the optimal set-point at time instant \( t \) is given by the optimal value of the variable \( r(t) \). The Lagrangian function is:

\[
L = J + \sum_{i=1}^{N_u} \mu_i \left[ u(t+i-1) - u(t+i -2) - \Delta u_{max} \right] + \sum_{i=1}^{N_u} \eta_i \left[ u(t+i-1) - u(t+i -2) - \Delta u_{max} \right] + \sum_{i=1}^{N_y} \sum_{j=1}^{N_y} \gamma_i^j \left[ y(t+j) - y_{max} \right] - \sum_{i=1}^{N_y} \sum_{j=1}^{N_y} \sigma_i^j \left[ y_{min} - y(t+j) \right]
\]

where \( J \) is the objective function defined by equation (3) and the other terms are related to constraints given by equations (4) to (6) and (8) and (9). \( \lambda_j, \mu_i, \eta_i^\min, \eta_i^\max, \sigma_i^\min, \sigma_i^\max, \gamma_i^\min, \gamma_i^\max \) are Lagrange multiplier vectors.

The solution to the optimisation task is given by the following Kuhn–Tucker conditions. In this method, the optimal set-point at time instant \( t \) is given by the optimal value of the variable \( r(t) \).

\[
\frac{\partial L}{\partial \mathbf{u}(t+i-1)} = 2(u(t+i-1) - u(t+i-2)) \mathbf{u}_{max}^T \Psi_{u} \Psi_{w}
\]

\[
-2(u(t+i) - u(t+i-i)) \mathbf{w}_{max}^T \Psi_{w} \Psi_{w} + \sum_{j=1}^{N_y} \hat{y}(t+j) \mathbf{y}_{max}^T \Psi_{y} \Psi_{y} + \hat{e}_y^T
\]

\[
+ \lambda_j + \sum_{k=1}^{N_v} \lambda_{k,h} b_i = 0
\]

\[
\frac{\partial L}{\partial \mathbf{e}(t+j)} = 2\gamma(t+j) \mathbf{y}_{max}^T \Psi_{x} \Psi_{x} + \sum_{i=1}^{N_u} (u(t+i-1) - u(t+i -2)) \mathbf{u}_{max}^T \Psi_{u} \Psi_{u} + \hat{e}_y^T
\]

\[
+ \lambda_j + \sum_{k=1}^{N_v} \lambda_{k,h} b_i = 0
\]

\[
\frac{\partial L}{\partial \mathbf{e}(t+i-1)} = -\sum_{i=1}^{N_u} \lambda_i \mathbf{b}_i = 0
\]

for \( i = 1, ..., N_u, j = 1, ..., N_y \)

Also, the following Kuhn-Tucker conditions must be fulfilled:

\[
\lambda_j \left[ A_1(q^{-1}) \hat{y}(t+j) + A_2(q^{-1}) y(t) \right] - B_1(q^{-1}) u(t+j) - B_2(q^{-1}) u(t+i-1) - C_1(q^{-1}) e(t) = 0
\]

\[
\mu_i \left[ A_1(q^{-1}) u(t+i-1) + A_2(q^{-1}) u(t) - B_1(q^{-1}) r(t+i-1) - B_2(q^{-1}) r(t-1) - B_3(q^{-1}) y(t+i-1) - B_4(q^{-1}) y(t) \right] = 0
\]

Notice the difference between this Lagrangian and the Lagrangian considered previously (equation (10)) is the lack of constraints imposed by the regulatory level. The solution to the optimisation task, (the optimal value of the manipulated variable \( u(t) \)) is given by the following Kuhn–Tucker conditions:

\[
\hat{y}(t+i) = \lambda_j + \sum_{k=1}^{N_v} \lambda_{k,h} b_i
\]

\[
\hat{e}(t+i) = \gamma(t+i) \mathbf{y}_{max}^T \Psi_{x} \Psi_{x} + \sum_{i=1}^{N_u} (u(t+i-1) - u(t+i -2)) \mathbf{u}_{max}^T \Psi_{u} \Psi_{u} + \hat{e}_y^T
\]

\[
\hat{e}_y(t+i) = \lambda_j + \sum_{k=1}^{N_v} \lambda_{k,h} b_i = 0
\]
The j-step ahead prediction for the drum level is the j-step ahead prediction for the superheated steam pressure and the Langrange multipliers are the optimisation variables. Therefore, the total number of optimisation variables is 4Ny + 5Nu. For i = 1, ..., Ny, u

Also, the Kuhn-Tucker conditions must be fulfilled:

\[ \lambda_i \left[ A_i(q^{-1})y(t + j) + A_j(q^{-1})y(t) - B_i(q^{-1})u(t + i - 1) - B_j(q^{-1})u(t) - C_i(q^{-1})c(t) \right] = 0 \]

\[ \eta_{i, \max} (u_{i, \min} - u(t + i - 1)) = 0 \]

\[ \eta_{i, \max} (u(t + i - 1) - u_{i, \max}) = 0 \]

\[ 0 \leq \gamma_{i, \min} \leq 0, \quad \sigma_{i, \min} \leq 0, \quad \sigma_{i, \max} \geq 0, \quad \eta_{i, \max} \geq 0, \quad \gamma_{i, \min} \geq 0, \quad \gamma_{i, \max} \geq 0 \]

for j = 1, ..., Ny. The expressions (24) to (33) generate 4Ny + 5Nu equations.

Next, substitute equation (37) to equations (11), (12) and (13). After this substitution, it is easy to confirm that equation (11) is equivalent to (24), equation (12) to (25) and equations (14), (16) to (22) are equivalent to (26) to (33). Therefore, the optimal conditions for the optimisation problems are identical. Thus, when a general objective function with constraints is considered at the supervisory level, the supervisory controller modifies the control action of the fixed regulatory controller in such a way that the solution of general objective function with constraints is generated at the regulatory level. However, necessary assumption here is that the optimal solution exists and is stable.

3. APPLICATION TO THE THERMAL POWER PLANT BOILER SIMULATOR

3.1 Problem statement

In this work, the objective function considers both an economic and a regulatory level objective. That is, the minimisation of the operational total costs (JCf) and the minimisation of the set-point trajectory error together with the control action effort (JCr). Then, the total objective function is:

\[ J = J_{CF} + J_{CR} \]

Also, considering the fuel flow to the boiler \( w_f \) and the feedwater flow \( w_e \) as the main process supplies, the economic objective function (JCf) is given by:

\[ J_{CF} = \sum_{i=1}^{N} C_{w}(t + i - 1) + \sum_{i=1}^{N} C_{w}(t + i - 1) + C_{F} \]

\[ \text{with } C_{w} \text{ are the fuel costs, } C_{F} \text{ the feedwater supply cost and } C_{F} \text{ fixed costs. } N \text{ is the prediction horizon.} \]

The regulatory level objective (JCr) is given by:

\[ J_{CR} = C_{ps} \left( \sum_{i=1}^{N} \left( \hat{P}_s(t + j) - p_s^* \right)^2 + \lambda_{ps} \sum_{i=1}^{N} \Delta w_s(t + i - 1) \right) + C_{rl} \left( \sum_{i=1}^{N} \left( \hat{L}(t + j) - L^* \right)^2 + \lambda_{we} \sum_{i=1}^{N} \Delta w_e(t + i - 1) \right) \]

where \( \hat{P}_s(t + j) \) is the j-step ahead prediction for the superheated steam pressure \( p_s \) and \( \hat{L}(t + j) \) is the j-step ahead prediction for the drum level. Also, \( C_{ps} \) and \( C_{rl} \) are the cost factors of the regulatory level, chosen by a trade-off criteria. \( \lambda_{ps} \) and \( \lambda_{we} \) are the control weightings, selected by a regulatory criteria. The external set-point trajectories \( p_s^* \) and \( L^* \) for the superheated steam pressure and the drum level respectively are constant and previously fixed by an operational criteria. Considering the objective function defined in equation (3), the corresponding variables and parameters of the objective function defined by equation (39), are:

\[ y(t) = [p_s(t) \ L(t)]^T, \quad u(t) = [w_f(t) \ w_e(t)]^T, \quad \psi_{\mu}^\lambda = \begin{bmatrix} C_{ps} & 0 \\ 0 & C_{rl} \end{bmatrix}, \quad \psi_{\lambda} = \begin{bmatrix} C_{ps} \lambda_{ps} & 0 \\ 0 & C_{rl} \lambda_{we} \end{bmatrix}, \quad \xi_{\mu}^\lambda = \begin{bmatrix} 2C_{ps} p_s^* & 2C_{rl} L^* \end{bmatrix}, \quad \xi_{\lambda} = \begin{bmatrix} C_{f} \ C_{e} \end{bmatrix} \]

Also, \( \psi_{\mu}^\lambda = 0, \psi_{\lambda} = 0, \xi_{\mu}^\lambda = 0 \).

Next, two solution algorithms are proposed in order to solve the optimisation problem.
3.2 Supervisory controller

In the proposed supervisory control strategy, all the PI controllers are considered. The economic optimiser shown in Figure 2 will provide the optimum set-points for the regulatory level. The optimisation variables proposed here are the set-points of the superheated steam pressure $p_s^*$, the drum water level $L^*$, and the mean value of superheated steam flow $w_s$, as they directly depend on the main process inputs: the fuel flow to the boiler $w_f$, and the feedwater flow $w_e$, which are included in the objective function proposed. The set-points $T_i^*$ will be constant as they do not affect the economic optimiser.

The supervisory control strategy minimises the objective function defined in the equation (39), giving the optimum set-points for the PI controllers. The process models (equations (42) and (43)) and the PI controllers models (equations (44) and (45)) are considered as constraints. The optimum set-points are calculated by numerically solving the optimisation problem (K-T conditions of section 2.2).

Fig. 2. Supervisory control strategy for the boiler

3.3 Regulatory controller

The regulatory control strategy determines directly the optimal control actions of the process. As Figure 3 shows, the economic optimiser provides the optimum control actions for the boiler. The optimisation variables proposed here are the fuel flow to the boiler $w_f$ and the feedwater flow $w_e$ as the optimiser directly depend on those variables. In this case, the PI controllers for the superheated steam pressure and for the drum level are replaced by the proposed regulatory level.

The regulatory control strategy minimises the objective function defined in the equation (39), giving the optimal control actions. The process models (equation (42) and (43)) are considered as constraints. The optimum control actions are calculated by numerically solving the optimisation problem (K-T conditions of section 2.3).

Fig. 3. Regulatory control strategy for the boiler

3.4 Comparative analysis

We assume that the superheated steam flow ($w_s$) changes, corresponding to changes in power produced by the steam turbine, and, this is treated as a disturbance. The proposed supervisory and regulatory controllers are compared with a control strategy in which the optimum set-points are constant and calculated from a static optimisation of the objective function defined in equation (40). Figures 4 and 5 show the performance of the system with the constant set-points, regulatory and supervisory controllers. The fuel flow to the boiler decreases when using the optimal control strategies proposed, while the feedwater flow remains similar to the control strategy with constant set-points. This is caused by the higher value of the fuel cost coefficient ($C_f$) compared with the feedwater supply cost coefficient ($C_e$) in equation (39).

Both controllers give a similar performance (Theorem). This is because the supervisory control strategy eliminates the action of the low level PI controller and the algorithm implicitly replaces the PI controller by the predictive control action given by the regulatory objective function (equation (41)). Also, the regulatory control strategy directly provides the control action based on the same regulatory objective function.

In Table 1, the mean values of the objective functions (equations (40), (41)) are evaluated using the data presented in the Figures 4 and 5. Also, the savings
for the fuel costs comparing with the control strategy with constant set-point are defined by:

\[
\text{Savings} = \left( 100 - 100 \frac{J_{Cr} \text{ with supervisory/ regulatory level}}{J_{Cr} \text{ with constant set-points}} \right) \% \quad (46)
\]

The savings obtained with the supervisory and regulatory control strategies are around 4%. The performances of the two proposed controllers are equivalent as previously explained (Theorem). The minimal differences are due to numerical accuracy.

5. CONCLUSIONS AND DISCUSSION

In this paper, optimality conditions were derived for a supervisory optimal controller, based on the theory of predictive control and a regulatory predictive controller. The optimality conditions were based on Lagrange theory with Kuhn –Tucker conditions, for system with constraints. Both proposed controllers are based on the same general objective function that can represent regulatory criterion, economic criterion, or others, and operational constraints are considered.

In the theorem, the equivalence between the proposed supervisory and regulatory controllers was established. Also, it was mentioned that the supervisory control cannot make up for poor lower loop if for example non-minimum phase controller is considered at regulatory level.

The theorem was applied to the economic optimisation of the boiler simulator of the combined cycle power plant, obtaining equivalent performance with supervisory and regulatory controllers.

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