NON-MINIMAL MODEL PREDICTIVE CONTROL WITH AN INTEGRAL-OF-ERROR STATE VARIABLE

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Abstract: This paper considers Model Predictive Control (MPC) using a Non-Minimal State Space (NMSS) form, in which the state vector consists only of directly measured system variables. Compared to previous research on NMSS/MPC, the new approach includes an explicit *integral-of-error* state variable to preserve steady state tracking of the reference input. The paper suggests that tuning techniques previously developed for linear Proportional-Integral-Plus (PIP) control can now be used for straightforward NMSS/MPC design.

Keywords: control system design; model predictive control; non-minimal state space; constraints; optimal control; digital control

1. INTRODUCTION

Model Predictive Control (MPC) is a widely used control technique with numerous applications, especially in the chemical engineering industry (Morari and Lee, 1999). Despite the relatively high online computational burden, the inherent handling of constraints has made it an attractive method in process applications, where constraint handling is vital for the success of the control design. Ongoing research in this field is largely focused on reducing the online computational load, while maintaining the desirable closed-loop performance and robustness properties of the controller (Bemporad *et al.*, 2002; Bacic *et al.*, 2003; Imsland *et al.*, 2005).

Here, the dependence of the controller on the estimated model of the system has particular importance. In this regard, (Wang and Young, 2005) recently proposed an approach to MPC based on the definition of a Non-Minimal State Space (NMSS) model. This approach eliminates the need for an observer such as a deterministic state reconstructor or a stochastic Kalman filter.

Such NMSS models have previously been used for linear (fixed gain) Proportional-Integral-Plus (PIP) control system design (Young *et al.*, 1987; Taylor *et al.*, 2000). For NMSS/PIP control, the state vector consists only of the present and past values of the output variable, past values of the input variable and an integral-of error state variable, introduced to ensure type one servomechanism performance. All these state variables are directly measurable, making the controller potentially less sensitive to the problems of model mismatch than the minimal state equivalent using an observer.

Such NMSS/PIP control algorithms have been utilised in numerous applications; e.g. (Taylor *et al.*, 2004; Taylor and Shaban, 2006). However, in all these cases, any system and actuator constraints have been handled in a rather *ad-hoc* manner. For example, the control law is always implemented in an incremental form to avoid the problem of 'integral wind-up' caused by input saturations. By contrast, (Wang and Young, 2005) develop an approach for inherent handling of system and actuator constraints for NMSS models using MPC methods. Furthermore, they show that the NMSS/MPC approach leads to improved performance in comparison to conventional 'minimal' MPC with an observer.

In order to ensure steady state tracking, (Wang and Young, 2005) define a state vector consisting of the *differences* of past output and input values, a very common approach in chemical and process control engineering. However, since a number of useful tuning techniques have been developed for NMSS/PIP design, including multi-objective optimisation of the linear quadratic cost function weights (Chotai *et al.*, 1998), the present paper evaluates MPC using this alternative framework.

In this regard, Section 2 of the paper develops a new approach for MPC based on the standard NMSS form (Young *et al.*, 1987), where type one servomechanism performance is obtained by the introduction of an explicit integral-of-error state variable. Section 3 proves the required steady state tracking and disturbance rejection properties. This is followed in Sections 4 and 5 by simulation examples and the conclusions, respectively.

2. MPC FORMULATION

2.1 System Description

Let the system be described by the following discrete time transfer function:

$$y(k) = \frac{B(z^{-1})}{A(z^{-1})}u(k) \tag{1}$$

where $u(k)$ and $y(k)$ are the system input and output respectively, and the polynomials,

$$\begin{aligned} B(z^{-1}) &= b_1 z^{-1} + b_2 z^{-2} + \cdots + b_m z^{-m} \\ A(z^{-1}) &= 1 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_n z^{-n} \end{aligned}$$

are represented in terms of the backward shift operator z^{-1}, i.e. $z^{-1}y(k) = y(k-1)$. For brevity, only the Single-Input, Single-Output (SISO) model will be considered here. However, these results have been extended to the more general case of Multi-Input, Multi-Output (MIMO) models and this will be reported in a future publication. Equation (1) is written in a Non-Minimal State Space (NMSS) form, as described by e.g. (Young *et al.*, 1987; Taylor *et al.*, 2000):

$$\begin{aligned} \boldsymbol{x}(k) &= \boldsymbol{A}\boldsymbol{x}(k-1) + \boldsymbol{b}u(k-1) + \boldsymbol{d}y_d(k) \\ y(k) &= \boldsymbol{c}\boldsymbol{x}(k) \end{aligned} \tag{2}$$

where the state vector is defined as follows,

$$\begin{aligned} \boldsymbol{x}^T(k) = [&y(k) \;\; \cdots \;\; y(k-n+1) \\ &u(k-1) \;\; \cdots \;\; u(k-m+1) \;\; z(k)] \end{aligned}$$

and $z(k) = z(k-1) + y_d(k) - y(k)$ is the *integral-of-error* state variable. Here $y_d(k)$ is the reference level (or set-point) at sample k. The various matrices and vectors in equation (2) are:

$$\boldsymbol{A} = \begin{bmatrix} -a_1 & \cdots & -a_n & b_2 & \cdots & b_{m-1} & b_m & 0 \\ 1 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & \cdots & 0 & 1 & \cdots & 0 & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & \cdots & 0 & 0 & \cdots & 1 & 0 & 0 \\ a_1 & \cdots & a_n & -b_2 & \cdots & -b_{m-1} & -b_m & 1 \end{bmatrix}$$

$$\begin{aligned} \boldsymbol{b} &= [b_1 \;\; 0 \;\; \cdots \;\; 0 \;\; 1 \;\; 0 \;\; \cdots \;\; 0 \;\; -b_1]^T \\ \boldsymbol{d} &= [0 \;\; \cdots \;\; 0 \;\; 1]^T \\ \boldsymbol{c} &= [1 \;\; 0 \;\; \cdots \;\; 0] \end{aligned}$$

2.2 Control Description

In the following, N_p is the number of samples ahead for which the output is predicted, i.e. the prediction horizon; and N_c the number of samples ahead that the input signal is predicted, i.e. the control horizon. In general $N_c < N_p$ and the control signal is considered to be zero after the control horizon, i.e. $u(k+i) = 0$, $i = N_c, \ldots, N_p$. The future (predicted) state vector is defined as,

$$\boldsymbol{X} = \left[\boldsymbol{x}^T(k+1|k) \; \boldsymbol{x}^T(k+2|k) \; \cdots \; \boldsymbol{x}^T(k+N_p|k)\right]^T$$

where $\boldsymbol{x}(k+i|k)$ is the i-step ahead prediction of the state vector $\boldsymbol{x}$ at sampling instant k, while the future control trajectory vector is defined as,

$$\boldsymbol{U} = [u(k) \;\; u(k+1) \;\; \cdots \;\; u(k+N_c-1)]^T$$

where $u(k+i)$, $i = 0, \ldots, N_c - 1$ are the future control signal values to be optimised. The equation that associates $\boldsymbol{X}$ and $\boldsymbol{U}$ is obtained from manipulation of the state equation (2):

$$\boldsymbol{X} = \boldsymbol{F}\boldsymbol{x}(k) + \boldsymbol{\Phi}\boldsymbol{U} + \boldsymbol{H}\boldsymbol{S}$$

in which,

$$\boldsymbol{F} = \begin{bmatrix} \boldsymbol{A} \\ \boldsymbol{A}^2 \\ \vdots \\ \boldsymbol{A}^{N_p} \end{bmatrix}, \; \boldsymbol{H} = \begin{bmatrix} \boldsymbol{d} & \boldsymbol{0} & \cdots & \boldsymbol{0} \\ \boldsymbol{A}\boldsymbol{d} & \boldsymbol{d} & \cdots & \boldsymbol{0} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{A}^{N_p-1}\boldsymbol{d} & \boldsymbol{A}^{N_p-2}\boldsymbol{d} & \cdots & \boldsymbol{d} \end{bmatrix}$$

$$\boldsymbol{\Phi} = \begin{bmatrix} \boldsymbol{b} & \boldsymbol{0} & \cdots & \boldsymbol{0} \\ \boldsymbol{A}\boldsymbol{b} & \boldsymbol{b} & \cdots & \boldsymbol{0} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{A}^{N_p-1}\boldsymbol{b} & \boldsymbol{A}^{N_p-2}\boldsymbol{b} & \cdots & \boldsymbol{A}^{N_p-N_c}\boldsymbol{b} \end{bmatrix}$$

$$\boldsymbol{S} = [y_d(k+1) \;\; y_d(k+2|k) \;\; \cdots \;\; y_d(k+N_p|k)]^T$$

where $y_d(k+i)$ is the i-steps ahead reference signal. Similarly, the plant output prediction equation is derived by manipulation of the observation equation (2):

$$\boldsymbol{Y} = \bar{\boldsymbol{C}}\boldsymbol{X} = \bar{\boldsymbol{C}}\boldsymbol{F}\boldsymbol{x}(k) + \bar{\boldsymbol{C}}\boldsymbol{\Phi}\boldsymbol{U} + \bar{\boldsymbol{C}}\boldsymbol{H}\boldsymbol{S} \tag{3}$$

where,

$$\boldsymbol{Y} = [y(k+1|k) \;\; y(k+2|k) \;\; \cdots \;\; y(k+N_p|k)]^T$$

is the vector of predicted values of the output and $\bar{\boldsymbol{C}}$ is the following $(N_p) \times ((n+m)N_p)$ matrix,

$$\bar{\boldsymbol{C}} = \begin{bmatrix} \boldsymbol{c} & \boldsymbol{0} & \cdots & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{c} & \cdots & \boldsymbol{0} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{c} \end{bmatrix}$$

The controller is subsequently derived from numerical minimization of the following index at each sampling instant,

$$J(\boldsymbol{x}(k), y_d(k)) = \sum_{i=1}^{N_p} \boldsymbol{x}(k+i|k)^T \boldsymbol{Q} \boldsymbol{x}(k+i|k) + \sum_{i=0}^{Nc-1} ru(k+i|k)^2 \quad (4)$$

subject to the system constraints,

$$\begin{cases} \underline{u} \leq u \leq \overline{u} \\ \underline{\Delta u} \leq \Delta u \leq \overline{\Delta u} \\ \underline{y} \leq y \leq \overline{y} \end{cases}$$

where the notation $\underline{\cdot}$ and $\overline{\cdot}$ is used to represent the lower and upper allowed value of a variable, $\boldsymbol{Q}$ is a positive definite square matrix of dimention $(n+m) \times (n+m)$ and r a positive constant. The cost function (4) is written in matrix form,

$$J_s = \boldsymbol{X}^T \bar{\boldsymbol{Q}} \boldsymbol{X} + \boldsymbol{U}^T \bar{\boldsymbol{R}} \boldsymbol{U}$$

where $\bar{\boldsymbol{Q}}$ and $\bar{\boldsymbol{R}}$ are positive definite matrices of dimensions $(n+m)N_p \times (n+m)N_p$ and $N_c \times N_c$, with the matrix $\boldsymbol{Q}$ and scalar r on their diagonals respectively, for which many algorithms for direct numerical solution exist, e.g. (Luenberger, 1973). The examples below utilise the quadprog function from the optimisation toolbox in Matlab®, although other options are clearly possible.

2.3 Constraints

Input constraints take the form $-\underline{\boldsymbol{U}} \leq \boldsymbol{U} \leq \overline{\boldsymbol{U}}$, where $\underline{\boldsymbol{U}}$ and $\overline{\boldsymbol{U}}$ are $Nc \times 1$ vectors of the minimum and the maximum allowed values for the control input signal. For the case of input rate-of-change, the constraints take the form,

$$\underline{\Delta \boldsymbol{U}} + \boldsymbol{C}_1 u_{\text{act}} \leq \boldsymbol{C}_2 \boldsymbol{U} \leq \overline{\Delta \boldsymbol{U}} + \boldsymbol{C}_1 u_{\text{act}}$$

where the $N_c \times 1$ vector $\boldsymbol{C}_1$ and the $N_c \times N_c$ matrix $\boldsymbol{C}_2$ are defined as follows,

$$\boldsymbol{C}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}, \; \boldsymbol{C}_2 = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 & 0 \\ -1 & 1 & 0 & \cdots & 0 & 0 \\ 0 & -1 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 & 0 \\ 0 & 0 & 0 & \cdots & -1 & 1 \end{bmatrix}$$

Here, $\underline{\Delta \boldsymbol{U}}$ and $\overline{\Delta \boldsymbol{U}}$ are $Nc \times 1$ vectors with the minimum and maximum values allowed for Δu and u_{act} is the current value of the input. Finally the output constraints are represented by,

$$-\bar{\boldsymbol{C}} \boldsymbol{\Phi} \boldsymbol{U} \leq -\underline{\boldsymbol{Y}} + \bar{\boldsymbol{C}} \boldsymbol{F} \boldsymbol{x}(k) + \bar{\boldsymbol{C}} \boldsymbol{H} \boldsymbol{S}$$
$$\bar{\boldsymbol{C}} \boldsymbol{\Phi} \boldsymbol{U} \leq \overline{\boldsymbol{Y}} - \bar{\boldsymbol{C}} \boldsymbol{F} \boldsymbol{x}(k) + \bar{\boldsymbol{C}} \boldsymbol{H} \boldsymbol{S}$$

where the $\underline{\boldsymbol{Y}}$ and $\overline{\boldsymbol{Y}}$ $N_p \times 1$ vectors consist of the minimum and maximum output values allowed. In this case, the control optimisation problem can be conveniently summarised as follows,

$$\begin{cases} \min \{J_s\} \\ s.t. \; \begin{bmatrix} \boldsymbol{M}_1 \\ \boldsymbol{M}_2 \\ \boldsymbol{M}_3 \end{bmatrix} \boldsymbol{U} \leq \begin{bmatrix} \boldsymbol{N}_1 \\ \boldsymbol{N}_2 \\ \boldsymbol{N}_3 \end{bmatrix} \end{cases}$$

where,

$$\boldsymbol{M}_1 = \begin{bmatrix} -\boldsymbol{I} \\ \boldsymbol{I} \end{bmatrix}, \; \boldsymbol{M}_2 = \begin{bmatrix} -\boldsymbol{C}_2 \\ \boldsymbol{C}_2 \end{bmatrix}, \; \boldsymbol{M}_3 = \begin{bmatrix} -\bar{\boldsymbol{C}} \boldsymbol{\Phi} \\ \bar{\boldsymbol{C}} \boldsymbol{\Phi} \end{bmatrix}$$

$$\boldsymbol{N}_1 = \begin{bmatrix} -\underline{\boldsymbol{U}} \\ \overline{\boldsymbol{U}} \end{bmatrix}, \; \boldsymbol{N}_2 = \begin{bmatrix} -\Delta \underline{\boldsymbol{U}} - \boldsymbol{C_1} u_{\text{act}} \\ \Delta \overline{\boldsymbol{U}} + \boldsymbol{C_1} u_{\text{act}} \end{bmatrix}$$

$$\boldsymbol{N}_3 = \begin{bmatrix} -\underline{\boldsymbol{Y}} + \bar{\boldsymbol{C}} \boldsymbol{F} x(k) + \bar{\boldsymbol{C}} \boldsymbol{H} \boldsymbol{S} \\ \overline{\boldsymbol{Y}} - \bar{\boldsymbol{C}} \boldsymbol{F} x(k) - \bar{\boldsymbol{C}} \boldsymbol{H} \boldsymbol{S} \end{bmatrix}$$

The conventional receding horizon approach is employed to implement the control algorithm. Here, only the first element of $\boldsymbol{U}$ is used as an input to the system, while the remaining elements are discarded. The optimization process is repeated at each control sample.

3. PROPERTIES OF THE CONTROL SCHEME

In the present section, the required steady state tracking of the reference input, along with the disturbance rejection properties of the proposed MPC formulation, will be presented for the *unconstrained* case. The unconstrained solution to the problem takes the form of a straightforward, fixed gain controller,

$$\boldsymbol{U} = -\boldsymbol{K_s} \boldsymbol{x}(k) - \boldsymbol{R_s} \boldsymbol{S} \quad (5)$$

where,

$$\boldsymbol{K_s} = \left(\boldsymbol{\Phi}^T \bar{\boldsymbol{Q}} \boldsymbol{\Phi} + \bar{\boldsymbol{R}} \right)^{-1} \boldsymbol{\Phi}^T \bar{\boldsymbol{Q}} \cdot \boldsymbol{F}$$

$$\boldsymbol{R_s} = \left(\boldsymbol{\Phi}^T \bar{\boldsymbol{Q}} \boldsymbol{\Phi} + \bar{\boldsymbol{R}} \right)^{-1} \boldsymbol{\Phi}^T \bar{\boldsymbol{Q}} \cdot \boldsymbol{H}$$

Letting $\boldsymbol{k_s^1} = \begin{bmatrix} k_1 & \cdots & k_n & k_{n+1} & \cdots & k_{n+m-1} & k_{n+m} \end{bmatrix}$ be the first row of $\boldsymbol{K_s}$ and $\boldsymbol{r_s^1} = \begin{bmatrix} r_1 & \cdots & r_{Np} \end{bmatrix}$ the first row of $\boldsymbol{R_s}$, the control law, i.e. the first element of $\boldsymbol{U}$ in equation (5), can be written as,

$$\begin{aligned} u_k = & -(k_1 y_k + k_2 y_{k-1} + \cdots + k_n y_{k-n+1}) \\ & -(k_{n+1} u_{k-1} + \cdots + k_{n+m-1} u_{k-m+1}) \\ & -(r_1 y_{d,k+1} + r_2 y_{d,k+2} \cdots + r_{Np} y_{d,k+Np}) \\ & -k_{n+m} z_k \end{aligned}$$

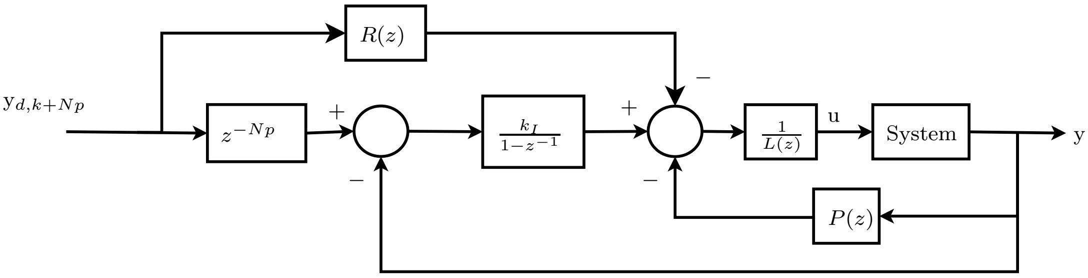

Fig. 1. Proposed MPC in block diagram form.

Defining the polynomials,

$$P(z) = k_1 + k_2 z^{-1} + \cdots + k_n z^{n-1}$$
$$L(z) = k_{n+1} z^{-1} + k_{n+2} z^{-2} + \cdots + k_{n+m-1} z^{m-1}$$
$$R(z) = r_{Np} + r_{Np-1} z^{-1} + \cdots + r_1 z^{-Np+1}$$

and substituting k_{m+n} with $-k_I$ and $z_k = \frac{y_{d,k} - y_k}{1-z^{-1}}$, the control law takes the form,

$$u_k = \frac{1}{L(z)} \left[\frac{k_I}{1-z^{-1}} (y_k - y_{d,k}) - P(z) y_k - R(z) y_{d,k+N_p} \right]$$

In this case, the block diagram representation of the closed-loop system is illustrated in Figure 1.

Letting the system be represented by $\frac{B(z)}{A(z)}$, the closed-loop transfer function can be found by manipulation of Figure 1:

$$T(z) = \frac{-(1-z^{-1}) R(z) B(z) - k_I B(z) z^{-Np}}{(1-z^{-1}) \left(A(z) L(z) + B(z) P(z) \right) - k_I B(z)}$$

for which $\lim_{z \to 1} T(z) = 1$. This means that the closed-loop will follow any constant reference input with no steady state error.

Similarly, the transfer function from a control input disturbance to the output variable is:

$$S_i(z) = \frac{(1-z^{-1}) B(z) L(z)}{(1-z^{-1}) \left(A(z) L(z) + B(z) P(z) \right) - k_I B(z)} \quad (6)$$

and from an output (load) disturbance to the output:

$$S_o(z) = \frac{(1-z^{-1}) A(z) L(z)}{(1-z^{-1}) \left(A(z) L(z) + B(z) P(z) \right) - k_I B(z)} \quad (7)$$

From equations (6) and (7) it follows that,

$$\lim_{z \to 1} S_i(z) = 0$$
$$\lim_{z \to 1} S_o(z) = 0$$

Hence, the closed-loop system rejects any constant input or output disturbances.

Remark 1. At the steady state, and since the desired output is in the feasible region, the solution to the quadratic program will be the one of the unconstrained case. Therefore the steady state tracking and disturbance rejection properties hold for the constrained case as well.

4. SIMULATION EXAMPLES

This section considers two simulation examples. The first example will illustrate the reference following and disturbance rejection properties of the controller, while the second example will compare the proposed NMSS/MPC controller with the approach of (Wang and Young, 2005).

4.1 Double integrator plant

Consider a continuous time double integrator plant. Using a sample time of $1s$, the discrete time transfer function is:

$$G(z^{-1}) = \frac{0.5z^{-1} + 0.5z^{-2}}{1 - 2z^{-1} + z^{-2}}$$

The non-minimal state vector is given by,

$$\boldsymbol{x}(k)^T = [y(k) \;\; y(k-1) \;\; u(k-1) \;\; z(k)]$$

while the system equations (2) are defined by,

$$\boldsymbol{A} = \begin{bmatrix} 2 & -1 & 0.5 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -2 & 1 & -0.5 & 1 \end{bmatrix}, \; \boldsymbol{b} = \begin{bmatrix} 0.5 \\ 0 \\ 1 \\ -0.5 \end{bmatrix}$$

$$\boldsymbol{d} = [0 \;\; 0 \;\; 0 \;\; 1]^T, \; \boldsymbol{c} = [1 \;\; 0 \;\; 0 \;\; 0]$$

The prediction and control horizons are chosen as $N_p = 15$ and $N_c = 3$ respectively. For the purposes of the present example, $\bar{\boldsymbol{Q}}$ and $\bar{\boldsymbol{R}}$ are assigned values of $\frac{5}{15}$ and $\frac{1}{3}$ on their diagonals, defined here as ratios between the 'weight' and the prediction or control horizon. The control objectives are to keep the output of the plant at the origin, despite the presence of input and output disturbances, while also satisfying the input saturation ($-10 \le u \le 10$) and rate of change ($-7 \le \Delta u \le 7$) constraints.

For results shown in Figure 2, a step output disturbance of magnitude 15 is applied to the system after 5 seconds from the start of the experiment, while a step input disturbance of -5 is applied after 20 seconds. It is clear that the controller rejects both disturbances, successfully maintaining the output signal close to the origin.

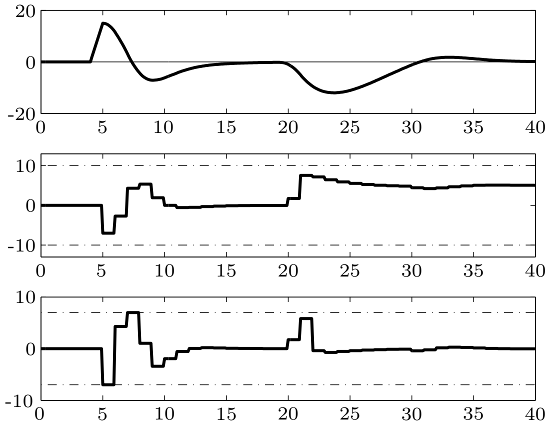

Fig. 2. NMSS/MPC applied to the double integrator plant. Top subplot: reference level and output. Middle subplot: control input (solid trace) and constraints (dash-dot); Lower subplot: input rate-of-change (solid) and constraints (dash-dot).

4.2 The IFAC '93 Benchmark

The IFAC '93 benchmark is a challenging problem that includes stringent closed-loop performance requirements, despite the fact that the plant parameters vary randomly within a certain range, defined by one of three stress levels. The control objective is to have a fast system response, with a bounded control signal, while the output should preferably stay within a soft output constraint and always within a hard constraint. Full system information and control objectives are given by (Whidborne *et al.*, 1995), while (Taylor *et al.*, 2001) describe the application of NMSS/PIP methods to the problem. For brevity, these details will not be repeated here.

In order to obtain the control model, the deterministic 7th order continuous-time transfer function represention of the system is simulated in open-loop, data are collected with a sample time of $0.5s$ and the Simplified Refined Instrumental Variable (SRIV) algorithm (Young, 1984) is used to estimate the model (1) for this system. Here, selection of an appropriate model structure is based on the coefficient of determination R_T^2, representing the response error, coupled with the Young Identification Criterion (YIC), which provides a combined measure of fit and parametric efficiency. The discrete time transfer function model obtained in this manner takes the form:

$$H(z^{-1}) = \frac{0.0946z^{-2}}{1 - 0.9055z^{-1}}$$

The prediction and control horizons are $N_p = 50$ and $N_c = 10$ respectively, while the matrices $\bar{\boldsymbol{Q}}$ and $\bar{\boldsymbol{R}}$ are assigned values $\frac{50}{50}$ and $\frac{1}{10}$ on their diagonals. The MPC of (Wang and Young, 2005) is tuned to produce a similar closed-loop output response in the zero mismatch case, so that the two controllers are comparable. However, they yield different responses when model mismatch is subsequently introduced, as illustrated in Figures 3 and 4, which represent the controller proposed in the present paper and that described by (Wang and Young, 2005) respectively. In this robustness test, which includes the input constraints defined by (Whidborne *et al.*, 1995), the system parameters are based on stress level 2. In the ideal case, the output should lie within the bounds marked by the dashed trace in Figures 3 and 4.

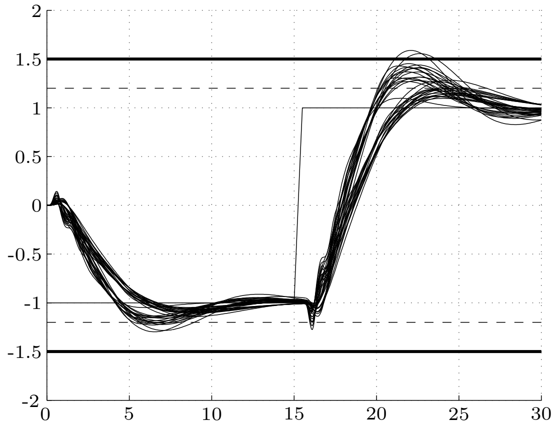

Fig. 3. Robustness test of the IFAC '93 benchmark system for the NMSS/MPC controller with an explicit integral-of-error state variable.

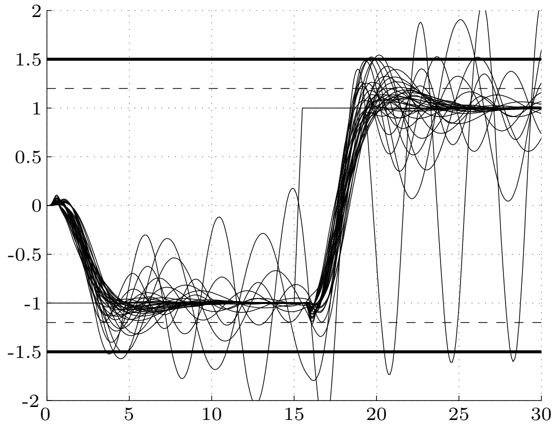

Fig. 4. Robustness test of the IFAC '93 benchmark system for the NMSS/MPC controller based on differenced input and output variables (Wang and Young, 2005)

It is evident from the figures that the proposed NMSS/MPC controller, utilising an explicit integral-of-error state variable, yields a higher proportion of well behaved responses than the equivalent controller of (Wang and Young, 2005). Of course, these figures represent just one simulation example – the authors are presently investigating the relative advantages of the two NMSS/MPC approaches for other examples.

5. CONCLUSIONS

This paper was motivated by earlier research into MPC using a Non-Minimal State Space (NMSS) form (Wang and Young, 2005), which demonstrated potential performance and robustness benefits when compared to conventional MPC using a minimal state space model with observer. However, the present paper develops an alternative framework for NMSS/MPC based on the directly measured (rather than differenced) values of the input and output variables. The new approach has close parallels with linear Proportional-Integral-Plus (PIP) methods, particularly with regard to the tuning of the weighting matrices and the inclusion of an *integral-of-error* state variable. Of course, the advantage of NMSS/MPC design is constraint handling.

In the simplest 'trial and error' case, the control engineer can adjust the total weights assigned to the input and output variables, together with the integral-of-error state, to achieve satisfactory performance; e.g. (Taylor *et al.*, 1998; Taylor *et al.*, 2004; Taylor and Shaban, 2006). More advanced methods, such as multi-objective optimisation of the weighting matrices in $\bar{\boldsymbol{Q}}$ and $\bar{\boldsymbol{R}}$ are also possible (Chotai *et al.*, 1998).

In both cases, the explicit integral-of-error state variable may provide additional degrees of freedom allowing for improved control, although this requires further research. In fact, a full comparison of the proposed NMSS/MPC algorithm, the approach of (Wang and Young, 2005) and conventional minimal MPC methods, is the subject of ongoing research by the authors. However, preliminary simulation results suggest that the robustness and performance characteristics of the new approach are at least as good, or better, as those obtained by (Wang and Young, 2005).

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