MODEL-BASED PERFORMANCE ASSESSMENT FOR CONTROL OF MIMO PROCESSES

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Abstract: In this paper, methods are proposed to assess the performances between multivariable control (abbr. MVC) and multi-loop control (abbr. MLC) for disturbance rejection. Indices for assessment are defined, and, guidelines for selection between these two control structures are presented for easy application. Copyright © 2006 UKACC

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1. INTRODUCTION

Two control structures, MLC and MVC, are used to control chemical plants which have MIMO dynamics. It used to be considered that MVC systems have better control performance than MLC systems. But, due to the controller being aimed to invert the process dynamics, some limitation may lead MVC to perform worse than MLC. Methods such as RGA, RGD, RDGA and so on, have been proposed guide the selection of control structure (Chang, 1992; Skogestad, 1987; Hansen, 1998; Hovd, 1992; Pomerleau, 2001). Most of them merely use steady-state gains. Thus, they usually fail to take into accounts the effects of process dynamics. In addition, the performance competition can’t be directly presented in their method.

In this paper, systematic approaches are proposed to assess performances in terms of disturbance rejection of the two control structures to facilitate the selection of control structure between MLC and MVC. In MLC systems, an effective disturbance input (EDI) is defined. Based on the gain and dynamics of this EDI, the best achievable performance (abbr. BAP) of a given MLC can be estimated. In addition, the estimations of responses of MVC systems are determined by a simple determinant of the process transfer function matrix. Next, the indices are defined to assess the performances between MLC and MVC based on simple gains, ideal responses, achievable responses, and so on. Finally, some guidelines are applied suggestions of selecting a control structure.

2. EQUIVALENT DISTURBANCES AND LOOPS IN MLC SYSTEMS

The assessment using EDI is first illustrated using a 2-loop system, and, then, extends the results to systems with higher dimensions. Furthermore, a new design of multi-loop controllers is also presented.

2.1 Equivalent disturbances and loops for 2-loop systems.

Consider a 2×2 system of following:

\[ Y(s) = G(s)U(s) + G_L(s)L(s) \]  

where \( Y(s) \), \( U(s) \) and \( L(s) \) are outputs and inputs and a load, respectively. \( G(s) \) and \( G_L(s) \) are an

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open-loop transfer function matrices for manipulation input and disturbance. They are:

\[
G(s) = \begin{bmatrix}
g_{1t}(s) & g_{1d}(s) 
g_{2t}(s) & g_{2d}(s)
\end{bmatrix}
\]

(2)

and

\[
G_i(s) = \begin{bmatrix}
g_{i1}(s) 
g_{i2}(s)
\end{bmatrix}
\]

(3)

As shown in Fig. 1, when the second loop is closed, the input from \( L \) to \( y_1 \) has two transmission paths. The combination of the transfer functions through these two paths is considered as the effective disturbance dynamics of the first equivalent disturbance and is designated as \( g_{1t}(s) \). With this definition, the EDIs of a 2-loop system are given as:

\[
g_{L1,1} = g_{1t} - g_{1d}G_{i1}G_{i2}h_2
\]

\[
g_{L1,2} = g_{1d} - g_{2d}G_{i1}G_{i2}h_2
\]

\[
G_i = \begin{bmatrix}
g_{i1}(s) 
g_{i2}(s)
\end{bmatrix}
\]

(4)

where

\[
h_i = \frac{G_{i1}h_1}{1+G_{i2}h_2}, \quad i = 1, 2
\]

(5)

and \( g_{i1} \) is the controller of \( i \)th loop.

Similarly, the EOPs are written as:

\[
g_i = g_{i1} - g_{i2}G_{i1}G_{i2}h_2
\]

(6)

By a similar mathematical manipulation to the work of Huang et al. (2003), the EDIs for higher dimensional systems can be written as:

\[
g_{L1,1} = g_{1t} - G_{i1}G_{i2}h_2
\]

\[
G_i = \begin{bmatrix}
g_{i1}(s) 
g_{i2}(s)
\end{bmatrix}
\]

(7)

Notice that the matrix \( G(s) \) has been partitioned into the following:

\[
G(s) = \begin{bmatrix}
g_{1t}(s) & g_{1d}(s) 
g_{2t}(s) & g_{2d}(s)
\end{bmatrix}
\]

(8)

The matrix product designated by \( \otimes \) in Eq. (7) is known as the Hadamard product, which means element to element product of two matrices. The vector \( H \) consists of all complementary sensitivity functions \( h_i(s) \) except \( h_1(s) \). Meanwhile, \( \text{Cs} \{1\} = [1, 1, \ldots, 1]^T \). Similarly, other EDIs for \( i = 2, 3 \) can be derived in the same way after permuting the matrix to move \( g_{i1} \) to \( g_{i1} \) and \( g_{i2} \) to \( g_{i1} \), and defined the sub-systems of the permuted \( G(s) \) and \( G_i(s) \) accordingly. After the EDI is derived as above mentions, the EOP will also be used (Leitch and O’Reilly, 1991; Huang et al., 2003). That is:

\[
g_i = g_{i1} - G_{i1}G_{i2}h_2
\]

\[
G_i = \begin{bmatrix}
g_{i1}(s) 
g_{i2}(s)
\end{bmatrix}
\]

(9)

2.3 Controller synthesis for disturbance rejection

Referring to Fig. 2, \( g_{i1}(s) \) represents the counteract response generated by the system to compensate for the open-loop response from \( g_{i1}(s)l(s) \). Thus, the closed-loop transfer function for disturbance rejected becomes:

\[
y_i(s) = \frac{g_{i1}(s)}{l(s)} \quad \text{where} \quad y_i(s) = g_{i1}(s) - g_{i1}(s)
\]

(10)

We can assume that \( g_{i1}(s) \) consists of \( g_{i1}(s) \) and \( g_{i1}(s) \) in cascade. Thus, the closed-loop transfer function for disturbance rejected is rewritten as:

\[
y_i(s) = \frac{g_{i1}(s)}{l(s)} = g_{i1}(s)\left[1 - g_{i1}(s)\right]
\]

(11)

Where, \( g_i \) is the EOP of \( i \)th loop. Re-arranging the both sides give an expression for the feedback controller:

\[
g_{i1}(s) = \frac{g_{i1}(s)}{g_i(s)\left[1 - g_{i1}(s)\right]}
\]

(12)

The perfect control that can be achieved is to select \( g_{i1}(s) \) as one. By this, it means perfect predictive control, i.e. \( g_{i1}(s) = e^{\theta t}, \) is required. In practice, a reasonable \( g_{i1}(s) \) of first order or second order dynamics is assigned so that the peak gain of the resulting sensitive function denoted as \( M_S \) has a value in the range of 1.2-2.0 (Chen and Seborg, 2002). Where, \( M_S \) is:

\[
M_S = \max_{\omega} \left[\|S(j\omega)\|\right] = \max_{\omega} \left[\|1 - g_{i1}(j\omega)e^{-\theta t}\|\right]
\]

(14)

Where,

\[
S_i = \frac{1}{1 + g_i(s)} = 1 - g_{i1}(s) - g_{i1}(s)e^{-\theta t}
\]

(15)

In other words,

\[
h_i(s) = g_{i1}(s) = g_{i1}(s)e^{-\theta t}
\]

(16)

And

\[
g_{L1,1} = g_{1t} - G_{i1}G_{i2}h_2
\]

\[
G_i = \begin{bmatrix}
g_{i1}(s) 
g_{i2}(s)
\end{bmatrix}
\]

(17)

\[
g_i = g_{i1} - G_{i1}G_{i2}h_2 \quad \text{and} \quad G_i = \begin{bmatrix}
g_{i1}(s) 
g_{i2}(s)
\end{bmatrix}
\]

(18)

Where \( h_i(s) \), i.e. \( g_{i1}(s) \), is determined by using above methods for \( g_{L1,1} \) and \( g_i \) instead of \( g_{i1}(s) \) and \( g_{i1}(s) \) in a single loop system. In general, models of \( g_{L1,1} \) and \( g_i \) can be found by fitting their frequency responses. Then, the controller \( g_{i1}(s) \) can be synthesized by Eq. (12).
and are the gains of \( g_{l,i} \). The MVC has better control than \( G_s \), designated as \( G_s \). \( -\) can be also represented as:

\[
\frac{\partial y_i}{\partial l_i} = \frac{\partial y_i}{\partial l_i} \quad \text{all loops open}
\]

When \( \varepsilon_i^p > 1 \), the MVC has better control performance with \( n \)th loop. On the other hand, MLC is better. For the 2-loop system, \( \varepsilon_i^p < 1 \) implies

\[
\left| k_{i,1} \right| > \left| k_{i,1} - k_{i,2}/k_{i,2}k_{i,1} \right|
\]

The definition of \( \varepsilon_i^p \) in Eq. (20) also implies the following:

(i) when \( |\beta/\lambda| < 1 \), MLC is better than MVC.

(ii) On the other hand, MVC has better performance. Furthermore, if \( k_{i,1} \) has the same sign to \( k_{i,2}/k_{i,2}k_{i,1} \) and \( \left| k_{i,1} - k_{i,2}/k_{i,2}k_{i,1} \right| \) is smaller than \( 2|k_{i,1}| \), the loop interactions, \( \lambda \), far from one, benefit MLC systems.

4.2 Theoretical best achievable performance

The theoretical best achievable responses for disturbance rejection can be regarded as those as shown in Fig. 4, where \( \theta_{d,i} \) and \( \theta_{p,i} \) are the time delays associated with the equivalent disturbance and the equivalent \( n \)th loop, respectively. Initially, the controller acts at \( \theta_{d,i} \). Then, the disturbance can be rejected completely after the time when process starts to respond, \( \theta_{p,i} + \theta_{d,i} \). The similar responses can also be obtained by a 2-df Smith control (Wang, 2005). For MLC systems, \( \theta_{d,i} \), \( \theta_{p,i} \) equals to the dead time of \( g_{l,i} \), designated as \( \theta_{d,i} \). The responses for MLC systems can be represented as:
\[ y_{i,MLC}(t) = \begin{cases} 0 & t < 0_{Li} \\ \hat{g}_{i,i}(t)u(t) & 0_{Li} \leq t \leq (0_{Li} + \theta_{pi}) \\ 0 & t > (0_{Li} + \theta_{pi}) \end{cases} \]  
where \( \hat{g}_{i,i} \) is the model of the EDI in time domain for MLC systems. Assume that \( u(t) \) is a unit step change, a response in Eq. (22) is written as:

\[ y_{i}(t) = \hat{g}_{i,i}(t)u(t) - u\left(t - (0_{Li} + \theta_{pi})\right) \]  

The square of 2-norm of \( y_{i,MLC}(t) \) is defined as:

\[ \left\| y_{i,MLC}(t) \right\|_{2}^{2} = \int_{0_{Li}}^{\infty} \left[y_{i,MLC}(t)\right]^{2} dt \]  

By substituting Eq. (23) into Eq. (24), \( \left[y_{i,MLC}(t)\right]^{2} \) is rewritten as:

\[ \left[y_{i,MLC}(t)\right]^{2} = \int_{0_{Li}}^{\infty} \left[\hat{g}_{i,i}u(t)\right]^{2} dt \]  

Similarly, for MVC systems an ideal response \( y_{i,MVC}(t) \) is also represented as:

\[ y_{i}(t) = \hat{g}_{i,i}(t)u(t) - u\left(t - (0_{Li} + \theta_{pi})\right) \]  

where \( \hat{g}_{i,i}(t) \) is the time domain model of \( g_{i,i}(s) \), and \( \theta_{i,i} \) and \( \theta_{pi} \) are the delay times of \( g_{i,i} \) and \( q_{i} \), respectively. Then, the square of 2-norm of \( y_{i,MVC}(t) \) is given as:

\[ \left[y_{i,MVC}(t)\right]^{2} = \int_{0_{Li}}^{\infty} \left[\hat{g}_{i,i}u(t)\right]^{2} dt \]  

Then, a comparative index \( \varepsilon_{i}' \) of ideal responses for \( i \)th loop is defined as:

\[ \varepsilon_{i}' = \frac{\left[y_{i,MVC}(t)\right]^{2}}{\left[y_{i,MLC}(t)\right]^{2}} \]  

When \( \varepsilon_{i}' > 1 \), the MVC has better ideal control performance with \( i \)th loop. On the other hand, the ideal control performance of MLC is better.

### 4.3 Competitions for achievable and desired performances

A more practical control as shown in Fig. 5 is that the disturbance \( g_{d,i} \) is rejected by a reasonable dynamic \( g_{d,i}g_{d,i}^{-1} \). For example, we assume that \( g_{d,i} \) has the following transfer function model:

\[ g_{d,i}(s) = \frac{z_{i}s^{2} + z_{i}s + 1}{p_{i}s^{2} + p_{i}s + 1}e^{-\alpha_{i}t} \]  

In this design, the practical achievable performances or system robustness is considered. On the other hand, a predictive control isn’t used. By selecting an adequate \( g_{d,i} \), the system is designed to reach the desired robustness or practical achievable responses which satisfy the performance criteria (e.g. the 2-norm of errors).

**Desired responses:** The desired responses that achieve the certain system robustness in Eq. (14) can be represented as:

\[ y_{d,i}(s) = \frac{\left[ g_{d,i}(s) \right]^{-1}(1 - g_{d,i}(s))}{\left[ g_{d,i}(s) \right]^{-1}(1 - g_{d,i}(s))}l(s) \]  

\[ y_{d,i} \] is so called as a desired response. For MLC systems, \( y_{d,i} \) is rewritten as \( y_{d,i,MLC} \) and \( y_{d,i} \) is re-designated as \( g_{d,i} \). Similarly, \( y_{d,i,MVC} \) and \( g_{d,i} \) are defined in the same way for MVC systems. Then, an index used to compare desired control performance with each other for MLC and MVC systems in \( i \)th loop is defined as:

\[ \varepsilon_{i}^{a} = \frac{\left[y_{d,i,MVC}(t)\right]^{2}}{\left[y_{d,i,MLC}(t)\right]^{2}} \]  

Then, a comparative index \( \varepsilon_{i}' \) of ideal responses for \( i \)th loop is defined as:

\[ \varepsilon_{i}' = \frac{\left[y_{d,i,MVC}(t)\right]^{2}}{\left[y_{d,i,MLC}(t)\right]^{2}} \]  

When \( \varepsilon_{i}' > 1 \), the MVC has better ideal control performance with \( i \)th loop. On the other hand, the ideal control performance of MLC is better.

**Achievable responses:** For MVC systems, the achievable performance can be obtained by minimizing the 2-norm of responses of each individual loop. But, as shown in Fig. 6, in MLC systems each performance interacts in each loop. In the following, we illustrate a 2-loop case to easy explanation. First, we defined the total objective as:

\[ E = W_{1}E_{1} + W_{2}E_{2} \]  

Where \( W_{i} \) is a weighting factor and \( E_{i} \) is the 2-norm of errors in \( i \)th loop. Then, responses which are used to achieve the best performance are represented as:

\[ y_{d,i} = \left[ g_{d,i}^{-1}g_{d,i,MLC}(s) \right]^{-1}l(s) \]  

and

\[ y_{d,i} = \left[ g_{d,i}^{-1}g_{d,i,MVC}(s) \right]^{-1}l(s) \]  

So \( g_{d,i}(s) \) and \( g_{d,i,MLC}(s) \) can be determined to satisfy the performance criteria in Eq. (32). Similarly, the index \( \varepsilon_{i}^{a} \) which represents the competition of achievable responses in \( i \)th loop is defined as the ratio of \( \left[y_{d,i}^{MLC}\right]^{2} \) to \( \left[y_{d,i}^{MLC}\right]^{2} \). Similarly, the desired and achievable responses in MLC systems are better for \( \varepsilon_{i}' > 1 \) and \( \varepsilon_{i}^{a} > 1 \), respectively.
4.4 Competitions for resulting responses by rough and precise estimation

Although the competitions between MLC and MVC for extreme ideal, achievable, and general desired responses have been proposed to give the suggestion of selecting control structure, the specific situations in controller design may confuse engineers when they practically implement in real processes. The more closed to actual responses so called as apparent responses are applied here and a simple $2 \times 2$ process is used in order to explain conveniently.

In a $2 \times 2$ system, a rough and a precise approximations of the control system are shown in Fig. 7 and Fig. 8, respectively. A $[g_{MLC}]^p$ is obtained by the simplified procedures that the controller is synthesized for $g_{i}(s)$ and the delay time of $g_{i}(s)$. Similarly, $[g_{MLC}]^p$ is determined for more complex $g_{i}$ and $\theta_{i}$ but the results are more accurate.

In Fig. 8, the transfer functions from $l$ to $y_1$ and $y_2$ are derived as:

$$H_{P,1}^{MLC} = \frac{g_{i1}(1-\Lambda_{11}) + g_{i2} g_{21} \Lambda_{12}}{\Delta}$$

$$H_{P,2}^{MLC} = \frac{g_{i2}(1-\Lambda_{11}) + g_{i1} g_{21} \Lambda_{12}}{\Delta}$$

where

$$\Lambda_{11} = -\frac{g_{11} g_{21} \rho_{d,1} g_{d,1}}{g_{i1} (1- \rho_{d,1})}$$

$$\Lambda_{12} = -\frac{g_{12} g_{22} \rho_{d,1} g_{d,2}}{g_{i2} (1- \rho_{d,2})}$$

$$\Delta = 1 - \Lambda_{11} - \Lambda_{12} - \Lambda_{13} + \Lambda_{11} \Lambda_{12}$$

Furthermore, in Fig. 6 the transfer functions from $l$ to $y_1$ and $y_2$, $H_{R,1}^{MLC}$ and $H_{R,2}^{MLC}$, are derived in the same way except that $g_{i,j}$ and $[g_{MLC}]^p$ are used instead of $g_{i}$ and $[g_{MLC}]^p$ in Eqs. (38)-(40). For higher dimensional systems (e.g. $m \times m$ processes), the transfer function matrices from disturbances to outputs in Fig. 6 and Fig. 7 are derived as:

$$H_{R,1}^{MLC} = (I + L_{R})^{-1} G_L$$

$$H_{R,2}^{MLC} = (I + L_{R})^{-1} G_L$$

where

$$L_{R}(i,j) = \frac{g_{i,j} [g_{MLC}]^p}{g_{i,j} - [g_{MLC}]^p} \forall i, j \in m$$

In both of cases, 2-norms of $y_i$ for unit step load input are calculated by:

$$\|y_{R,1}^{MLC}\|_2 = \frac{1}{2\pi} \int_{0}^{\pi} \|H_{R,1}^{MLC}(j\omega)\|_{2} \frac{1}{j\omega} d\omega$$

$$\|y_{R,2}^{MLC}\|_2 = \frac{1}{2\pi} \int_{0}^{\pi} \|H_{R,2}^{MLC}(j\omega)\|_{2} \frac{1}{j\omega} d\omega$$

Furthermore, for MVC systems both transfer functions in two cases from $l$ to $y$, are defined as:

$$H_{R,1}^{MVC} = g_{i,j} \left(1 - [g_{MLC}]^p\right)$$

$$H_{R,2}^{MVC} = g_{i,j} \left(1 - [g_{MLC}]^p\right)$$

where $[g_{MLC}]^p$ is found for $g_{i,j}$ and the dead time of $g_{i,j}$, and $[g_{MLC}]^p$ is determined for $g_{i,j}$ and $\theta_{i}$.

Then, in MVC systems 2-norms of $y_i$ for unit step load input are also calculated by:

$$\|y_{R,1}^{MVC}\|_2 = \frac{1}{2\pi} \int_{0}^{\pi} \|H_{R,1}^{MVC}(j\omega)\|_{2} \frac{1}{j\omega} d\omega$$

$$\|y_{R,2}^{MVC}\|_2 = \frac{1}{2\pi} \int_{0}^{\pi} \|H_{R,2}^{MVC}(j\omega)\|_{2} \frac{1}{j\omega} d\omega$$

Then, the comparative indices for $i$th loop in both cases are defined as:

$$e_i^R = \frac{\|y_{R,i}^{MLC}\|_2}{\|y_{R,i}^{MVC}\|_2}$$

$$e_i^R = \frac{\|y_{R,i}^{MLC}\|_2}{\|y_{R,i}^{MVC}\|_2}$$

As earlier mentions, a MVC structure is suggested when the index is greater than one, or a MLC structure is recommended.
5. AN ILLUSTRATIVE EXAMPLE

Consider the well-known Wood and Berry (WB) process (1973). The transfer function matrices of this process are given as follows:

\[ G(s) = \begin{bmatrix} 12.8e^{-3s} & -18.9e^{-3s} \\ 16.7s + 1 & 21s + 1 \\ 6.6e^{-3s} & -19.4e^{-3s} \\ 10.9s + 1 & 14.4s + 1 \end{bmatrix} \]

(50)

\[ G_c(s) = \begin{bmatrix} 3.8e^{-3s} \\ 14.9s + 1 \\ 4.9e^{-3s} \\ 13.2s + 1 \end{bmatrix} \]

(51)

First, the proposed indices are calculated by the above definitions and the results are given as:

\[ e_{1}^R = 0.26, e_{2}^R = 0.6, e_{1}^I = 0.75, e_{2}^I = 1.0, e_{3}^I = 1.04, e_{4}^I = 1.03, e_{5}^I = 0.434, e_{6}^I = 1.06, e_{7}^I = 0.487, e_{8}^I = 1.02, e_{9}^I = 0.697 \text{ and } e_{10}^I = 1.05. \]

It is shown that the MLC is better in first loop and they have the similar results in another loop. Then, we simulate the process for both MVC and MLC systems according to the proposed design methods at \( Ms_0 = Ms_1 = 1.6 \). The resulting controllers with reduced PID form are determined as:

\[ g_{\text{MLC}}^{M} = 0.323 \frac{(1.525s^2 + 2.475s + 1)}{(s(0.01s + 1))}; \]

\[ g_{\text{MLC}}^{I} = -0.0276 \frac{(12.29s^2 + 5.66s + 1)}{(s(0.03s + 1))}; \]

\[ g_{\text{MVC}}^{M} = -0.026 \frac{(1.295s^2 + 3.40s + 1)}{(s(0.01s + 1))}; \]

\[ g_{\text{MVC}}^{I} = -0.00377 \frac{(9.405s^2 + 8.11s + 1)}{(s(0.03s + 1))} \]

(52)

The simulating responses and their individual ISE values are all shown in Fig. 9. From the results, we can find that the proposed estimations can give us a previous suggestion to select the control structure. But \( e_{n}^R \) have the much larger error to estimate the performance competitions due to the more assumptions. Moreover, for MLC systems the achievable performances improve slightly in second loop but damage seriously in first loop. The reason is that the error in second loop is relatively larger than another.

6. CONCLUSIONS

In this paper, an effective disturbance input (EDI) transmission from \( l \) to \( y \) is defined and is formulated to decompose the disturbances of a multi-loop system into a number of equivalent disturbances. By using a proposed controller synthesis to approximate the dynamics of single loops (i.e. \( h_{j}(s) \)), EDIs and EOPs are formulated to bypass the needs for controller dynamics in the other loops. Then, in disturbance rejection the design for multi-loop controllers for as the design of equivalent disturbances and loops is also proposed. Then, the competitive indices are defined and are determined which one has the better performance. Then, these results give the guideline to select a control structure. The simulation results are also applied to show that the proposed methods are effectively used to determine the performance competitions and to select the control structure.

REFERENCES


