Equalisation Tuning Method

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Abstract: The paper presents a novel tuning method for different types of controllers. Since the tuning method is trying to equalise the closed-loop response to the open-loop response, it is named "Equalisation tuning method". The main advantage of this method is that it does not require any additional data from the operator except the measurement of the process steady-state change in an open-loop experiment. The equalisation method is also relatively insensitive to process output noise. Simplicity and efficiency of the method is demonstrated on several process models and on a hydraulic laboratory plant. Matlab and Simulink files are provided.

1. INTRODUCTION

Tuning of PID controllers has been attracting interest for six decades. Numerous methods suggested so far try to accomplish the task by using different information from the process.

In tuning, like in other decision-making tasks, it is very important to know the amount of information required from the process in order to accomplish the task. Achieving maximal performance at minimal requested knowledge is the key motivation for most of the tuning methods.

In general, the tuning methods can be divided into the model-based and non-model-based methods. The former ones require an explicit process model, while the later use non-parametric data (usually time-domain measurements) for controller tuning. Non-model-based methods are therefore simpler for implementation in practice.

The representatives of the non-model-based methods are Magnitude Optimum Multiple Integration (MOMI) method (Vrančić et al., 2001) and Direct Adaptive Controller (DIRAC) method (DeKeyser, 2000). Both methods are very simple and straightforward for implementation in practice.

The MOMI tuning method calculates controller parameters by consecutive integrations of the process input and output signals during steady-state change. However, the method is sensitive to process disturbances during identification stage.

The DIRAC method calculates the controller parameters from the process step-response and is less sensitive to disturbances and noise, but it requires a-priori definition of desired closed-loop transfer function. If the desired closed-loop transfer function is not chosen properly, the resulting closed-loop performance may be deteriorated.

In this paper it is shown that the mentioned shortcoming of DIRAC method can be avoided by equalising the closed-loop and the open-loop response of the process. The proposed equalisation method requires only measurement of the process steady-state change in time-domain. Therefore, the operator is implicitly giving desired closed-loop transfer function by driving the process from initial to the final steady-state (not necessarily the process step-response!). The equalisation method does not depend on controller structure.

The paper is organized as follows. Section 2 shortly describes continuous-time DIRAC method. The proposed equalisation method is described in Section 3. Next section presents the results on several process models and on one laboratory plant. The last section gives some conclusions and comments.

2. CONTINUOUS 2-DOF DIRAC METHOD

The original Direct Adaptive Controller method (DIRAC) has been developed for discrete 1-degree-of-freedom controllers (DeKeyser, 2000). In this section a continuous version of DIRAC method for 2-degrees-of-freedom (2-DOF) controllers will be derived.

Figure 1 shows 2-DOF controller (G_{CR} and G_{CY}) and the process (G_P) in the closed-loop configuration.

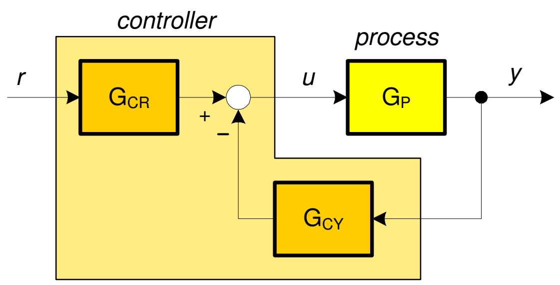

Fig. 1. Two-degrees-of-freedom controller and the process in the closed-loop configuration

The closed-loop transfer function (from the reference signal r to the process output variable y) is:

$$G_{CL} = \frac{Y}{R} = \frac{G_{CR}G_P}{1+G_{CY}G_P}, \quad (1)$$

where capital letters denote Laplace transforms. Expression (1) can be rewritten as follows:

$$(G_{CR} - G_{CY}G_{CL})G_P = G_{CL} \quad (2)$$

Then multiplying the expression (2) by the transform of the process input signal U yields the following expression:

$$(G_{CR} - G_{CY}G_{CL})G_P U = G_{CL} U . \quad (3)$$

From the process input-output relation

$$Y = G_P U , \quad (4)$$

the expression (3) can be rewritten into:

$$(G_{CR} - G_{CY}G_{CL})Y = G_{CL} U \quad (5)$$

Multiplication with signal U is the most important step, since the process model (G_P) disappears from expression (5)!

By defining the filtered signals y_F and u_F:

$$\begin{aligned} Y_F &= G_{CL} Y \\ U_F &= G_{CL} U \end{aligned}, \quad (6)$$

expression (5) becomes even simpler:

$$G_{CR} Y - G_{CY} Y_F = U_F . \quad (7)$$

Controller parameters can therefore be calculated by using least-squares identification method on signals u_F, y and y_F. Note that signals u and y can be obtained from the process step-response.

The main disadvantage of DIRAC tuning method is that desired closed-loop transfer function (G_{CL}) should be chosen a-priori by the user (operator). If the desired closed-loop transfer function is not chosen properly, the resulting closed-loop performance may be deteriorated.

3. EQUALISATION METHOD

According to Fig. 1, controller output signal is:

$$U_{CL} = RG_{CR} - Y_{CL}G_{CY} , \quad (8)$$

where U_{CL}, R and Y_{CL} denote Laplace transforms of the controller output, the reference and the process output in the closed-loop configuration, respectively.

If desired closed-loop responses (u_{CL} and y_{CL}) are chosen to be the same as the measured open-loop responses (u_{OL} and y_{OL}), similar to the Balanced tuning approach proposed by Klán and Gorez (2000), then the following equation also holds:

$$U_{OL} \cong RG_{CR} - Y_{OL}G_{CY} . \quad (9)$$

Transfer functions G_{CR} and G_{CY} can be calculated from expression (9) by using the least-square method or by some optimisation method. Note that the last expression contains "more or less equal" sign, since degree of equality depends on controller order and structure as well as on degree of process non-linearity.

Signals u_{OL} and y_{OL} are measured during the experiment. However, the reference signal (r) is not measured and should be defined as follows: r should have the same initial and final values as y_{OL} (if controller has integral term) and should make step-change (from the initial to the final value) at time instant when u_{OL} changes for the first time (see Figure 2). Therefore, the reference signal r can be easily obtained from measured u_{OL} and y_{OL}.

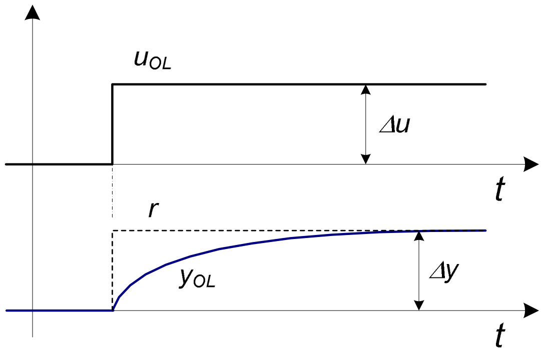

Fig. 2. Process input (u_{OL}), process output (y_{OL}) and the calculated reference (r) during an open-loop experiment.

It should be pointed out that the proposed Equalisation method does not depend on controller structure. Moreover, it can be applied to non-linear controllers as well. However, in the subsequent derivations the PID controller structure will be assumed.

The following 2-DOF PID controller has been chosen:

$$\begin{aligned} G_{CR} &= \frac{K_i}{s} + K \\ G_{CY} &= \frac{K_i}{s} + K + K_d s \end{aligned} , \quad (10)$$

where K_i, K and K_d are integral, proportional and derivative gains of the PID controller. Note that the derivative term is not connected to the reference in order to avoid excessive kicks of signal u_{CL} during reference changes. Namely, the mentioned kicks are usually not present in the process open-loop responses (u_{OL}).

Expression (9) can be rearranged as follows:

$$\frac{K_i}{s}(R - Y_{OL}) + K(R - Y_{OL}) - K_d s Y_{OL} = U_{OL} \quad (11)$$

In time-domain, expression changes into:

$$K_i\left[\int_0^t (r(\tau) - y_{OL}(\tau))d(\tau)\right] + K[r(t) - y_{OL}(t)] - K_d\left[\frac{dy_{OL}(t)}{dt}\right] = u_{OL}(t) . \quad (12)$$

Since expression (12) includes pure derivation, all the signals (r, y_{OL}, and u_{OL}) should be appropriately filtered by using simple first-order filter. The filter time constant is not critical and can be defined as a fraction of time of experiment.

The controller parameters can be calculated by using least-squares method or any other appropriate method including optimisation. An advantage of using the least squares method is that variance of parameters K_i, K and K_d can be estimated.

The variance can serve as an indicator of the quality of controller tuning.

The PID controller tuning proceeds as follows:

1. Make experiment on the process so as to change the steady-state of the process (not necessarily the step response!) and measure process input (u_{OL}) and process output (y_{OL}) signals.
2. Calculate reference signal (r) according to Figure 2
3. Filter all three signals by the first-order filter (time constant can be e.g. 5% of time of experiment)
4. Calculate controller parameters (K_i, K and K_d) from expression (12) by using least-squares method
5. If needed, calculate variance of controller parameters as an indicator of the quality of controller tuning.

The tuning procedure is simple and straightforward. The Matlab files performing tuning from the process time-responses are available on-line (Vrančić, 2007).

4. EXAMPLES

The equalisation tuning method is tested on four process models and on a hydraulic laboratory plant.

4.1 Experiments on process models

The proposed method is tested on the first-order process (G_{P1}), fourth-order process (G_{P2}), delayed process (G_{P3}) and non-minimum phase process (G_{P4}):

$$\begin{aligned} G_{P1} &= \frac{1}{1+4s} \\ G_{P2} &= \frac{1}{(1+s)^4} \\ G_{P3} &= \frac{e^{-2s}}{1+2s} \\ G_{P4} &= \frac{1-2s}{(1+s)^2} \end{aligned} \qquad (13)$$

The open-loop response of the process G_{P1} is given in Figure 3. The reference signal is calculated from the measured process input and output signals (see upper plot in Figure 3). The controller parameters have been calculated by using the least-squares method:

$$K_i = 0.261,\ K = 0.991,\ K_d = 0\ . \qquad (14)$$

The closed-loop responses are shown in Figure 3. It can be seen that the closed-loop responses are very similar to the open-loop responses.

Matlab and Simulink files, performing calculations and simulations for all the processes (13), are available on-line (Vrančić, 2007).

The open-loop response of the fourth-order process (G_{P2}) is given in Figure 4. The controller parameters have been calculated by using the least-squares method:

$$K_i = 0.251,\ K = 0.846,\ K_d = 0.937\ . \qquad (15)$$

Note that the derivative term in (11) should be filtered by the first-order filter for implementation in practice. The time constant of the filter is chosen as 0.2 times derivative time constant:

$$\begin{aligned} G_{CR} &= \frac{K_i}{s} + K \\ G_{CY} &= \frac{K_i}{s} + K + \frac{K_d s}{1 + 0.2 s \frac{K_d}{K}} \end{aligned} \ . \qquad (16)$$

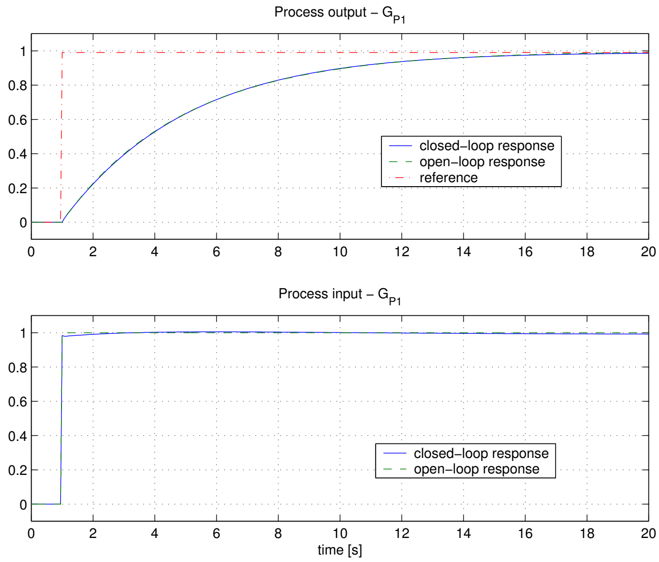

Fig. 3. Process G_{P1}: Open-loop and closed-loop responses.

The closed-loop responses are shown in Figure 4. It can be seen that the closed-loop response is again very similar to the open-loop response.

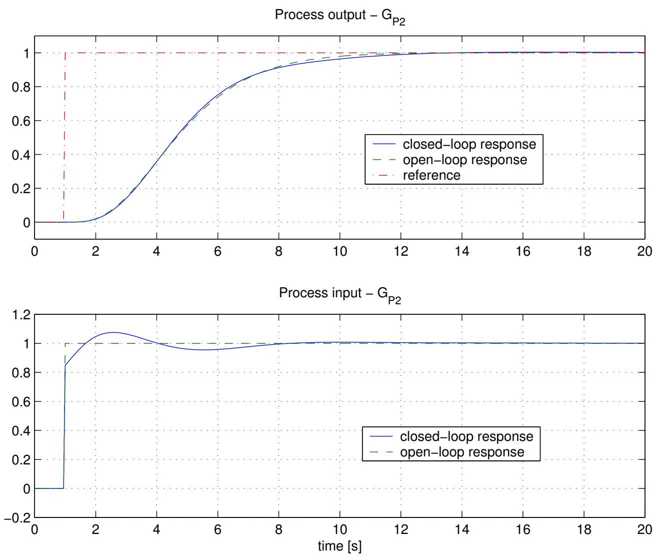

Fig. 4. Process G_{P2}: Open-loop and closed-loop responses.

The open-loop response of delayed process (G_{P3}) is given in Figure 5. The calculated controller parameters are:

$$K_i = 0.251,\ K = 0.769,\ K_d = 0.625\ . \tag{17}$$

The closed-loop responses are also shown in Figure 5. The closed-loop responses are still similar to the open-loop responses.

The open-loop response of the non-minimum phase process (G_{P4}) is given in Figure 6. The calculated controller parameters are:

$$K_i = 0.250,\ K = 0.501,\ K_d = 0.252\ . \tag{18}$$

The closed-loop responses are shown in Figure 6 (solid lines). The closed-loop responses are again very similar to the open-loop responses, like in all the previous cases.

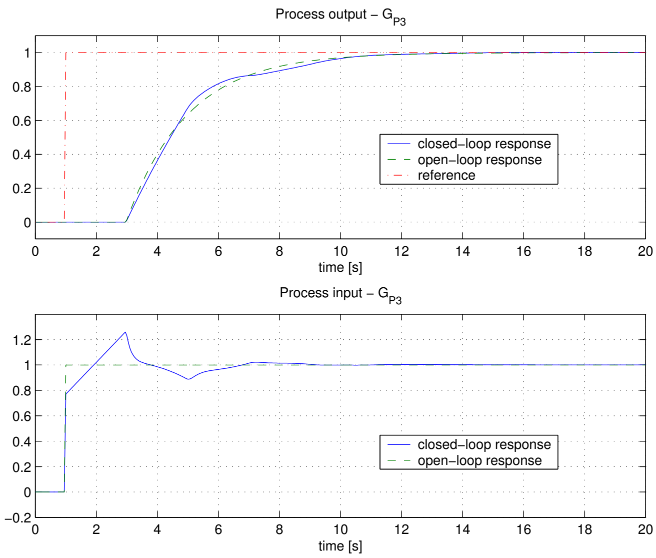

Fig. 5. Process G_{P3}: Open-loop and closed-loop responses.

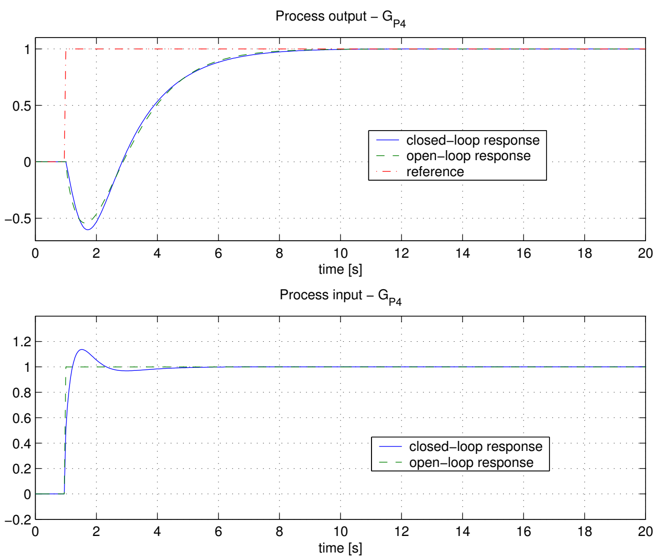

Fig. 6. Process G_{P4}: Open-loop and closed-loop responses.

Although the tested processes vary in their nature (low-order, higher-order, delayed and non-minimum phase processes), the equalisation method makes the closed-loop responses very similar to the open-loop ones. There is no additional data required by the user, except simple steady-state change of the process.

4.2 *Sensitivity to noise*

Before equalisation method has been implemented on the real plant, a sensitivity of the method to the noise has been tested on the following second-order process:

$$G_P = \frac{1}{(1+s)^2} \tag{19}$$

A noise with power 0.0001 (Simulink block "Band limited white noise") has been added to the process output. Sampling time was 0.05s. Typical open-loop and closed-loop responses are shown in Figure 7.

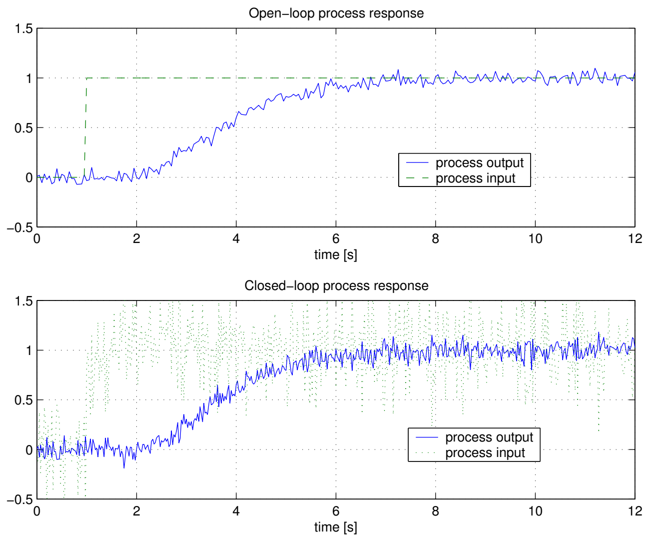

Fig. 7. Typical open-loop and closed-loop responses of the second-order system with added noise.

The closed-loop response of the process output is still similar to the open-loop response.

Two hundred noise signals with different seeds and the same noise power have been added to the process open-loop response. The closed-loop experiments were performed without noise in order to clearly show the difference between the closed-loop responses.

Figure 8 shows all the 200 open-loop and closed-loop responses. It is clear that the method is quite robust to process noise. Figure 9 gives histogram of all three controller parameters. Standard deviation for all three parameters is relatively low.

Matlab and Simulink files which perform all the calculations and simulations are available on-line (Vrančić, 2007).

4.3 Experiment on hydraulic plant

The last experiment was performed on a hydraulic laboratory set-up consisting of three water tanks (Figure 10). A schematic diagram of the set-up is given in Figure 11. The process input is the voltage on pump P_1 and the process output is the water level in the second tank (h_2). Valves V_2 and V_3 are closed. The laboratory set-up is non-linear due to non-linear relation between the liquid level and the flow.

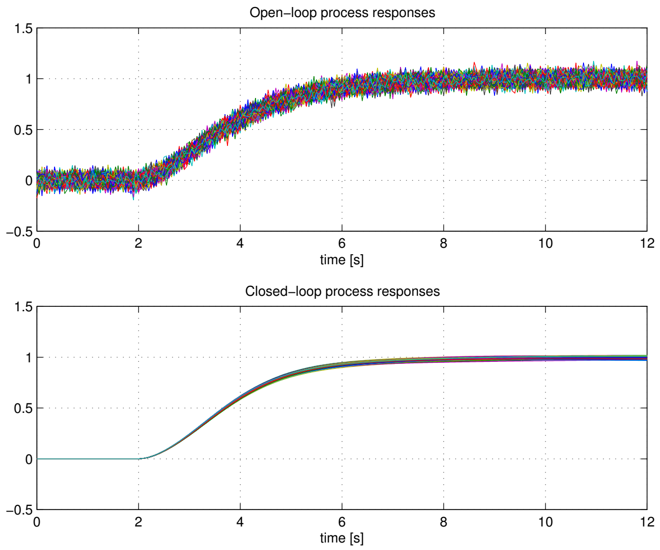

Fig. 8. Two hundred open-loop and closed-loop responses of the second-order system with added noise.

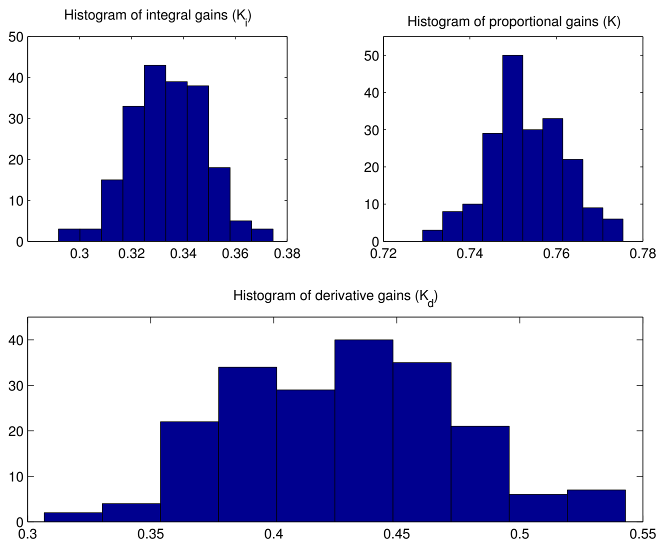

Fig. 9. Histogram of PID controller parameters for the second-order system with added noise.

Two experiments were performed on the set-up. First, a voltage step change was applied to pump P_1. The process output (water level h_2 in volts) is shown in Figure 12. From measured open-loop response, the following PID controller parameters were obtained:

$$K_i = 0.0091,\ K = 0.64,\ K_d = 11.25\ . \tag{20}$$

note that all the measurements, Matlab and Simulink files are given on-line in Vrančić (2007).

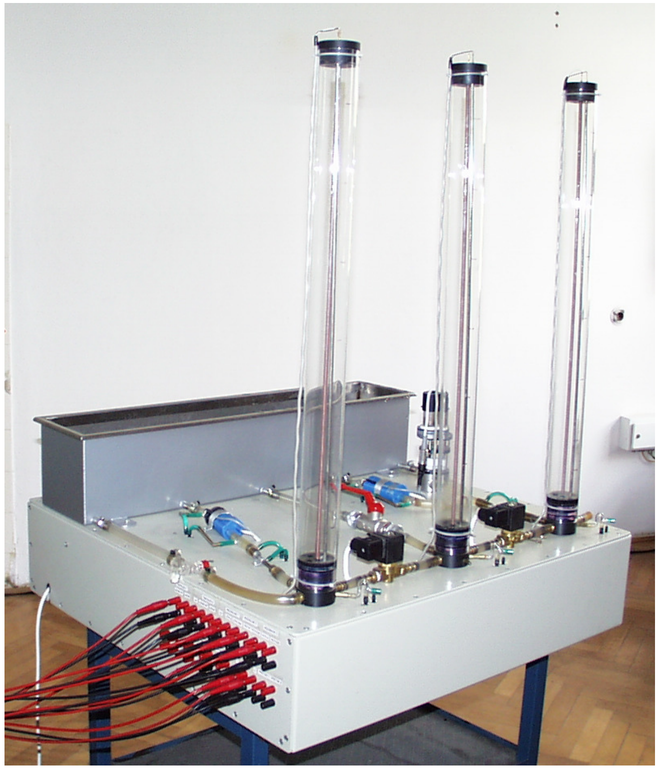

Fig. 10. Hydraulic laboratory set-up.

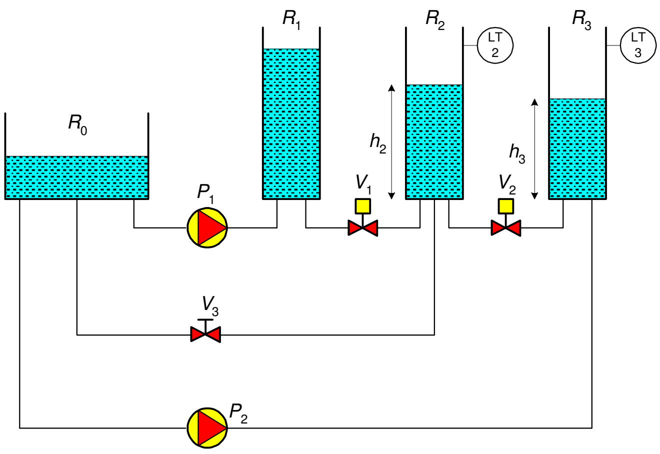

Fig. 11. Schematic diagram of the hydraulic laboratory set-up.

According to Figure 12, the closed-loop response is very similar to the open-loop response, all according to the equalisation approach.

The second experiment was performed on the same plant. This time the voltage on the plant was manipulated manually so as to increase the speed of response. The open-loop response is shown in Figure 13 (broken lines).

From measured open-loop response the following PID controller parameters were obtained (also given on-line in Vrančić, 2007):

$$K_i = 0.0245\,,\ K = 1.76,\ K_d = 0\ . \qquad (21)$$

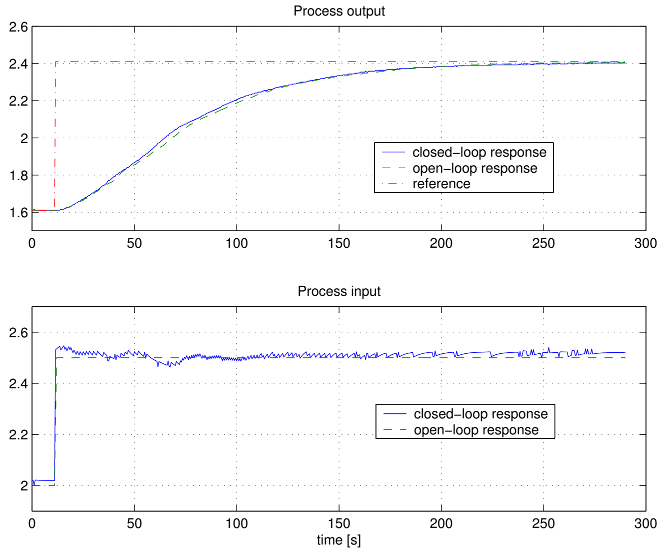

Fig. 12. The open-loop and the closed-loop response of the hydraulic set-up during the first experiment.

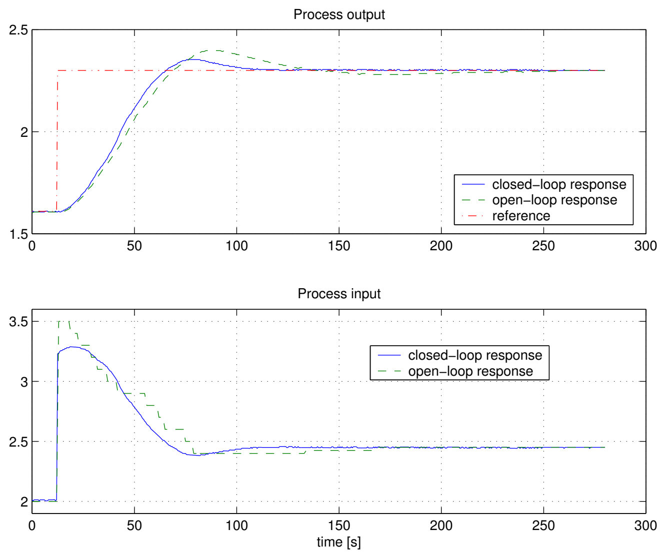

Fig. 13. The open-loop and the closed-loop response of the hydraulic set-up during the second experiment.

Although there are some differences between the open-loop and the closed-loop responses, both responses are still quite similar.

5. CONCLUSIONS

The paper has shown that controllers designed to keep the closed-loop response near to the process open loop dynamics can be derived directly from the measured process data achieved by simple experiments and by solving relatively simple sets of equations. This can reasonably simplify the plant identification and process parametrization usually required by other methods leading to the equivalent closed loop dynamics (Huba, 2006; Klán and Gorez, 2000).

Simplicity of the proposed approach has also been demonstrated on the hydraulic laboratory plant. Another advantage of the method is that the measured open-loop response of the process is not limited to process step-response.

Application of the method to some unstable processes and some other controller structures and types of controllers is foreseen. Including of derivative filter in identification stage and generalisation of DIRAC method is also under our investigation.

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