Identification Applied to Dual Sensor Transient Temperature Measurement ⋆

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Abstract: The harsh environment presented by engines, particularly in exhaust systems, necessitates the use of robust and therefore low bandwidth temperature sensors. Consequently, high frequencies are attenuated in the sensor output. A number of techniques for addressing this problem involve measurement of the gas temperature using two thermocouples with different time-constants and mathematical reconstruction of the true gas temperature from the resulting signals. Many of these methods rely on the assumption that the ratio of the thermocouple time-constants is invariant and known *a priori*. In addition, they are generally subject to singularities and sensitive to noise. A recently proposed two-thermocouple sensor characterization method which utilises system identification techniques and is much more generally applicable is described. Previous offline methods for constant velocity flow are extended using polynomial parameter fitting on a sliding data window to accommodate variable velocity. These methods have been successfully tested and proven for the first time in variable velocity flow with experimental data produced from a novel and highly instrumented test rig. Results show that the increase in bandwidth arising from the dual sensor technique allowed accurate measurement of fluctuating temperatures with relatively robust thermocouples. The introduction of sliding windows is shown to be effective, while the inclusion of polynomial fitting within the window produces marginal improvements in performance.

Keywords: Temperature Measurement; Sensor Characterisation; System Identification.

1. INTRODUCTION

Sensor modeling and characterisation is increasingly important in many engineering applications, due to the requirement to capture fast and accurate transient inputs. In the automotive industry for example, accurate dynamic measurement of exhaust gas temperature (EGT), is required for onboard diagnosis of catalyst malfunction and for gaining valuable insight into engine combustion, allowing conclusions on engine performance and efficiency to be drawn (Kee and Blair, 1994). However, performing accurate, reliable and cost-effective measurement of rapidly changing gas temperature is a challenging problem.

In order to fully characterise such flows, high bandwidth sensors are required to resolve the high frequency fluctuations. This can be achieved using such techniques as Coherent anti-Stokes Raman spectroscopy, Laser-Induced Fluorescence, and infrared Pyrometry. Such methods often exceed the requirements of automotive EGT measurements, are expensive and prove difficult to install, maintain and calibrate and are therefore not practical for wide scale deployment outside the laboratory.

Thermocouples are commonly used for temperature measurement due to their simplicity, robustness, relatively low cost, and ease of manufacture and installation. The design of a thermocouple does however represent a compromise between accuracy, robustness and rapidity of response and as such poses major problems when measuring high frequency temperature fluctuations. From thermodynamic considerations, it can be shown that the bandwidth of a thermocouple (ω_B) is dependent on its diameter according to (1), where k and m are constants (Tagawa and Ohta, 1997; Kee *et al.*, 1999), d is the diameter of the thermocouple wire and v is the velocity of the gas

$$\omega_B = kd^{m-2}v^m. \tag{1}$$

Harsh environments such as automotive exhaust systems require the use of robust thermocouples, resulting in low bandwidth sensors and consequently severely attenuated and lagged measurements. Dynamic compensation is therefore necessary if the true gas temperature is to be reconstructed. Forney and Fralick (1994) showed that if certain criteria regarding the mechanical construction and placement of thermocouples are met, they can be assumed to have first-order dynamics with time constant τ and unity gain:

$$T_g(t) = T_m(t) + \tau\dot{T}_m(t). \tag{2}$$

This model allows the true gas temperature $T_g(t)$ to be reconstructed from the measured gas temperature $T_m(t)$ and its derivative. Since τ is a function of the bandwidth

⋆ This work was supported in part by the Virtual Engineering Centre, Queen's University Belfast.

ω_B ($\tau = 2\pi\omega_B^{-1}$), from (1) it varies with gas velocity and is generally unknown *a priori*. Furthermore, noise corrupted measurements make accurate computation of $\dot{T}_m(t)$ impractical. It is clear that a single thermocouple will therefore not provide sufficient information to accurately determine sensor characteristics *in situ*.

Time constant estimation using a two-thermocouple probe (TTP) of varying thermocouple diameters was first suggested by Pfriem (1936). Numerous articles on time constant estimation and subsequent temperature reconstructions on this basis have been published e.g. Kee *et al.* (1999); Tagawa and Ohta (1997); Forney and Fralick (1994). These methods rely on the restrictive assumptions that the ratio of the time constants α is invariant and known *a priori* and/or are subject to computational singularities and noise sensitivity.

Hung *et al.* (2005a) presented a novel difference equation based method that makes no such assumption, yet looks superior to existing methods in terms of time constant estimation accuracy and noise tolerance. Furthermore, this analytical technique is more generally applicable to other sensor types. Hung *et al.* (2005b) extended this offline method by using a sliding data window to accommodate changes in gas velocity. This approach exploits the invariance of time constant ratio with respect to gas velocity. To improve the tracking ability of this algorithm and to therefore improve the accuracy of parameter estimations, a modification that incorporates polynomial parameter fitting is introduced. This method seeks to accurately track time constant variation within each sliding window. The performance of these methods are experimentally verified against previous methods in varying flow environments for the first time with data collected from a novel and highly instrumented test rig.

The remainder of the paper is organised as follows. Section 2 gives an overview of the TTP temperature reconstruction technique. The difference equation based method is introduced in Section 3 and the sliding window and polynomial fitting extensions are described in Section 4. The experimental test rig and data collection procedures are detailed in Section 5. Results are then presented in Section 6. Finally, Section 7 provides a summary and conclusions.

2. TWO-THERMOCOUPLE PROBE (TTP) RECONSTRUCTION TECHNIQUES

The basis for TTP temperature reconstruction techniques is that both thermocouples are subject to the same environmental conditions; most importantly the gas temperature $T_g(t)$ and gas velocity v. From (1), it can be shown that the time constant ratio is a function of thermocouple geometry only (3) and therefore approximately invariant:

$$\alpha = \frac{\tau_1}{\tau_2} = \frac{\omega_{B2}}{\omega_{B1}} = \frac{kd_2^{m-2}v^m}{kd_1^{m-2}v^m} = \left(\frac{d_1}{d_2}\right)^{2-m}. \quad (3)$$

Subscripts 1 and 2 are used to label the two thermocouples. The corresponding thermocouple models are then given by

$$T_g(t) = T_{m1}(t) + \tau_1\dot{T}_{m1}(t) \quad (4)$$

and

$$T_g(t) = T_{m2}(t) + \tau_2\dot{T}_{m2}(t). \quad (5)$$

Noting that

$$\frac{T_g(t) - T_{m1}(t)}{T_g(t) - T_{m2}(t)} = \alpha\frac{\dot{T}_{m1}(t)}{\dot{T}_{m2}(t)} \quad (6)$$

and assuming knowledge of α, $T_g(t)$ can be estimated directly using

$$T_g(t) = \frac{T_{m1}(t)\dot{T}_{m2}(t) - \alpha\dot{T}_{m1}(t)T_{m2}(t)}{\dot{T}_{m2}(t) - \alpha\dot{T}_{m1}(t)}. \quad (7)$$

Although simple, this solution leads to numerical difficulties when reconstructing $T_g(t)$ from noisy measurements due to the requirement for instantaneous derivative values and a possible singularity due to the form of the denominator. Tagawa and Ohta (1997) proposed a continuous-time reconstruction technique that minimised the time-averaged, mean-squared difference between the reconstructions from (4) and (5), thus allowing τ_1 and τ_2 to be estimated. This avoids the singularity problems of (7) and performs well on noise-free data but proves unreliable under noisy conditions, often producing infeasible results. Kee *et al.* (1999) later proposed a modification to this technique, TDR-Kee, which improved robustness, but singularities were still possible.

3. SYSTEM IDENTIFICATION BASED SENSOR CHARACTERISATION

Hung *et al.* (2002) proposed an approach based on discrete-time system identification that assumes nothing about the sensor time constants. Here the ARX difference equation equivalent to the single thermocouple model (2) is given by

$$T_m(k) = aT_m(k-1) + bT_g(k-1), \quad (8)$$

where a and b are parameters and k is the sample instant. Assuming ZOHs and a sampling interval τ_s, the parameters of the discrete and continuous time thermocouple models are related by

$$a = \exp(-\tau_s/\tau), b = 1 - a. \quad (9)$$

3.1 Three-parameter ARX model (γ-model)

The parameters a and b cannot be identified for data using (8) alone because T_g is unknown. However, using two thermocouples that are assumed to be subject to the same gas temperature T_g and gas velocity v, T_g can be eliminated to produce a 3-parameter ARX representation:

$$T_{m2}(k) = \gamma_1 T_{m2}(k-1) + \gamma_2 T_{m1}(k) + \gamma_3 T_{m1}(k-1) \quad (10)$$

where

$$\gamma_1 = a_2, \gamma_2 = \frac{1-a_2}{1-a_1}, \gamma_3 = -\frac{a_1(1-a_2)}{1-a_1}. \quad (11)$$

This model was first proposed by Hung *et al.* (2003) and the unknown parameter can be estimated using linear least-squares (LS). In practice, however, biased estimates are produced by conventional LS as noise is present on both the $T_{m1}(k)$ and $T_{m2}(k)$ measured data sequences. More recently, McLoone *et al.* (2006) showed that unbiased parameter estimates can be achieved by employing Total

Least-Squares (TLS). However estimation performance deteriorated rapidly with increasing noise levels. This was due to the extra degree of freedom introduced by a 3-parameter model for a system with only two unknowns.

3.2 *Two-parameter ARX model (β-model)*

Hung *et al.* (2005a) reduced (10) to a linear two-parameter formulation by expressing it in terms of b_2 using (9) and introducing a new parameter β, defined as $\beta \triangleq b_2/b_1$

$$\Delta T^k_{m2} = \beta \Delta T^k_{m1} + b_2 \Delta T^{k-1}_{m12}, \tag{12}$$

where ΔT^k_{m1}, ΔT^k_{m2}, and ΔT^{k-1}_{m12} are temperature differences defined as

$$\begin{aligned} \Delta T^k_{m1} &= T_{m1}(k) - T_{m1}(k-1) \\ \Delta T^k_{m2} &= T_{m2}(k) - T_{m2}(k-1) \\ \Delta T^{k-1}_{m12} &= T_{m1}(k-1) - T_{m2}(k-1) \end{aligned} \tag{13}$$

For an M-sample data set (12) can be expressed in vector-matrix form as

$$Y = X\theta, \tag{14}$$

with $Y = \Delta T^k_{m2}$, $X = \left[\, \Delta T^k_{m1} \;\, \Delta T^{k-1}_{m12} \,\right]$ and $\theta = [\, \beta \;\, b_2 \,]^T$. Here ΔT^k_{m1}, ΔT^k_{m2} and ΔT^{k-1}_{m12} are now $(M-1)$-vectors.

4. SLIDING WINDOW CHARACTERISATIONS

Hung *et al.* (2005b) extended the difference-equation approach to incorporate variable time-constant scenarios by introducing a sliding data window (β-GTLS). Recalling (3) and (9), it can be shown that

$$\beta = \frac{1 - \exp(\tau_s/\tau_2)}{1 - \exp(\tau_s/\tau_1)} \approx \frac{\tau_1}{\tau_2} = \alpha < 1 \tag{15}$$

provided $\tau_s << \tau_1$. Since α is known to be invariant (Kee *et al.*, 1999), even under variable flow conditions, it follows that β is also approximately invariant and can therefore be assumed to be constant over large data windows even if the time constants are not. The constraints on time constant variation can then be relaxed in the following generalization of (13):

$$\Delta T^k_{m2} = \beta \Delta T^k_{m1} + b_2(k) \Delta T^{k-1}_{m12} \tag{16}$$

with

$$b_2(k) = b_{20} + k b_{21} + k^2 b_{22} + k^3 b_{23} \tag{17}$$

where b_{2j} is the polynomial coefficient of the jth power. Here the constant b_2 in (12) has been replaced by a third-order polynomial, $b_2(k)$ in order to capture the parameter variation within the data window (β-GTLS PS). Time constant estimates are then obtained by evaluating $b_2(k)$ at the centre of each sliding data window. The matrix-vector representation in (14) is then modified to incorporate $b_2(k)$:

$$Y_P = X_P \theta_P \tag{18}$$

where

$$\begin{aligned} X_P &= \left[\Delta T^k_{m1} \; \Delta T^{k-1}_{m12} k \; \Delta T^{k-1}_{m12} k^2 \; \Delta T^{k-1}_{m12} k^3 \; \Delta T^{k-1}_{m12}\right] \\ Y_P &= \Delta T^k_{m2}, \; \theta_P = [\beta \; b_{20} \; b_{21} \; b_{22} \; b_{23}]^T \end{aligned} \tag{19}$$

5. EXPERIMENTAL WORK

5.1 *Test Rig*

Previously, the sliding window algorithm had only been tested in simulation. Experimental verification of the algorithms in Sections 3 and 4 required the specific design of a test rig (Fig. 1), capable of producing periodic temperature fluctuations. Air was supplied to the test rig via a pressure regulator and needle valve, ensuring that the mass flow rate was approximately constant. The flow was then divided into two streams; one was heated, while the other remained at the supply temperature. Both streams were then passed into small reservoirs which supplied two orifices. After leaving the orifices, the streams entered a mixing chamber in order that a temperature gradient would be produced across the exit of the chamber. The input streams were then balanced using ball valves, to ensure a uniform velocity profile across the air outlet.

The mixing chamber then reciprocated about a pivot under the control of a crank and linkage. A temperature measurement probe was fixed above the mid-point of the chamber and as it reciprocated, varying proportions of hot and cold air were supplied to the probe at a constant gas velocity. A step change in gas velocity was achieved by adding an additional balanced air supply before the air entered the reservoirs. Gas velocity was measured using a pitot-static tube connected to a fast response pressure transducer which was fixed directly above the temperature measurement probe.

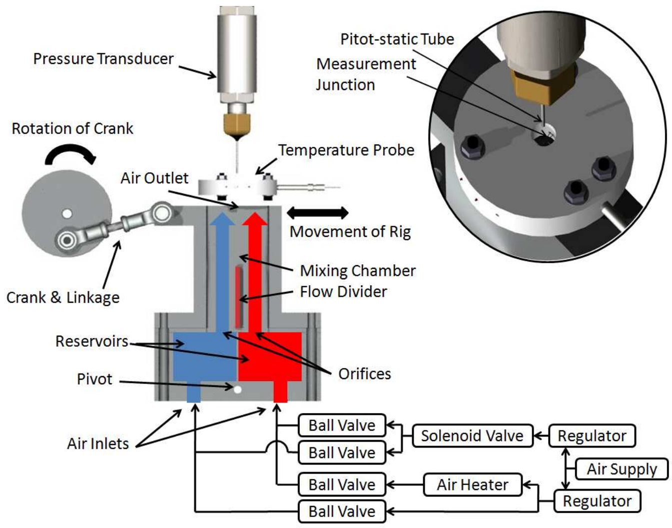

Fig. 1. Experimental test rig schematic.

5.2 *Temperature Measurement Probe*

Temperature measurements were produced from two thermocouples of unequal diameters (50 and 127 μm), with a constant-current anemometer (3.8 μm) providing a reference measurement. The mounting of such delicate sensors posed a significant design challenge, as stress on the sensors must be minimised, as well as intrusion and disturbance of flow. Furthermore, a basic assumption of the TTP method is that both thermocouples are subject to identical measurement environments. Figure 2 plots the variation in cross-correlation coefficient between data from two thermocouples (50 μm) with junction separation under constant a velocity flow. The inverse relationship produced shows that as junction separation is minimised, the chances of obtaining good data are increased as the average values of coefficients are more tightly distributed and approach unity.

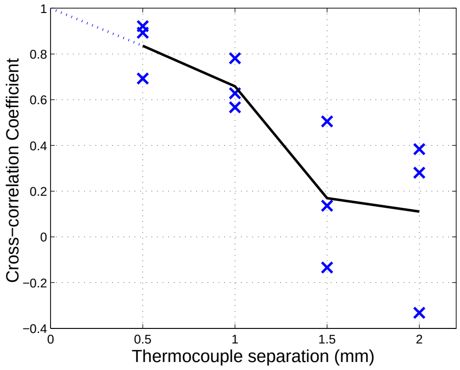

Fig. 2. Variation in correlation-coefficient between TTP thermocouple data with spacing.

For this study, a temperature measurement probe (Fig. 3) was manufactured to ensure that the thermocouple junction spacing was less than 0.5 mm. The sensors are precisely located between two nylon discs, with an orifice allowing the gas flow to pass through.

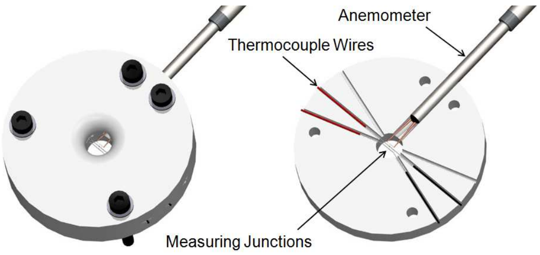

Fig. 3. Temperature measurement probe assembly.

Thermocouples do not provide a single point measurement of the gas temperature in which they are immersed; they provide a measure of the temperature at their junction, which may or may not be equal to that of the gas. If a thermocouple is exposed to a thermal gradient along its length, whether in steady-state or transient flows, thermal conduction may cause the junction to be at a different temperature than that of the gas around the junction. The temperature at the junction is therefore a function of thermal conductivity, the magnitude of the temperature gradient and the thermocouple diameter.

In order to investigate the effect of thermocouple diameter on gas measurement accuracy, a Computational Fluid Dynamics (CFD) simulation was performed, where 35 mm long cylinders of diameters 25, 50 and 127 μm (shown as a, b and c respectively in Fig. 4) were subjected to a steady-state temperature gradient of approximately 3 °C/mm along their length at a gas velocity of 30 m/s.

Figure 4 shows that only the temperature of a 25 μm cylinder accurately represented that of the gas temperature along its entire length. Therefore a thermocouple

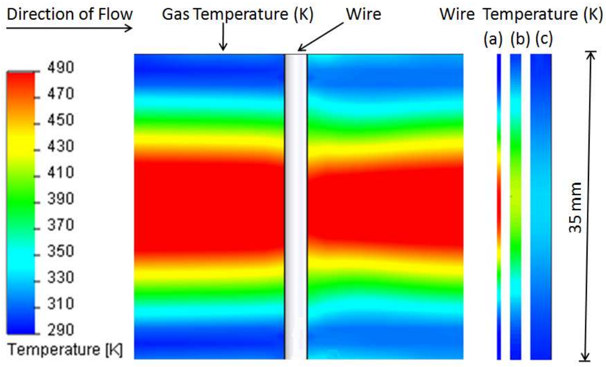

Fig. 4. CFD simulation of fine wires exposed to a steady-state temperature gradient (colour diagram).

junction at any point along the 35 mm cylinder would give an accurate temperature estimate for the surrounding gas. In contrast, the simulation shows that the temperatures of the 50 and 127 μm cylinders were almost constant due to conduction in the wires, resulting in the wire temperature in the central region being significantly different than the gas temperature. These results clearly demonstrate that as thermocouple diameter increases, so too does conduction in the wire, leading to erroneous gas temperature estimates. Therefore, in addition to being closely spaced, in the presence of temperature gradients the thermocouples must also be relatively fine.

6. RESULTS

The sensor assembly was mounted in the test rig, and subjected to a cyclic variation in temperature (Fig. 5) and a step change in gas velocity. A step increase in velocity occurs in the time interval 2.1 to 2.3 s (Fig. 6), causing a corresponding decrease in thermocouple time constants (3). Although there is a small fluctuation in gas velocity before and after the step, it is deemed insignificant, as τ_1 and τ_2 are approximately proportional to the square root of gas velocity (3).

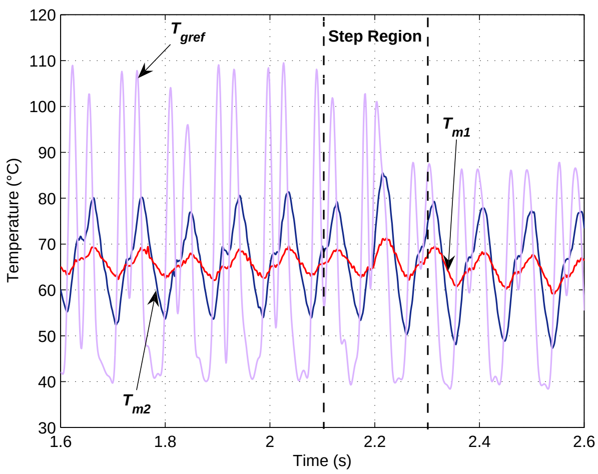

Fig. 5. Temperature variation of TTP under a step change in gas velocity.

The β-GTLS and β-GTLS PS sliding window algorithms were both used to produce time constant estimates with

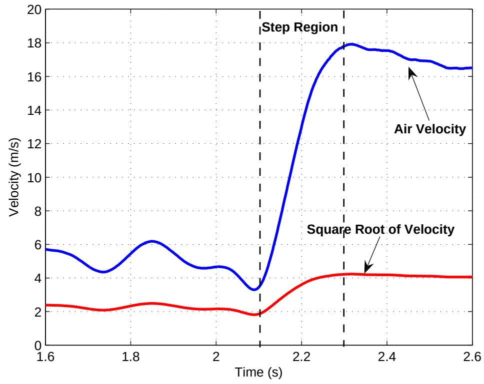

Fig. 6. Step change gas velocity variation.

varying sizes of data window, as shown in Figs. 7 and 8 respectively. The thermocouple data series was pre-filtered at 200 Hz before the sliding window algorithms were applied with window sizes (n) of between 20 and 1000 samples. For clarity, estimates made under window sizes of 200, 400 and 600 samples (0.2, 0.4 and 0.6 s respectively) only are shown. Offline estimates of the pre- and post-step time constants, obtained by applying β-GTLS to the pre- and post-step data respectively, are also plotted for comparison.

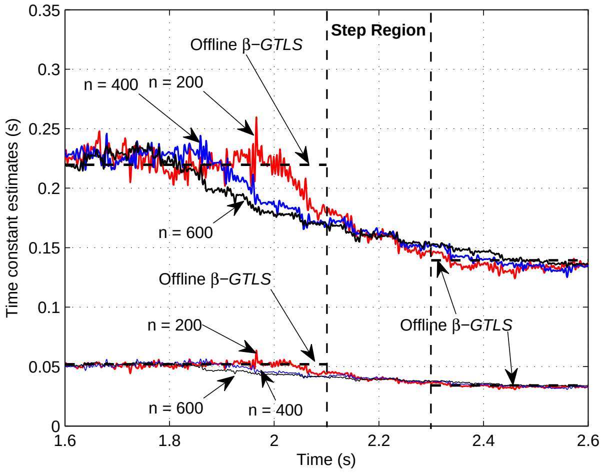

Fig. 7. Time constant estimates using β-GTLS sliding window algorithm with varying sized data windows.

The low-pass filtering effect of the sliding window is shown to be more apparent as sliding window size increases. Furthermore, tracking ability is slowed with increasing window size. When compared to β-GTLS, β-GTLS PS produces estimates with a much greater variation. This variation is due to the ability of the β-GTLS PS algorithm to track time constant variations within each window. Conclusions on accuracy however, cannot be drawn as the true time constant profiles are of course unknown. Time constant estimates predicted by the two algorithms were therefore used to reconstruct the unknown gas temperature and the similarity between the reference (T_{mref}) and reconstructed (T_{gr}) temperatures was quantified by a cross-correlation coefficient:

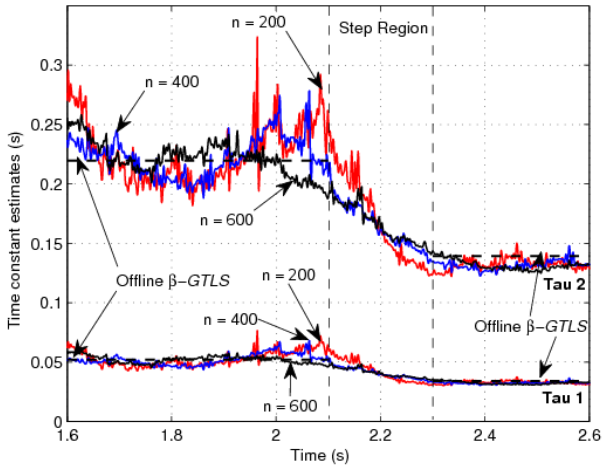

Fig. 8. Time constant estimates with varying sized data windows using β-GTLS PS sliding window algorithm.

$$r = \frac{E\left[T_{mref}T_{gr}\right]}{\sqrt{E\left[(T_{mref})^2\right]}\sqrt{E\left[(T_{gr})^2\right]}}. \tag{20}$$

Figure 9 shows cross-correlation coefficients against window size for the data presented in Fig. 5. With window sizes of below 200 samples, poor correlation-coefficients are produced. When the window size increases above 200 samples, accuracy is improved greatly, although there is little improvement in correlation coefficient in the range above this. This is probably due to the filtering effect of increasing the size of a sliding window and highlights the inability of these algorithms to handle noisy data. Although β-GTLS is shown to give superior correlation coefficients than β-GTLS PS at lower window sizes, this can probably be attributed to the filtering effect of the former, which will reduce noise on data. However, both β-GTLS and β-GTLS PS show significantly higher coefficients than TDR-Kee. Table 1 shows the highest correlation coefficient achieved with each of the algorithms and the corresponding window size. Fig. 10 then shows temperature reconstructions using this data.

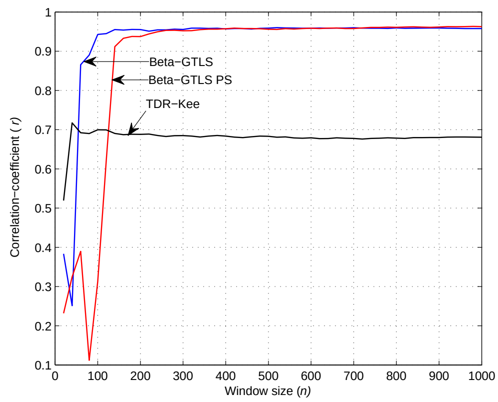

Fig. 9. Correlation-coefficient against sliding window size.

When compared to the original thermocouple data, it is observed that the reconstructed temperatures show a

Table 1. Cross-correlation coefficient for various characterisation algorithms

	TDR-Kee	β-GTLS	β-GTLS PS
n	40	700	980
r	0.7173	0.9602	0.9632

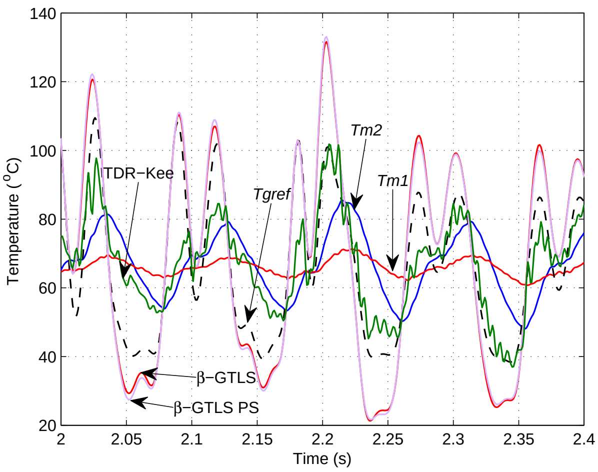

Fig. 10. Gas temperature reconstructions using various algorithms.

marked improvement in correlation with the reference temperature and a dramatic increase in sensor bandwidth. This is particularly evident with β-GTLS and β-GTLS PS, with the latter giving marginal improvements in accuracy. In terms of amplitude, it can be seen that T_{gr} does not precisely match the reference temperature. This may be due to sensor separation or alternatively, the sensor for the reference temperature T_{gref} might not be fast enough to follow the true gas temperature. Finally, Fig. 11 shows the reconstructed temperature profile when using β-GTLS PS and the optimum window size of 980 samples.

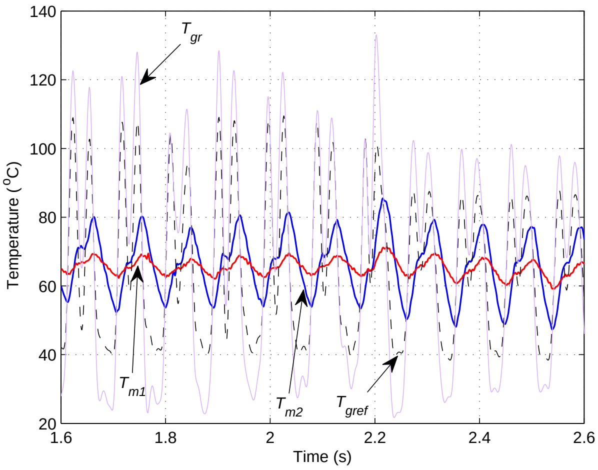

Fig. 11. Gas temperature reconstruction using β-GTLS PS algorithm with window size of 980 samples.

7. CONCLUSION

A novel sliding window system identification based two-thermocouple sensor characterisation algorithm has been successfully tested in variable velocity flow applications for the first time. Application to experimental data has confirmed the superiority of the sliding window approach to continuous time domain based methods. A polynomial fitting extension to this algorithm, capable of tracking time constant fluctuations over a window has shown marginal performance benefits. The increase in sensor bandwidth arising from the TTP method combined with the β-GTLS PS algorithm has allowed accurate measurement of fluctuating temperatures with relatively robust thermocouples. Experimental tests have highlighted that both sensors must be exposed to the same conditions and additionally that relatively fine thermocouples are required.

ACKNOWLEDGEMENTS

The first author wishes to acknowledge the financial support of both the Virtual Engineering Centre, Queen's University Belfast and the European Social Fund. The practical assistance of Mr Maurice Doherty is also gratefully acknowledged.

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