LFT-BASED OPTIMAL ROBUST STABILIZATION OF PLANTS WITH USE OF H_∞-BOUNDED NLC FACTOR UNCERTAINTY DESCRIPTION

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Abstract— Optimal robust stabilization of a family of linear systems is explicitly considered. The family is characterized by H_∞ bounded perturbations, which is created with the use of NLCF (Normalized Left Coprime Factorization) factors. The problem is first analyzed using LFT (Linear Fractional Transformations); it is then proved that based on the LFT the optimal H_∞ index used in the past is identical to nominal performance criterion. Finally, based on the LFT an optimal H_∞ index is derived for the robust stabilization problem. The index is derived as a standard 2×2 block optimal H_∞ problem.

Keywords: Uncertainty, Robust Control, NLCF, Stabilization, LFT, H_infinity.

1. INTRODUCTION

One can never be able to construct a model that includes the physical plant. Often there are some characteristics which can not easily be modeled. Furthermore phenomenon like noise and disturbance are unavoidable. Therefore robustness to modeling inaccuracy and disturbances is a vital character for any controller design.

The analysis and design of robust controllers have attracted large number of researchers for more than a decade. For SISO controller design, robustness is achieved by ensuring good gain and phase margins. Gain and phase margins are no longer adequate for multivariable control systems. Bad robustness of linear quadratic Gaussian (LQG) controllers (Doyle, 1978) is another cause of necessity of multivariable robust control theory development. In this regard, an important step was taken by Zames (1981).

The H_∞ synthesis, which is a fundamental tool for robust controller design, was a rather difficult problem until the advent of two-Riccati-equation method (Doyle, et al., 1989). Then robust design became easier to use and found its way into many new applications.

The main advantage of robust control is the fact that uncertainty becomes important. The model uncertainty has been categorized into two main groups: structured and unstructured. In 1981 Doyle and Safonov independently introduced similar concepts for systems with special kind of structured uncertainty. Doyle called it structured singular value or µ (Doyle, 1982), while Safonov called it stability margins of diagonally perturbed systems (Safonov, 1982).

Unstructured uncertainty in a process is that there is no information available except upper-bound on its magnitude as a function of frequency. In optimal H_∞-design, it is necessary to model plant uncertainty as a separated transfer function/matrix from the nominal plant model. There are several descriptions of model uncertainty, and the differences between the descriptions depend on the specific applications under consideration. Many descriptions of uncertainty and its characteristics have been described in (Sanchez-Pena and Sznaier, 1998). Two common descriptions for model uncertainty are additive and multiplicative.

The problem of robust stabilization of plants with use of a specific kind of unstructured uncertainty description has been solved in (Glover and McFarlane, 1989). It is shown that the coprime factors of the controller can be generated directly from the normalized coprime factors of the plant as a
Nehari extension problem. This controller must be applied to a well shaped plant. A classical loop shaping procedure was used to specify performance in presence of this controller for a VSTOL aircraft (Glover and Hyde, 1993).

The robust performance problem must be solved to achieve performance in an automatic procedure as an $\text{H}_\infty$ optimization problem. It is possible to analyze the problem in a standard structure by using LFT. It is necessary to solve the robust stabilization problem before following the robust performance problem and this is the major contribution of this paper. The LFT structure obtained from robust stabilization is used in robust performance problem. In order to solve robust performance problem, with use of LFT structure, a new criterion is obtained for robust stabilization. This is the main concern of this paper.

The rest of the paper is organized as follows. The basic concepts are briefly reviewed in section 2. In section 3, the optimal nominal performance criterion will be derived for regulator systems with usage of the LFT. The result obtained shows that the criterion used in (Glover and McFarlane, 1989) is identical to the nominal performance criterion of the system. In section 4, uncertainty modeling is described by NLCF factors. The optimal robust stability criterion for the family is proposed through LFT concepts in section 5. Conclusion will be declared in section 6.

2. BASIC CONCEPTS

In order to get familiar with principal concepts used in this paper, first describe NLCF factors then the LFT structure and its characteristics are covered.

2.1. Normalized Left Coprime Factorization

(Glover and McFarlane, 1989)

A state space realization of system $G = C(sI - A)^{-1} B + D$ is denoted by (1).

$G = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$ (1)

Definition 1: Matrices $\tilde{M}, \tilde{N} \in \text{RH}_\infty$ constitute a left coprime factorization (LCF) of $G$ iff
a) $\tilde{M}$ is square, and nonsingular.
b) $G = \tilde{M}^{-1}\tilde{N}$
c) There exist $V, U \in \text{RH}_\infty$ such that

$\tilde{M}V + \tilde{N}U = I$ (2)

Notice that these factors are not unique and $\text{RH}_\infty$ is real rational subspace of $\text{H}_\infty$.

Definition 2: A left coprime factorization of $G$ as defined above is normalized iff:

$\tilde{N}^* + \tilde{M}\tilde{M}^* = I$ (3)

That $N^*$ is $N^*(-s)$.

Definition 3: The generalized filtering algebraic Riccati equation (GFARE) is defined as:

$$(A - BD'R^{-1}C)Z + Z(A - BD'R^{-1}C) - ZC'R^{-1}CZ + B(I - D'R^{-1}D)B' = 0$$ (4)

$R = I + DD'$

Definition 4: The generalized control algebraic Riccati equation (GCARE) is defined as:

$$\begin{aligned}
(A - BS'D'C)X + X(A - BS'D'C) - XBS'BX + C'(I - DS'D)C &= 0 \\
S &= I + D'D
\end{aligned}$$ (5)

Lemma (Vidyasagar, 1988): Let $G = C(sI - A)^{-1} B + D$ with minimal realization $(A,B,C)$. Given $H$ and $R$

$$H = -(ZC^* + BD^*)R^{-1}$$ (6)

$$R = I + DD'$$

that $Z$ is the unique, positive definite solution to GFARE, then the following matrices represent a normalized left coprime factorization of $G$ i.e.,

$$\tilde{N} = R^{-1}C(sI - A - HC)'(B + HD) + R^{-1}D$$

$$\tilde{M} = R^{-1}C(sI - A - HC)'H$$ (7)

2.2. Linear Fractional Transformations

(Sanchez-Pena and Sznaier, 1998)

One of the usual structures used in robustness and performance analysis of a system is the LFT. This structure was used in digital control design. It is utilized in $\text{H}_\infty$ control theory because of its simplicity. With usage of this structure the robust stability and performance will clearly formulated. A general system in LFT structure is shown in Fig. 1.

This structure includes nominal model, uncertainty, controller, external signals and errors. Usually this structure is decomposed into two structures in use. In the Fig. 2, the nominal model and controller and in Fig. 3, the nominal model and uncertainty are decomposed. In these figures, the matrix $P$ is a 2×2 block that is as following:

$$P = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}$$ (8)

For the system shown in Fig. 2, may write:

$$\frac{e'}{d'} = P_{p_1}(s) + P_{p_2}(s)K(s)[I - P_{p_2}(s)K(s)]^{-1}P_{p_1}(s)$$ (9)

$$F_s(P(s), K(s))$$

Thus, for optimal $\text{H}_\infty$ minimization of $(e'/d')$ the following criterion is used.

$$\text{Min} \left\| F_s(P(s), K(s)) \right\|_{\infty}, K \in \text{RH}_\infty$$ (10)
This criterion will guarantee the robust stability and/or nominal performance by an appropriate selection of terminals given in Fig. 2. Another relation for the LFT structure, see Fig. 3, given below.

$$\frac{e'}{d'} = P_{ii}(s) + P_{ii}(s)\Delta(s)[I - P_{ii}(s)\Delta(s)]^{-1}P_{ii}(s) = F_i(P(s), \Delta(s))$$  \hspace{1cm} (11)

3. NOMINAL PERFORMANCE

In this section, the nominal performance criterion for a general system, as shown in Fig. 4, will be extracted by using LFT. It will be proven that the robust stability criterion, which was used in (Glover and McFarlane, 1989), is identical to the nominal performance criterion of the system. This index is shown in (12).

$$\min_{\Delta \in \mathcal{K}} \left| \begin{bmatrix} K & I \\ I & G \\ I & G \\ I & G \end{bmatrix} \left[ I - G K \right]^{-1} \tilde{M}^{-1} \right|_\infty$$  \hspace{1cm} (12)

Based on Fig. 4, a suitable LFT structure for solving the nominal performance problem is illustrated in Fig. 5. According to this figure the transformation matrix $P$ is obtained as follows:

$$P = \begin{bmatrix} K & KG & KG \\ I & G & G \\ I & G & G \end{bmatrix}$$  \hspace{1cm} (13)

From (9) and (10) the optimal nominal performance criterion for linear regulator system can be formulated as:

$$\min_{\Delta \in \mathcal{K}} \left| F_i(P(s), K(s)) \right|_\infty$$

$$F_i(P(s), K(s)) = P_{ii}(s) + P_{ii}(s)K(s)[I - P_{ii}(s)K(s)]^{-1}P_{ii}(s)$$

$$= \begin{bmatrix} K & KG & KG \\ I & G & G \\ I & G & G \end{bmatrix} \left[ I - G K \right]^{-1} [I \ G]$$  \hspace{1cm} (14)

For the system of Fig. 4, the output sensitivity functions are described by (15).

$$(I - GK)^{-1} = S_o$$

$$GK(I - GK)^{-1} = T_o$$  \hspace{1cm} (15)

Thus, in (14), $(I + GK(I - GK)^{-1})$ can be replaced with $(I - GK)^{-1}$ to obtain:

$$F_i(P(s), K(s)) = \left\| \begin{bmatrix} K \\ I \\ I \end{bmatrix} \left[ I - G K \right]^{-1} [I \ G] \right\|_\infty$$  \hspace{1cm} (16)

Because right multiplication of a co-inner function has no effect on $H_\infty$-norm, the following inequalities are identical.

$$\left\| \begin{bmatrix} K \\ I \end{bmatrix} \left[ I - G K \right]^{-1} \tilde{M} \right\|_\infty \leq \gamma \iff$$

$$\left\| \begin{bmatrix} K \end{bmatrix} \left[ I - G K \right]^{-1} \tilde{M} \tilde{N} \right\|_\infty \leq \gamma \iff$$

Therefore, the optimal nominal performance criterion becomes as:
Thus, based on the LFT, the index used in (Glover and McFarlane, 1989) as a robust stabilization criterion will guarantee the nominal performance of the system.

4. LCF DESCRIPTION OF MODEL UNCERTAINTY

One of the fundamental steps in solving $H_{\infty}$ problems is determination of model uncertainty for the system at hand. Uncertainties beside noise and disturbance are the factors affecting the stability and performance of systems.

In 1960's, the usual way to treat these phenomena was LQG method. In this method, uncertainty is exerted to the system as an external signal with white noise characteristics. Adequately the measurement noise can be modeled as white noise; uncertainty cannot be modeled in this way. The main problem of this method is in that LQG controllers can have bad robustness (Doyle, 1978).

In the optimal $H_{\infty}$ method, the uncertainty is modeled as a separated transfer matrix from nominal plant. One of the important factors in selection of the uncertainty model is that it must well express the uncertainty behavior in both low and high frequencies. Fig. 6 shows an appropriate structure exposing this property. The gain matrices $W_T$ and $W_S$ in Fig. 6 can be adjusted so that the satisfactory uncertainty behavior is obtained in both low and high frequencies. Use of NLCF factors ensures that these gain matrices will no conflict. It is shown in (Glover and Hyde, 1993) that this description is able to changes the number of unstable system poles.

In section 2.1, it shown that through NLCF, the system transfer matrix can be factorized as:

$$G = \tilde{M}^{-1}\tilde{N}$$

The open loop transfer matrix for the system of Fig. 6 becomes:

$$L = (1 - W_s\delta_s)^{-1}G(1 + W_s\delta_s)K$$
$$= (1 - W_s\delta_s)^{-1}\tilde{M}^{-1}\tilde{N}(1 + W_s\delta_s)K$$

In order to ensure consistency of gain matrices, the NLCF factors are used as follows:

$$W_T = \tilde{N}^{-1}$$
$$W_s = \tilde{M}^{-1}$$

Thus:

$$L = (\tilde{M} - \delta_s)^{-1}(\tilde{N} + \delta_s)K$$

Based on (22), the close loop family is shown in Fig. 7.

5. THE PROPOSED OF OPTIMAL ROBUST STABILIZATION

Based on the LFT concepts, the problem of optimal robust stabilization is considered. The appropriate terminals for construction of LFT structure are shown in Fig. 7. According to these terminals and the underlying problem, the LFT structure corresponding to Fig. 7, is shown in Fig. 8. From these figures, the transfer matrix $P$ becomes:

$$P = \begin{bmatrix}
K\tilde{M}^{-1} & K\tilde{M}^{-1} & K\tilde{M}^{-1}\tilde{N} \\
\tilde{M}^{-1} & \tilde{M}^{-1} & \tilde{M}^{-1}\tilde{N} \\
\tilde{M}^{-1} & \tilde{M}^{-1} & \tilde{M}^{-1}\tilde{N}
\end{bmatrix}$$

Based on the small gain theorem (Zhou, et al., 1996), the gain of $(z/w)$ must be minimized in order to achieve the maximum stability margin. Therefore, the optimal robust stability criterion for the system of Fig. 8 is:

$$\min_{K} \| F_1(P(s),K(s)) \|_{\infty}$$

Use (23) and (9), to obtain:

$$F_1(P(s),K(s)) = \begin{bmatrix}
K\tilde{M}^{-1} & K\tilde{M}^{-1} \\
\tilde{M}^{-1} & \tilde{M}^{-1}
\end{bmatrix} + \begin{bmatrix}
K\tilde{M}^{-1}\tilde{N} \\
\tilde{M}^{-1}\tilde{N}
\end{bmatrix}K(1 - GK)^{-1} \begin{bmatrix}
\tilde{M}^{-1} & \tilde{M}^{-1}
\end{bmatrix}$$

The criterion will be finally obtained by simplifying (25) as follows:
From (15) and some algebraic manipulations the criterion is rewritten as:

$$\text{Min}_{\text{Stab}} K \left[ \begin{bmatrix} K \\ I \end{bmatrix} I^{-1} + \begin{bmatrix} K \\ I \end{bmatrix} G (I - GK)^{-1} \begin{bmatrix} K \\ I \end{bmatrix} \right]_{\infty}$$ (26)

This expression is derived in such a way to fulfill a standard two-block $H_\infty$ problem. This problem was suboptimally solved in (Glover and McFarlane, 1989) as a Nehari extension problem. Because the $H_\infty$ norm is invariant under right multiplication by a co-inner function, the following inequalities are identical.

$$\left[ \begin{bmatrix} K \\ I \end{bmatrix} S_0 \bar{M}^{-1} \right]_{\infty} \leq \gamma \iff \left[ \begin{bmatrix} K \\ I \end{bmatrix} S_0 \bar{M}^{-1} [M^{-1} \bar{N}] \right]_{\infty} \leq \gamma$$ (28)

Therefore, the optimal robust stabilization criterion can be expressed as follows:

$$\text{Min}_{\text{Stab}} K \left[ \begin{bmatrix} KS_0 \\ S_0 \end{bmatrix} [I \ G] \right]_{\infty}$$ (29)

This is a standard 2×2-block optimal $H_\infty$ problem, which is suboptimally solved in (Doyle and Glover, 1989). Optimal solution of (27) and (29) obtained using iterative methods (Foo, 1984, Chu, et al., 1986).

6. CONCLUSION

In this paper, optimal robust stabilization of plants with the NLCF uncertainty description has been analyzed. This problem has been attacked previously in (Glover and McFarlane, 1989). There a particular index is used as robust stabilization criterion. It has been shown that based on the LFT, this criterion is identical to nominal performance criterion of the system. Finally, based on the LFT, an optimal $H_\infty$ index is derived for the robust stabilization problem. The index is derived so that it leads to a standard two-block $H_\infty$ problem. This problem was suboptimally solved in (Glover and McFarlane, 1989) as a Nehari extension problem. Furthermore, the index is formulated as a standard 2×2-block optimal $H_\infty$ problem (Doyle and Glover, 1989). Results of this paper would be used in optimal robust performance problem for systems with NLCF uncertainty description through LFT. This is the major contribution of this paper. The application of this method to some practical problem is under investigation and will be reported later.