RELIABLE GUARANTEED COST CONTROL OF LINEAR DESCRIPTOR TIME-DELAY SYSTEMS

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Abstract: This paper deals with the problem of reliable guaranteed cost control for linear descriptor time-delay systems with actuator failure. The actuator failure model adopted in this paper is in complete form to describe variation of the actuator. To design a controller such that, for admissible actuator failures, the plant remains asymptotically stable and guarantees an adequate level of a quadratic cost index. It derives a sufficient condition for the existence of such controller by Lyapunov inequality, provides its parameterized representation and gives the design method of the optimal controller. Furthermore, it derives the results for linear descriptor systems without delay. Finally, an example shows the potential of this method. Copyright © 2002 USTARTH

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1. INTRODUCTION

Descriptor systems (also referred to as differential-algebraic-equation (DAE) systems or singular systems) describe a broad class of systems which are not only of theoretical interest but also have great practical significances. Such systems arise in electrical networks (Newcomb, 1981), industry (Kumar, 1998), robotics (Newcomb, 1986), control problems (Verghese et al., 1981), economic systems and some population growth models (Zeeman, 1976; Luenberger, 1997). For example, in (Kumar, 1998), models of chemical processes typically consist of differential equations describing the dynamic balances of mass and energy while additional algebraic equations account for thermodynamic equilibrium relations, steady-state assumptions, empirical correlations, etc. Over the years, the problem of stabilizing time-delay systems has been explored because delay is commonly encountered various engineering systems, such as manufacturing systems, the above chemical processes, telecommunication or electric networks. Since time delay is an important source of instability and poor performance, considerable attention has been paid to the problem of stability analysis and controller synthesis for time-delay systems. Recently, some work dealt with the problems of robust stability analysis, and guaranteed cost control for uncertain normal delay systems has appeared in the literature, such as Magdi and Xie (2000), Souza et al. (1993), and others. In the study of the stability and guaranteed cost control of singular systems with delays, there are many new difficulties compared with that of normal systems with or without state delays as in (Campbell, 1982) and (Wang, 2005), respectively.

The problem of designing a reliable control system has drawn considerable attention in recent control system literature (Veilllette, 1995). In actual implementation, the actuators may be subjected to failures of one form or another. It is therefore of practical interest to design a control system which can tolerate some actuator failures. For linear systems with a given quadratic cost function, Yang et al. (2000) and Yu (2005) have studied the reliable guaranteed cost control in different approach, respectively. For linear descriptor systems with a given quadratic cost function, the study of reliable control has centred on the stability and $H_\infty$ control, and the actuator failure model adopt is as a scaling factor (Marx et al., 2004). Up to now, there are few results about the reliable guaranteed cost control of descriptor linear systems with/without state delay.

In this paper, we deal with the problem of reliable guaranteed cost control for linear descriptor time-
delay systems with actuator failure. The actuator mold adopted in this paper is in a complete form to describe variation of the actuator. The purpose is to design a memoryless state feedback controller such that, for admissible actuator failures occurring among the prespecified subset of actuators, the plant remains asymptotically stable and guarantees an adequate level of a quadratic cost index. A sufficient condition for the existence of reliable guaranteed cost controller is derived in the form of Lyapunov inequality. Furthermore, it is showed that a group of linear matrix inequalities provides a parameterized representation of reliable guaranteed cost controllers. Based on that, the design problem of the optimal guaranteed cost controller is formulated as a convex optimization problem, which can be solved by the existing convex optimization techniques. Finally, an example provided in this paper show the technical potential of this method.

**Notations.** The notations in this note are quite standard. $R^n$ and $R^{nm}$ denote, respectively, the $n$-dimensional Euclidean space and the set of all $n \times m$ real matrices. The superscript “ $T$ ” denotes the transpose and the notation $0_{n \times m}$ stands for the mathematical expectation operator. Symmetric terms in symmetric matrices are denoted by $\ast$; i.e.,

$$
\begin{bmatrix}
A & B \\
\ast & C
\end{bmatrix} = 
\begin{bmatrix}
A & B \\
B^T & C
\end{bmatrix}
$$

2. PROBLEM FORMULATION AND LEMMNAS

Consider the following uncertain linear descriptor linear system

$$
\begin{align}
Ex(t) &= Ax(t) + A_s x(t-d) + Bu(t), \\
x(t) &= \phi(t), t \in [-d, 0],
\end{align}
$$

where $x(t) \in R^n$ is the state system, $d > 0$ is time delay constant, and $u(t) \in R^n$ is the system control input. And $E, A, A_s$ and $B$ are constant matrices of appropriate dimensions. $E$ is singular matrix and $n > \text{rank } E = r > 0$. Here $\phi(t) \in C[-d, 0]$ is the system initial condition.

Because $\text{rank } E = r < n$, without loss generality, it is assumed that the matrix $E$ in equation (1) has the special form as Wang and Liu (2005):

$$
E = \begin{bmatrix}
I_r & 0 \\
0 & 0
\end{bmatrix}
$$

where $I_r$ is an $r \times r$ identity matrix.

Associated with system (1) is the following quadratic cost function

$$
J = \int [x^T(t)Qx(t) + u^T(t)Ru(t)]dt
$$

where $Q > 0$ and $R > 0$ are given state and control weighting matrices.

Generally, a memoryless state feedback $u(t) = Kx(t)$

where $K$ is feedback gain matrix, results the following closed-loop system

$$
Ex(t) = (A + BK)x(t) + A_s x(t-d)
$$

Hence, if the resulted closed-loop system (5) is asymptotically stable and there exists an upper bound for the closed-loop value of the cost function (3), then the state feedback (4) is named as guaranteed cost controller for system (1)-(2) with cost function (3).

The following actuator failure model is adopted in this paper

$$
u_i^r = \alpha_i u_i, i = 1, 2, \ldots, m,
$$

where

$$
0, < \alpha_i \leq \bar{\alpha}_i, i = 1, 2, \ldots, m,
$$

and

$$
\bar{\alpha}_i \leq \alpha_i \leq \bar{\alpha}_i, i = 1, 2, \ldots, m.
$$

In the above model of actuator failure, if $\bar{\alpha}_i = \bar{\alpha}_i$, then it corresponds to the normal case $u_i^r = u_i$. When $\bar{\alpha}_i = 0$, it covers the outage case. If $\alpha_i > 0$, it corresponds to the partial failure, i.e., partial degradation of the actuator.

Denote

$$
u^r = [u_1^r, u_2^r, \ldots, u_m^r]^T
$$

$$
\alpha = \text{diag}\{\alpha_1, \alpha_2, \ldots, \alpha_n\}
$$

$$
\bar{\alpha} = \text{diag}\{\bar{\alpha}_1, \bar{\alpha}_2, \ldots, \bar{\alpha}_m\}
$$

$$
\tilde{\alpha} = \text{diag}\{\tilde{\alpha}_1, \tilde{\alpha}_2, \ldots, \tilde{\alpha}_n\}
$$

$\alpha$ is said to be admissible if $\alpha$ satisfies $\alpha \leq \bar{\alpha} \leq \tilde{\alpha}$. Then the actual state feedback controller is in the following form

$$
u^r = \alpha Kx(t).
$$

**Definition 2.1** A control law $u(t) = Kx(t)$ is said to be a reliable guaranteed cost control of system (1) with cost function (3), if, for all admissible $\alpha$, the following resulted closed-loop system

$$
Ex(t) = (A + \alpha K)x(t) + A_s x(t-d)
$$

is asymptotically stable and there exists an upper bound $J^r$ for the closed-loop value of the cost function (3) such that $J < J^r$.

The objective of this paper is to develop a procedure for determining a state feedback gain matrix $K$ such that the control law (4) is a reliable guaranteed cost control of system (1) and cost function (3), and to give the upper bound $J^r$ for the closed-loop value of the cost function (3).

Define

$$
\beta = \text{diag}\{\beta_1, \beta_2, \ldots, \beta_n\}
$$

$$
\beta_i = \text{diag}\{\beta_{i1}, \beta_{i2}, \ldots, \beta_{in}\}
$$

where

$$
\bar{\alpha}_i + \frac{\alpha_i}{2}, \beta_i = \frac{\bar{\alpha}_i - \alpha_i}{\bar{\alpha}_i + \alpha_i}, i = 1, 2, \ldots, m.
$$

From the above, we have that

$$
\alpha = (I + \alpha_i) \beta
$$

and

$$
\alpha_i \leq \beta_i \leq I,
$$

where

$$
\alpha_i = \text{diag}\{\alpha_{i1}, \alpha_{i2}, \ldots, \alpha_{in}\}
$$

$$
\frac{\alpha_i}{\lambda_i} = \text{diag}\{\alpha_{i1}, \alpha_{i2}, \ldots, \alpha_{in}\}
$$
To end this section, we give some lemmas used in this paper.

**Lemma 2.1 (Schur complement) (Yu, 2005)** Given constant matrices $M$, $L$ and $Q$ of appropriate dimensions where $M$ and $Q$ are symmetric, then $Q > 0$ and $M + L'Q^{-1}L < 0$ if and only if

$$
\begin{pmatrix}
M & L' \\
L & -Q
\end{pmatrix} < 0
$$

or, equivalently

$$
\begin{pmatrix}
-Q & L' \\
L & M
\end{pmatrix} < 0.
$$

**Lemma 2.2 (Yu, 2005)** Given vectors $a$ and $b$ of appropriate dimensions, then, for any diagonal matrix $R_0 > 0$, the following inequality holds

$$
2a'b \leq a'R_0a + b' R_0^{-1}b
$$

3. MAIN RESULTS

The following theorem presents a sufficient condition for the existence of the reliable guaranteed cost controllers and a design procedure for such controllers.

Firstly, we give the following lemma to guarantee the asymptotical stability of the system (1) with $u(t) = 0$.

**Lemma 3.1 (Feng, 2002)** Consider system (1) with $u(t) = 0$ and system matrix $E$ in general form. If there exist matrix $P \in \mathbb{R}^{n \times n}$ and $0 < W \in \mathbb{R}^{n \times n}$ such that the following inequality holds

$$
PE = E'P^* \geq 0
$$

then system (1) has unique solution to any compactable initial condition $\phi(t), t \in [-d, 0]$, and the zero solution is asymptotically stable, i.e., system (1) has asymptotically stable.

Noting that matrix $E$ has been assumed to be in special form (2), we give the following sufficient condition of the existence of a reliable guaranteed cost control of system (1)-(2) with cost function (3).

**Theorem 3.1** Consider system (1)-(2) with cost function (3). For state feedback controller (4), if there exist matrices $0 < W \in \mathbb{R}^{n \times n}$ and $P \in \mathbb{R}^{n \times n}$ such that

$$
(\begin{array}{c}
\alpha \end{array})
\begin{bmatrix}
E & W
\end{bmatrix}
\begin{bmatrix}
A + BaK
\end{bmatrix}
+ P + P^* (A + BaK)
+ W + P^* A W^{-1} A' P + K' aR aK + Q < 0
$$

holds for all admissible $\alpha$, where $P$ is in the following form

$$
P = \begin{bmatrix}
P_1 & P_2 \\
0 & P_3
\end{bmatrix}
$$

with $0 < P_2 \in \mathbb{R}^{n \times n}$ is weighting matrix, $P_2 \in \mathbb{R}^{(n - n)^\times (n - n)}$, $P_1 \in \mathbb{R}^{(n - m)^\times (n - m)}$ and $P_3$ is invertible, then, the controller (4) is a reliable guaranteed cost control of system (1)-(2) with cost function (3) and the upper bound $J^*$ for the closed-loop value of the cost function (3) is given by:

$$
J^* = x_0^T P_1 x_0 + \int_{-d}^{0} x_0^T (\theta) Wx_0 (\theta) d\theta
$$

**Proof** Take the following Lyapunov functional candidate

$$
V(x) = x^T(t) E^T P x(t) + \int_{-d}^{0} x_0^T (\theta) Wx_0 (\theta) d\theta
$$

where $x(s) = x(t + s), -d \leq s \leq 0$, $W$ and $P$ are given in Theorem 3.1, $E$ takes the form of (2). Observe that $V(x) = 0$ for $x = 0$ and $V(x) > 0$ when $x \neq 0$. The time derivative of $V(x)$ along solution of system (1)-(2) is given by

$$
\dot{V}(x) = \dot{x}^T(t) E^T P x(t) + \dot{x}(t) E^T P x(t) + \dot{x}(t) Wx(t)
$$

$$
= x(t) A^T x(t) + x(t) B^T K x(t) + x(t) W x(t)
$$

$$
= x(t) (A + BaK) x(t) + P^* (A + BaK)
+ W + P^* A W^{-1} A' P + K' aR aK + Q
$$

$$
\leq x(t) (A + BaK) x(t) + P^* (A + BaK)
+ W + P^* A W^{-1} A' P
$$

then, according to (13), one obtains that

$$
\dot{V}(x) < x(t) [- K' aR aK - Q] x(t) < 0
$$

which, according to Lemma 3.1, shows that the closed-loop system (5) is asymptotically stable and

$$
V(x(t)) \to 0, \text{ as } t \to +\infty.
$$

Now, by calculating the time derivative of $V(x)$, we have

$$
\dot{V}(x) < -x(t) (Q x(t) - u(t) R u(t))
$$

which is equal to

$$
x(t) \dot{Q} x(t) + u(t) R u(t) < -\dot{V}(x).
$$

Noting (3) and (16), one obtains

$$
J < \int_{-d}^{0} -V(x) d\theta
$$

$$
= V(x) - V(x_0)
$$

$$
= x_0^T P_1 x_0 + \int_{-d}^{0} x_0^T (\theta) Wx_0 (\theta) d\theta
$$

The proof is done.

**Remark.** In system (1), when $A = 0$, system (1) reduces to the general descriptor linear system without state delay, that is, the system description is given by

$$
\dot{E} x(t) = A x(t) + B u(t),
$$

and the system matrix $E$ is still in the form of (2). Just like in the former section, the actuator failure model is still in the form of (6)-(7), then we get the following corollary.

**Corollary 3.1** Given system (17) and (2) with cost function (3). For state feedback controller (4), if there exists matrix $P \in \mathbb{R}^{n \times n}$ such that

$$
(\begin{array}{c}
\alpha \end{array})
\begin{bmatrix}
E & W
\end{bmatrix}
(\begin{array}{c}
A + BaK
\end{array})
+ P + P^* (A + BaK)
+ K' aR aK + Q < 0
$$

holds for all admissible $\alpha$, where $P$ is in the following form

$$
P = \begin{bmatrix}
P_1 & P_2 \\
0 & P_3
\end{bmatrix}
$$

with $0 < P_2 \in \mathbb{R}^{n \times n}$ is weighting matrix, $P_2 \in \mathbb{R}^{(n - n)^\times (n - n)}$, $P_1 \in \mathbb{R}^{(n - m)^\times (n - m)}$ and $P_3$ is invertible, then, the controller (4) is a reliable guaranteed cost control of system (17) and (2) with cost function (3) and the upper bound $J^*$ for the closed-loop value of the cost function (3) is given by:
\[ J^* = x_0^* P x_0. \]

**Proof** For system (17) and (2) with cost function (3), one takes the following Lyapunov functional candidate
\[ V(x) = x^T(t) E P x(t) \]
where \( P \) is just in the form of (14) satisfying the condition provided in the corollary. Then the following proof is similar to the proof of Theorem 3.1. The proof is done.

**Remark** Note that the sufficient condition in Theorem 3.1 can not be used to test the existence of reliable guaranteed cost controllers, however, according to Lemma 2.1, it can be rewritten in the form of linear matrix inequality as
\[ \begin{bmatrix} \Lambda & P^T A \alpha & Q & K^T \beta \\ * & -W & 0 & 0 \\ * & * & -Q & 0 \\ * & * & * & -R^T \end{bmatrix} < 0 \]
where
\[ \Lambda = A^T P + K^T \beta B^T P + P^T A + P^T B \alpha K + W. \]
Hence, (18) can be solved by LMI toolbox in MATLAB for fixed \( \alpha \). In fact, however, \( \alpha \) is still unknown with bound. At the same time, according to (10)-(12), we give the following theorem to design the reliable guaranteed cost controllers for system (1)-(2) with cost function (3).

**Theorem 3.2** Given system (1)-(2) with cost function (3). If there exist matrices \( 0 < U \in R^{n \times n} \), \( X \in R^{n \times n} \), diagonal matrix \( 0 < R \in R^{n \times n} \), and matrix \( Y \in R^{m \times n} \) such that
\[ \begin{bmatrix} \Pi_1 & A U & X \beta \beta & X^T \\ * & -U & 0 & 0 \\ * & * & -Q & 0 \\ * & * & * & \Pi_1 \\ * & * & * & * & -R \end{bmatrix} < 0 \]
holds, where \( X \) is in the following form (14) with \( 0 < X \in R^{n \times n} \) is weighting matrix, \( X \in R^{n \times n} \), \( X \in R^{n \times n} \) is invertible, and
\[ \begin{align*} 
\Pi_1 &= X^T A^T + Y^T \beta B^T + AX + B \beta Y + BK \beta + W \\
\Pi_2 &= Y^T \beta + BR \beta + R \beta + R \beta \\
\Pi_1 &= -R^T + R \beta 
\end{align*} \]
then the reliable guaranteed cost control of system (1)-(2) with cost function (3) is given by
\[ u(t) = K x(t) = Y X^{-1} x(t) \]
and the upper bound \( J^* \) for the closed-loop value of the cost function (3) is given by
\[ J^* = x_0^* X^{-1} x_0 + \int_0^T \varphi^T(\theta) U^{-1} \varphi(\theta) d\theta \]

**Proof** Substituting (11) into (18), one obtains that (13) in Theorem 3.1 is equal to
\[ \Gamma = \Omega + \begin{bmatrix} K' \alpha_3 \beta B^T P + P^T B \alpha_3 \beta K + W & 0 & 0 & K' \alpha_3 \beta \beta \\ * & 0 & 0 & 0 \\ * & * & 0 & 0 \\ * & * & * & 0 \end{bmatrix} \]
where
\[ \Pi_1 = A^T P + K^T \beta B^T P + P^T A + P^T B \beta K + W. \]
Using Lemma 2.2, we obtain that, for some diagonal matrix \( R \in R^{n \times n} \) and for all \( \alpha \) satisfying (12), the following inequality holds
\[ \Gamma = \Omega + \begin{bmatrix} P^T B & 0 & 0 & 0 \\ 0 & \alpha \beta K & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{bmatrix} \]
\[ \leq \Omega + \begin{bmatrix} P^T B & P^T B & K' \beta & K' \beta \\ 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{bmatrix} \]
\[ = \begin{bmatrix} \Pi_2 & P^T A \beta & Q & K' \beta + P^T B R \beta \\ * & -W & 0 & 0 \\ * & * & -Q & 0 \\ * & * & * & -R^T + R \end{bmatrix} \]
where
\[ \Pi_2 = A^T P + K^T \beta B^T P + P^T A + P^T B \beta K + W + P^T B R \beta + P + K^T \beta R \beta + R^T \beta K. \]
Hence, it is obvious that if \( \Gamma < 0 \), then \( \Gamma < 0 \). So it is sufficient to show that \( \Gamma < 0 \) is equal to (19). Pre- and post- multiplying \( \Gamma \) by diag \((P^T, I, I, I)\) and diag \((P^T, I, I, I)\), respectively, we obtain that
\[ \begin{bmatrix} \Pi_1 & A \beta & P^T \beta & K' \beta + BR \beta \\ * & -W & 0 & 0 \\ * & * & -Q & 0 \\ * & * & * & -R^T + R \end{bmatrix} < 0 \]
and
\[ \Pi_1 = A^T P + K^T \beta B^T P + P^T A + P^T B \beta K + W + P^T \beta \beta \beta + BR \beta + P + K^T \beta \beta \beta + R^T \beta K. \]
Pre- and post- multiplying (22) by diag \((I, W^T, I, I)\), and letting \( U = W^T \), \( X = P^T \) and \( Y = K P^T \), respectively, then applying Schur complement again, one concludes that the matrix inequality (22) is equivalent to (19). Noting that if the matrix \( P \) is in the form of (14), then \( X = P^T \) is still in the form of (14). As to (21), it is trivial from (15) in Theorem 3.1. The proof is done.

**Corollary 3.2** Given system (17) and (2) with cost function (3). If there exists matrix \( X \in R^{n \times n} \),
Y ∈ R^n×n, and diagonal matrix 0 < Rₙ ∈ R^n×n such that
\[
\begin{bmatrix}
\Pi & Y'β_0 & X'Q & Y'β + BR_0 \\
0 & -R_0 & 0 & 0 \\
0 & * & -Q & 0 \\
0 & * & * & -R^{-1} + R_0
\end{bmatrix} < 0
\] (23)
holds, where X is in the following form (14) with 0 < X ∈ R^{n×n} is weighting matrix, Xₙ ∈ R^{n×n}, Xᵢ ∈ R^{n×n} for the closed-loop cost function given in (3).

If (19) is feasible, then Theorem 3.2 gives a set of some additional requirements. In particular, the design the reliable guaranteed cost controllers with this parameterized representation can be exploited to construct reliable guaranteed cost controllers with cost function (3). Assume that 0

The proof of Corollary 3.4 is similar to that of Theorem 3.2, so it is omitted here.

Remark (24) and (25) are convex optimization problems with LMI constraints, therefore existing convex optimization techniques can be used to solve the two problem. The convexity of the optimization problem ensures that a global optimum, when it exists, is reachable.

4. NUMERICAL EXAMPLE

In this section, we present a simple numerical example for actuator failure to illustrate the proposed design method.

Consider the linear descriptor uncertain system (1)-(2) with cost function (3), where
\[
E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, A = \begin{bmatrix} -5 & 1 \\ 1 & 4 \end{bmatrix}
\]
\[
A_d = \begin{bmatrix} 0.2 & 0 \\ 0.1 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}
\]
\[
R = 0.5I_{2×2}, Q = I_{2×2}, h = 0.1, \quad \bar{α}_1 = \bar{α}_2 = 1, \bar{α}_3 = 1.1, \bar{α}_4 = 0.9
\]

It should be noted that the delay in the above system is small, which can also imply less conservativeness of the method. Then, according to (10), we get that
\[
β = I, β_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}
\]

According to Theorem 3.2, and applying the LMI toolbox in MATLAB, we get
\[
X = \begin{bmatrix} 0.0291 & 0 \\ -0.2660 & -0.2356 \end{bmatrix}, R_0 = \begin{bmatrix} 1.1693 & 0 \\ 0 & 0.8697 \end{bmatrix}
\]
and
\[
Y = \begin{bmatrix} -1.0324 & -0.9492 \\ -0.6090 & -1.4277 \end{bmatrix}, U = \begin{bmatrix} 1.4823 & 0.0000 \\ 0.0000 & 1.5117 \end{bmatrix}
\]
According to (20) in Theorem 3.2, we get the reliable guaranteed cost controller is given by
\[ K = YX^{-1} = \begin{bmatrix} -72.2970 & -4.0295 \\ -76.3196 & -6.0610 \end{bmatrix} \]
Furthermore, with guaranteed cost value is 0.0610.
And according to Corollary 3.5, the optimal guaranteed cost controller is given by
\[ J^* = \begin{bmatrix} 0.0610 \end{bmatrix} \]
we get that
\[ J^* = 34.4888, \]
And according to Corollary 3.5, the optimal guaranteed cost value is
\[ J^* = 28.0417. \]
For the limitation of length, the graph is omitted.

5. CONCLUSIONS

This paper deals with the problem of reliable guaranteed cost control for linear descriptor time-delay systems with actuator failure. The actuator failure mode adopted in this paper is in complete form to describe variation of the actuator. The purpose is to design a memoryless state feedback controller such that, for admissible actuator failures occurring among the prespecified subset of actuators, the plant remains asymptotically stable and guarantees an adequate level of a quadratic cost index. A sufficient condition for the existence of reliable guaranteed cost controller is derived in the form of Lyapunov inequality. Furthermore, it shows that a group of linear matrix inequalities provides a parameterized representation of reliable guaranteed cost controllers. Based on that, the design problem of the optimal guaranteed cost controller is formulated as a convex optimization problem, which can be solved by the existing convex optimization techniques. Finally, an example provided in this paper shows the technical potential of this method.

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