Sensor Location Based Optimum Design for Fault Detection System

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Abstract: This paper addresses the optimum design of fault detection systems based on sensor location. A multi-objective optimization problem based on optimal sensor location for fault detection is formulated for linear time invariant system. Measurement outputs are formed by selecting m variables of N available process measurements that ensures a high fault detetion performance. A minimum total measurement cost can be achieved when the system is designed to be as sensitive as possible to faults and simultaneously as robust as possible to the unknown inputs such as disturbance. The simulation results illustrate the effectiveness of the proposed approach.

1. INTRODUCTION

Since the late 70's, much research attention has been paid to the issues around optimal sensor location (Bagajewicz, 2001), as process monitoring in industrial processes has become increasingly important (Bhushan and Rengaswamy, 2000; Chmielewski et al., 2002; Yu et al., 2003; Löhner and Camelli, 2005). Lambert used fault-trees to analyze the problem of sensor location depending on the effect of basic units on the process variables (Lambert, 1977). Iri et al. were the first to introduce the cause-effect analysis of the system based on Signed Directed Graphs (SDG)(Iri et al., 1979). Later, Kramer, Chang, and Mohindra have systemically developed the rule-based approach using SDG technique for fault diagnosis (Kramer and Palowitch, 1987; Chang and Yu, 1990; Mohindra and Clark, 1993). Recently Raghuraj et al. and Bhushan et al. have applied this technique to study the problem of optimal sensor location based fault diagnosis (Raghuraj et al., 1999; Bhushan and Rengaswamy, 2002). These are knowledge-based approaches (Zamprohna et al., 2005). In Reference (Commault and Dion, 2003), a model-based approach has been presented, where the optimum for optimal sensor location is obtained by means of a digraph-based approach by transforming the analytical mode into a form of bi-digraph.

The major objective of this paper is to present a model-based optimum design approach to fault detection systems based on optimal sensor location. In this approach, a cost function, under the condition that the designed system is as sensitive as possible to faults and as robust as possible to the unknown inputs such as disturbance simultaneously, will be minimized.

2. OPTIMAL SENSOR LOCATION PROBLEM FORMULATION

Considering the following linear discrete-time systems described by

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) + E_d d(k) + E_f f(k) \\ y(k) &= Cx(k) + Du(k) + F_d d(k) + F_f f(k) \end{aligned} . \tag{1}$$

where $x(k) \in R^n$, $u(k) \in R^{k_u}$, and $y(k) \in R^m$ denote the vectors of states, inputs and outputs, respectively, $d(k) \in R^{k_d}$ denotes a unknown input vector that represents the disturbances, noise, and structured model uncertainty which is bounded by $\|d\|_2 \le \Delta_d$ or faults of no interest, and $f(k) \in R^{k_f}$ is the unknown vector that represents all possible additive faults that have to be detected and isolated from the faults of no interest. A, B, C, D, E_d, E_f, F_d and F_f are known and appropriate dimensions.

If there are M sensors available for N measure locations (measured variables) of process, then the available output equation can be described as:

$$z_a(k) = C_a x(k) + D_a u(k) + F_{ad} d(k) + F_{af} f(k) . \tag{2}$$

where $z_a \in R^N$, C_a, D_a, F_{ad} and F_{af} are known and of appropriate dimensions. The subscript a represents the parameters of the available measurement output. In our study, the following assumptions are made:

(A1) (C_a, A) is detectable;

(A2) $\begin{bmatrix} A - j\omega I & E_d \\ C_a & F_{ad} \end{bmatrix}$ has full row rank for all ω .

An introduced matrix Q is used to determine which m ($m < N$) sensors out of N locations available will be selected as measurement outputs. This selection would be done by an optimal trade-off between the fault detection performance and the total measurement cost. Entries q_{ij} of $Q \in R^{N\times N}$ is a binary variables defined by

$$q_{ij} = \begin{cases} q_{ij} = \begin{cases} 1 \text{ if } z_{ai} \text{ is selected as measure output} \\ 0 \text{ if } z_{ai} \text{ is not selected as measure output} \end{cases}, i, j = 1, \cdots, N. \\ q_{ij} = 0 \ (i \neq j) \end{cases} \quad (3)$$

As a result, the output equation can be described by

$$y_a(k) = Qz_a(k) = Q(C_a x(k) + D_a u(k) + F_{ad} d(k) + F_{af} f(k)). \quad (4)$$

According to Bagajewicz (Bagajewicz, 2001) and Chmielewski (Chmielewski, et al., 2002), the total measurement cost is now defined by

$$\phi = \sum_{i=1}^{N} q_{ii}\varphi_i \leq \phi_a \ . \quad (5)$$

where ϕ_a is the total cost of M sensors available process, φ_i is the cost of the sensor at i-th measure. The optimal sensor location problem becomes to determine the selection matrix Q. Q should be design to optimize the trade-off between the total measurement cost and the fault detection performance regarding to the sensor selection.

3. PROBLEM FORMULATION FOR FAULT DETECTION BASED ON OPTIMAL SENSOR LOCATION

A parity relation based residual generator for system (1) and (4) is expressed by

$$r(k) = V_s(y_s(k) - Q_s H_{au,s} u_s(k)) \quad (6)$$

which gives

$$r(k) = V_s Q_s (H_{ad,s} d_s(k) + H_{af,s} f_s(k)) \quad (7)$$

where $V_s \in R^{\gamma\times(s+1)N}$ is the so-called parity matrix belonging to parity space Π_s defined by

$$\Pi_s = \left\{ v_s^i \mid v_s^i Q_s H_{ao,s} = 0, v_s^i = \begin{bmatrix} v_{s,0}^i & v_{s,1}^i & \cdots & v_{s,s}^i \end{bmatrix}, i = 1, \cdots, \gamma \right\},$$

its row vector $v_s^i \in R^{1\times(s+1)N}$ is the parity vectors belonging to the parity space, s is the order of the residual generator and γ is the dimension of parity space (Ding and Guo,1998; Zhang et al., 2003; Ye et al., 2004). Choosing the parity vector v_s^i in some optimum sense can make the residual robust against the disturbances and sensitive to the fault simultaneously. In (6)-(7), the notations are defined as follows:

$$y_s(k) = \begin{bmatrix} y(k-s) \\ y(k-s+1) \\ \vdots \\ y(k) \end{bmatrix}, \quad u_s(k) = \begin{bmatrix} u(k-s) \\ u(k-s+1) \\ \vdots \\ u(k) \end{bmatrix},$$

$$f_s(k) = \begin{bmatrix} f(k-s) \\ f(k-s+1) \\ \vdots \\ f(k) \end{bmatrix}, \quad d_s(k) = \begin{bmatrix} d(k-s) \\ d(k-s+1) \\ \vdots \\ d(k) \end{bmatrix},$$

$$Q_s = \begin{bmatrix} Q & 0 & \cdots & 0 \\ 0 & Q & \ddots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & Q \end{bmatrix} \in R^{(s+1)N\times(s+1)N}, \quad H_{ao,s} = \begin{bmatrix} C_a \\ C_a A \\ \vdots \\ C_a A^s \end{bmatrix} \in R^{(s+1)N\times n},$$

$$H_{au,s} = \begin{bmatrix} D_a & 0 & \cdots & 0 \\ C_a B & D_a & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ C_a A^{s-1} B & \cdots & C_a B & D_a \end{bmatrix} \in R^{(s+1)N\times(s+1)k_u},$$

$$H_{ad,s} = \begin{bmatrix} F_{ad} & 0 & \cdots & 0 \\ C_a E_d & F_{ad} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ C_a A^{s-1} E_d & \cdots & C_a E_d & F_{ad} \end{bmatrix} \in R^{(s+1)N\times(s+1)k_d},$$

$$H_{af,s} = \begin{bmatrix} F_{af} & 0 & \cdots & 0 \\ C_a E_f & F_{af} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ C_a A^{s-1} E_f & \cdots & C_a E_f & F_{af} \end{bmatrix} \in R^{(s+1)N\times(s+1)k_f}.$$

Residual generator (7) can be further presented by

$$r(k) = P_s N_{b,s} Q_s (H_{ad,s} d_s(k) + H_{af,s} f_s(k)). \quad (8)$$

Where $N_{b,s} \in R^{\gamma N\times(s+1)N}$ is generalized basis matrix of the parity space Π_s, P_s is the parity matrix of basis $N_{b,s}$.

Thus, it is a natural way to evaluate the robustness to the unknown inputs using an induced norm of matrix $P_s N_{b,s} Q_s H_{ad,s}$:

$$R_d := \left\| P_s N_{b,s} Q_s H_{ad,s} \right\| = \sup_{d_s \neq 0} \frac{\left\| P_s N_{b,s} Q_s H_{ad,s} d_s(k) \right\|}{\left\| d_s(k) \right\|}. \quad (9)$$

In comparison, a worst-case for the sensitivity evaluation should, in fact, be the minimum influence of the faults on the residual vector which can be expressed in terms of

$$S_{f,-} = \inf_{f_s \neq 0} \frac{\left\| P_s N_{b,s} Q_s H_{af,s} f_s(k) \right\|}{\left\| f_s(k) \right\|}. \quad (10)$$

The residual evaluation function can be introduced by 2-norm of residual vector, threshold J_{th} is given by

$$J_{th} = \sup_{d_s, f_{s=0}} \left\| r \right\|_2 = \sup_{d_s(k)} \left\| P_s N_{b,s} Q_s H_{ad,s} d_s(k) \right\|_2. \quad (11)$$

where it is assumed that the unknown inputs are bounded by

$$\left(\sum_{i=0}^{s} d^T(k-i)d(k-i)\right)^{1/2} \le \Delta_d . \quad (12)$$

The problem of fault detection systems design can be formulated as a multi-criterion optimization problem: Given a residual generator (8), find matrix P_s and Q such that $R_d \to \min$, $S_{f,-} \to \max$.

A performance index was introduced by Ding et al. (Ding and Guo,1998), which takes the form

$$\min_{P_s} J_{R/S} = \min_{P_s} \frac{R_d}{S_{f,-}} . \quad (13)$$

The task of optimum design for fault detection systems based on optimal sensor location is described as: m ($m < N$) variables out of N available process measurements are selected for measurement outputs, so as to obtain the amount of information about the fault of process as more as possible, achieve the total measurement cost as less as possible simultaneously. That is, given a residual generator (8), find a optimal select matrix Q and parity space matrix P_s, such that system is asymptotically stable and

$$J_{R/S} \to \min . \quad (14)$$

$$\phi \to \min . \quad (15)$$

4. DESIGN SCHEMES

At first, consider solution of the optimal problem of fault detection performance. (14) can be written as

$$\begin{aligned}\min_{P_s} J_{R/S} &= \min_{P_s} \frac{R_d}{S_{f,-}} \\ &= \min_{p} \frac{p_{s,\min} N_{b,s} Q_s H_{ad,s} H_{ad,s}^T Q_s^T N_{b,s}^T p_{s,\min}^T}{p_{s,\min} N_{b,s} Q_s H_{af,s} H_{af,s}^T Q_s^T N_{b,s}^T p_{s,\min}^T}\end{aligned} . \quad (16)$$

The following theorem provides us with a solution for the optimization problem (16).

Theorem 1: Given a linear discrete-time systems described by (1) and (4). The optimal solution of (16) is given by

$$v_{s,opt} = p_{s,\min} N_{b,s} Q_s . \quad (17)$$

Where $p_{s,\min}$ is the eigenvector corresponding to the minimal eigenvalue $\lambda_{\min}$ of the generalized eigenvalue-eigenvector problem

$$v_{s,opt}(H_{ad,s} H_{ad,s}^T - \lambda_{\min} H_{af,s} H_{af,s}^T) = 0 . \quad (18)$$

Proof see for instance reference (Peng, 2005).

Secondly, consider the solution of system cost optimal problem. (15) can be described as the form of a single objective optimization with a constraint condition, that is

$$\begin{aligned}&\phi \to \min \\ &\text{s.t. } v_{s,opt} = p_{s,\min} N_{b,s} Q_s\end{aligned} . \quad (19)$$

The implementation procedure of the optimum design approach to fault detection systems based on optimal sensor location can be summarized as follows.

i) Set the order s of the residual generator, which is not small than $s_{max}=n-m+1$;

ii) Calculate $H_{ao,s}$, $H_{au,s}$, $H_{ad,s}$, $H_{af,s}$;

iii) Calculate the dimension γ of parity space, and the generalized basis matrix $N_{b,s}$ of parity space;

iv) Solve the generalized eigenvalue-eigenvector problem (18), and determine the optimal parity vector $v_{s,opt}$;

v) Solve the problem (19) of the single objective optimization with constraints, and determine Q;

vi) Construct of residual generator (8);

vii) Determine the threshold according to (11), and implement the residual evaluation.

5. NUMERICAL EXAMPLE

In order to illustrate the design approach proposed in the last section, a numerical example with model (20) is considered. It is assumed that the system sampling period is 0.1s and the disturbance $d(k)$ is white noise bounded by 0.5. Suppose that an impulse fault whose amplitude is 10 occurs. Set s=3, suppose that cost vector that sensors are placed at 4 measurement points is ϕ = [10.0 19.0 35.0 64.0].

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \\ x_4(k+1) \end{bmatrix} = \begin{bmatrix} 0.5 & -0.7 & 0.7 & 0 \\ 0 & 0.8 & 0 & 0 \\ -1 & 0 & 0 & 0.1 \\ 0 & 0 & 0 & 0.4 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \\ x_4(k) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} u(k) + \begin{bmatrix} 0 \\ 0.6 \\ 0 \\ -1 \end{bmatrix} d(k) + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} f(k)$$

$$\begin{bmatrix} y_1(k) \\ y_2(k) \\ y_3(k) \\ y_4(k) \end{bmatrix} = \begin{bmatrix} 1 & 0.5 & 0.4 & 1.2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \\ x_4(k) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} d(k) + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} f(k) . \quad (20)$$

By using the algorithm given in last section, we get $Q=diag(1\ 1\ 1\ 0)$. That is, sensors shown in model (20) are only placed at the 1st, the 2nd, and the 3rd measurement points, residual generator is constructed as

$$\begin{aligned} r(k) = &\ [-0.4195\ \ -0.2740\ \ -0.3343\ \ -0.2497] d(k) \\ &+ [-1.7841\ \ -0.9940\ \ -0.5217\ \ -0.2497] f(k) \end{aligned}$$

Its residual signal figure and residual evaluation result are shown by figures 1 and 2 respectively.

In order to validate the effectiveness of approach proposed, sensors shown in model (20) are placed at 4 measurement points, residual generator is constructed as

$$\begin{aligned} r(k) = &\ [-0.2414\ \ -0.3357\ \ -0.4139\ \ -0.0565] d(k) \\ &+ [-1.6835\ \ -1.0304\ \ -0.4195\ \ -0.0565] f(k) \end{aligned}$$

Its residual signal figure and residual evaluation result are shown by figures 3 and 4 respectively.

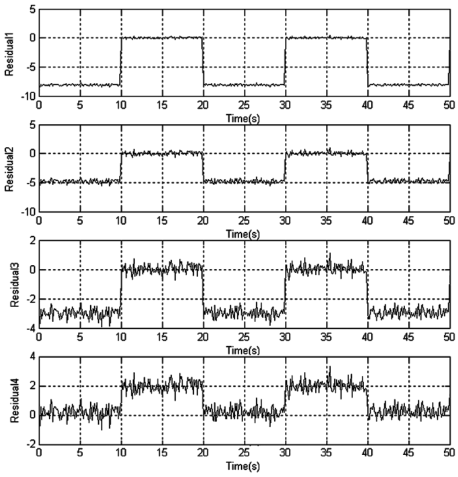

Fig.1. Residual signal figure by optimal sensor location

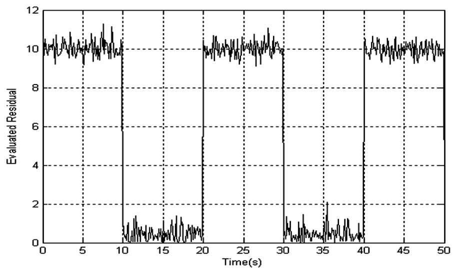

Fig. 2. Residual evaluation figure by optimal sensor location

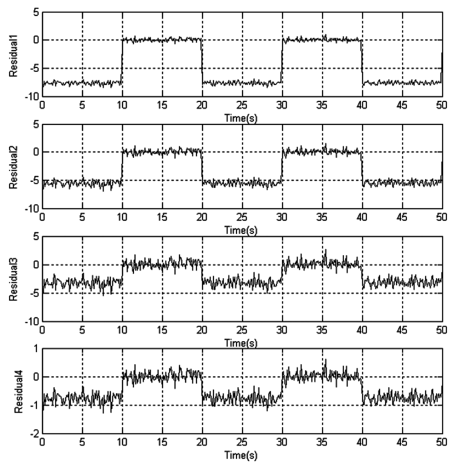

Fig. 3. Residual signal figure not by optimal sensor location

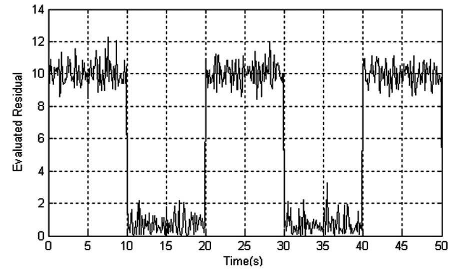

Fig. 4. Residual evaluation figure not by optimal sensor location

Comparing residual signals given in figures 3 and 4 with sensors being placed at 4 measurement points with ones in figures 1-2 with sensors being placed at 3 measurement points by optimal sensor location, it can be seen that both residual evaluation results are essentially consistent while residual signals are different. It demonstrates that the residual signal by optimal sensor location can be gotten as intensive as in normal case with a reduced cost.

6. CONCLUSIONS

In this paper, a model-based approach to fault detection systems based on optimal sensor location has been proposed for linear time invariant system. After a multi-objective optimization problem for fault detection system design is formulated, m variables, which ensure the desired fault detection performance, out of N available process measurements are selected as measurement outputs. In the case that the designed system is as sensitive as possible to faults and as robust as possible to the unknown inputs such as disturbance simultaneously, a minimization of the total measurement cost can be achieved. The simulation results illustrate the effectiveness of the proposed approach.

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