Model-Based Sensor Fault Diagnosis in General Stochastic Systems Using LMI Techniques

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Abstract: In this paper a method for sensor Fault Detection and Isolation (FDI) in non-Gaussian stochastic distribution control systems is proposed. As the output PDF is assumed measurable in probability density control methods, availability of a reliable output PDF measurements is vital. However, sensor faults occurred in practical cases can considerably affect the efficiency of the proposed PDF control algorithms. As such, studying sensor FDI in non-Gaussian stochastic distribution control systems is important. The purpose of this paper is to detect and diagnose PSD measurement sensors in a non-Gaussian system working normally under a PID control law. The proposed method is comprised of two stages, *a)* Nonlinear observer supervisory system design to continuously monitor Fault Detection Criteria (FDC). *b)* Nonlinear fault diagnosis filter design to estimate the value of the fault signal detected. Throughout the paper, the square-root PDF model has been applied and design methods are based on continuous-time Linear Matrix Inequalities (LMI) approach. Simulation results also confirm the effectiveness of the method proposed.

1. INTRODUCTION

Increasing demand for safety and reliability in process industries has attracted a great attention to the field of Fault Detection and Isolation (FDI) from 1970s Beard (1971), Jones (1973). Since then, numerous efforts have been made for FDI in different classes of systems Simani et al. (2003). While model-based FDI methods have been of more interest in stochastic control filed, most of them have considered linear systems where the residual generation is based on parameter estimation methods such as Kalman filtering Hassan et al. (1992). In addition, almost all of existing works have considered Gaussian systems which is restricting in cases where the plant is subject to non-Gaussian noise distributions Wang (2000). The output PDF control problem for non-Gaussian stochastic systems was simplified and further developed in Wang (2000), by employing B-Spline expansions to model the output PDF. More recently, Iterative Learning Control (ILC)-based methodologies have been applied to stochastic distribution control Wang and Afshar (2006), Wang et al. (2006), and Afshar et al. (2007) to improve the nonlinear tracking performance in a batch-by-batch model. FDI issues in non-Gaussian stochastic distribution control have been addressed in Guo and Wang (2005a) and Guo et al. (2006). where the output PDF measurement sensors are assumed precise and the faults are restricted to abrupt faults. Furthermore, the same level of significance is considered for all drifts over the time, which can result in more false alarm rates.

In this paper, a two-stage method has been introduced to detect and isolate the sensor faults affecting the stochastic distribution control systems. In the first stage, a Fault Detection Observer (FDO) works continuously to monitor the residual signal and reports the violations from the adaptive thresholds as faults. In the next stage, a nonlinear Fault Isolation Filter (FIF) is activated for identification of fault signal. The gains and parameters corresponding to FDO and FIF are calculated using a LMI-based convex optimization technique. Fig. 1 illustrates the scheme of the proposed FDI system.

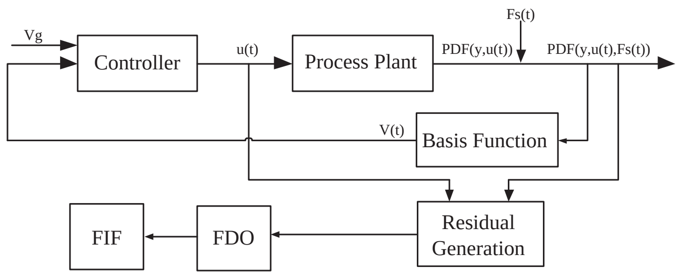

Fig. 1. The Scheme of proposed FDI system

During FDI operations, both abrupt and incipient faults are studied. In addition, a time-dependent window function is applied in FDO design to suppress the drifts caused by model uncertainties and non-Gaussian noises leading in reduction of the false alarms rates. Furthermore, RBFNN is used to model the output PDF which gives more generality over the cases which use fixed B-Spline expansions in terms of tune-able RBF activation function parameters, i.e., centers and widths.

This paper is organized as follows. In section 2, the problem of the output PDF control and square root models have been introduced. Section 3 is comprised of the FDO design procedure with guaranteed closed loop stability.

Then the FIF design procedures are proposed in section 4 and simulation results of both abrupt and incipient sensor faults make section 5. Finally, concluding notes are made paper in section 6. Throughout the paper, all matrices are assumed to have compatible dimensions. In addition,identity and zero matrices are shown by I and 0, respectively. Whenever used, $\|.\|$ represents the Euclidean norm of the real matrices. Also, function sym is defined as $sym(M) = M^T + M$. Negative definite matrices are also marked with < 0.

2. PROBLEM FORMULATION

Suppose $y \in [a, b]$, be the output of the stochastic process in Fig. 1. Let $u(t) \in \mathbb{R}^p$ be the which shapes the faultless output PDF denoted by $\gamma(y, u(t))$. The faulty output PDF is expressed as $\gamma(y, u(t), F_s(t))$, where $F_s(t)$ is the sensor fault (either abrupt or incipient).

2.1 Model Representation

The PDF of a random process y is defined as

$$P(a \leq y < \zeta) = \int_a^{\zeta} \gamma(y, u(t))dy \tag{1}$$

where $P(a \leq y < \zeta)$ is probability of y being between a and ζ. Similar to Wang et al. (2006) RBFs are be employed to model the square root of the output PDF to give

$$\begin{aligned}\sqrt{\gamma(y, u(t), F_s(t))} = R(y)V(t) + r_n(y)h(V(t)) \\ + \eta(y, u(t), F_s(t))\end{aligned} \tag{2}$$

where

$$R(y) = [r_1(y), \cdots, r_{n-1}(y)],\ V(t) = [v_1(t), \cdots, v_{n-1}(t)]^T$$

In (2), $v_l(t)$ and $r_l(y)$ are the l^{th} weight element and activation function corresponding to RBF neural network used for PDF modeling, respectively. Term $\eta(y, u(t), F_s(t))$ represents non-Gaussian noises and model uncertainties. Similar to Wang and Afshar (2006), and Wang et al. (2006) the RBFs are expressed as

$$r_l(y) = \exp\left(-\frac{(y_j - \mu_l)^2}{2\sigma_l^2}\right) \tag{3}$$

whilst μ_l, σ_l are RBF centres and widths, respectively. In (2), η must satisfy $|\eta(y, u(t), F_s(t))| \leq \delta$ where $\delta > 0$ is a known constant. When using square root models, it can be verified that the following constraint over the states should be satisfied Wang (2000).

$$V^T(t)\Lambda_4 V(t) - \Lambda_3(\eta_1(y, u(t)) - 1) \geq 0 \tag{4}$$

Denoting

$$\Lambda_4 = \Lambda_2^T \Lambda_2 - \Lambda_3 \Lambda_1$$

where

$$\Lambda_1 = \textstyle\int_a^b R^T(y)R(y)dy,\ \Lambda_2 = \int_a^b R(y)r_n(y)dy$$

and

$$\Lambda_3 = \textstyle\int_a^b r_n^2(y)dy$$

and also

$$\begin{aligned}\eta_1(y, u(t)) = \eta^2(y, u(t))(b - a) + 2\eta(y, u(t)) \\ \times \left[\left(\int_a^b R(y)dy\right)V(t) + \left(\int_a^b r_n(y)dy\right)h(V(t))\right]\end{aligned}$$

With the above definitions, it can be seen that

$$h(V(t)) = \frac{1}{\Lambda_3}(-\Lambda_2 V(t) \pm \sqrt{V^T(t)\Lambda_4 V(t) - \Lambda_3(\eta_1 - 1)}) \tag{5}$$

In (5) $h(.)$ should satisfy the following Lipschitz condition for any two $V_1(t)$ and $V_2(t)$ and known matrix U_1.

$$\|h(V_1(t)) - h(V_2(t))\| \leq \|U_1(V_1(t) - V_2(t))\| \tag{6}$$

Similar to Wang (2000) and Guo and Wang (2005a), the dynamic nonlinear model linking the weights to the control input can be written as follows.

$$\begin{aligned}\dot{x}(t) &= Ax(t) + Bu_k(i) + Gg(V(t)) \\ V(t) &= Ex(t)\end{aligned} \tag{7}$$

where $x(t) \in \mathbb{R}^n$ is the unmeasured state, and A, B, G, and E are model parameter matrices (known or identified). Model (7) is used together with 2 which has been fully described in Wang (2000)). The nonlinear function $g(V(t))$ represents nonlinear dynamics and satisfies $g(0) = 0$ and the following Lipschitz condition, similar to Hadjicostis (2003), Hibey and Charalambous (1999), and Guo and Wang (2005a).

$$\|g(x_1(t)) - g(x_2(t))\| \leq \|U_2(x_1(t) - x_2(t))\| \tag{8}$$

where U_2 is a known matrix.

3. FAULT DETECTION OBSERVER DESIGN

In this section FDO design for both abrupt and incipient faults is proposed.

3.1 Abrupt and incipient Faults Modeling

Similar to Zhang et al. (2005) and Demetriou (1998), a sensor fault $F_s(t)$ occurs at instant T and can be modeled as follows.

$$F_s(t) = \beta(t - T)f_{0s}(t) \tag{9}$$

where f_{0s} is a describes the fault to be a constant value (abrupt fault), or ramp (incipient fault). As such, abrupt and incipient faults can be expressed as follows.

$$f_{0s}(\tau) = \begin{cases} f_{0s} & \text{for abrupt faults,} \\ \int_0^t f_{0s}(\tau)d\tau & \text{for incipient faults.} \end{cases} \quad (10)$$

In (9), $\beta(.)$ denotes the time profile of the fault and is expressed as $\beta(\tau) = I_{no\times no} - \exp(-\Lambda\tau)$ when $\tau > 0$, and 0 otherwise. constant Λ is a symmetric matrix denoting the speed based on which the failure is occurred.

3.2 FDO design procedure

In order to detect a fault, a threshold (residual) should be defined from which drifts are regarded as faults. The residual signal is chosen as a weighted sum of drifts in the output PDFs in FDO dynamics as expressed as follows.

$$\begin{aligned} \dot{\hat{x}}(t) &= A\hat{x}(t) + Bu(t) + Gg(\hat{x}(t)) + L\epsilon(t) \\ \epsilon(t) &= \int_a^b \mu(t)[\sqrt{\gamma(y,u(t))} - \sqrt{\hat{\gamma}(y,u(t))}]dy \\ \sqrt{\hat{\gamma}(y,u(t))} &= R(y)Ex(t)\hat{x}(t) + h(E\hat{x}(t))r_n(y) \end{aligned} \quad (11)$$

where $\hat{x}$ is the estimated state, and L is the observer gain to be determined. The function $\mu(t)$ can be chosen as a sigmoidal function to signify desired portions of the PDF tracking error signal.

Denoting the estimation error as $e(t) = x(t) - \hat{x}(t)$, the dynamics of the estimation error will be expressed as

$$\begin{aligned} \dot{e}(t) = (A - L\Gamma_1)e(t) + [Gg(x(t)) - Gg(\hat{x}(t))] \\ - L\Gamma_2[h(Ex(t)) - h(E\hat{x}(t))] - L\Delta(t) \end{aligned} \quad (12)$$

where

$$\Gamma_1 = \int_a^b \mu(t)R(y)Edy,\ \Gamma_2 = \int_a^b \mu(t)r_n(y)dy$$
$$\Delta(t) = \int_a^b \mu(t)\eta(y,u(t))dy$$

The objective is to find the FDO gain L so that in the absence of sensor faults,i.e., $F_s(t) = 0$ the closed loop system comprised of the stochastic system and FDO is internally stable. For this purpose, the following Lyapunov function is considered.

$$\begin{aligned} &\Theta_1(e,x,\hat{x},t) = e^TPe(t) \\ &+ \frac{1}{\lambda_1^2}\int_a^b \left[\|U_1Ee(\tau)\|^2 - \|h(Ex(\tau)) - h(E\hat{x}(\tau))\|^2\right]d\tau \\ &+ \frac{1}{\lambda_2^2}\int_a^b \left[\|U_2e(\tau)\|^2 - \|g(x(\tau)) - g(\hat{x}(\tau))\|^2\right]d\tau \end{aligned} \quad (13)$$

Then it can be seen that

$$\begin{aligned} \dot{\Theta}_1(e,x,\hat{x},t) &= e^T[P(A - L\Gamma_1) + (A - L\Gamma_1)^TP]e(t) \\ &+ 2e^TP[Gg(x(t)) - Gg(\hat{x}(t))] \\ &- 2e^TPL\Gamma_2[h(Ex(\tau)) - h(E\hat{x}(\tau))] \\ &- 2e^TPL\Delta(t) \\ &+ \frac{1}{\lambda_1^2}[\|U_1Ee(\tau)\|^2 - \|h(Ex(\tau)) - h(E\hat{x}(\tau))\|^2] \\ &+ \frac{1}{\lambda_2^2}[\|U_2e(\tau)\|^2 - \|g(x(\tau)) - g(\hat{x}(\tau))\|^2] \end{aligned} \quad (14)$$

Using the typical inequality $2AB \leq (\|A\|^2 + \|B\|^2)$, it can be concluded that

$$\begin{aligned} \dot{\Theta}_1(e,x,\hat{x},t) \leq\ & e^T[P(A - L\Gamma_1) + (A - L\Gamma_1)^TP]e(t) \\ &+ e^T[\lambda_2^2PGG^TP + \lambda_1^2PL\Gamma_2\Gamma_2^TL^TP]e(t) \\ &+ e^T[\frac{1}{\lambda_1^2}E^TU_1^TU_1E + \frac{1}{\lambda_2^2}U_2^TU_2]e(t) \\ &- 2e^TPL\Delta(t) \end{aligned} \quad (15)$$

It can be verified that the satisfying the following LMI guarantees the closed loop stability when $F_s(t) = 0$.

$$\Upsilon_1 = \begin{bmatrix} \Pi_0 & \Pi_1 \\ \Pi_1^T & -I \end{bmatrix} < 0 \quad (16)$$

where

$$\Pi_0 = P(A - L\Gamma_1) + (A - L\Gamma_1)^TP + \tfrac{1}{\lambda_1^2}E^TU_1^TU_1E + \tfrac{1}{\lambda_2^2}U_2^TU_2$$
$$\Pi_1 = [\lambda_1PL\Gamma_2\ \ \lambda_2PG]$$

With the above considerations and also $R = PL$, it can be concluded that when $F_s(t) = 0$, the following threshold can be determined for $e(t)$.

$$\|\epsilon(t)\| > \sup(e(t))(\|\Gamma_1\| + \|\Gamma_2\|\|U_1\|) + \sup(\Delta(t)) \quad (17)$$

The feasible design procedure of FDO design can be summarized as follows.

(1) Solve the LMI (16) to find the observer gain L;
(2) Form the residual signal $\epsilon(t)$ as denoted in (11);
(3) Use the threshold (17) to detect sensor faults.

4. FAULT ISOLATION NONLINEAR FILTER DESIGN

After a sensor fault has been detected by FDO, the fault value must be estimated for diagnosis purposes. The following nonlinear model is considered for the nonlinear Fault Isolation Filter (FIF).

$$\begin{aligned} \dot{\hat{x}}(t) &= A\hat{x}(t) + Bu(t) + Gg(\hat{x}(t)) + L\epsilon_1(t) \\ \dot{\hat{F}}_s(t) &= \Lambda_1\hat{F}_s(t) + \Lambda_2\epsilon_1(t) \end{aligned} \quad (18)$$

where $\hat{F}_s(t)$ is the estimated value of the fault. Introducing $\tilde{F}(t) = F_s(t) - \hat{F}_s(t)$, the new auxiliary residual signal ϵ_1 is obtained as follows.

$$\epsilon_1(t) = \int_a^b \mu(t)\Big[R(y)Ee(t) + r_n(y)[h(Ex(t)) - h(E\hat{x}(t))] + \eta(y,u(t)) + \tilde{F}(t)\Big]dy \tag{19}$$

Denoting $\Gamma_3 = \int_a^b \mu(t)dy = \mu(t)(b-a)$, the residual signal can be re-written as follows.

$$\epsilon_1(t) = \Gamma_1 e(t) + \Gamma_2\Big[h(E(x(t))) - h(E\hat{x}(t))\Big] + \Delta(t) + \Gamma_3(\tilde{F}_s(t) \tag{20}$$

Thus, the estimation error dynamics can be expressed as

$$\begin{aligned}\dot{e}(t) &= (A - L\Gamma_1)e(t) + G[g(x(t)) - g(\hat{x}(t))] \\ &\quad - L\Gamma_2[h(x(t)) - h(\hat{x}(t))] - L\Delta(t)L\Gamma_3\tilde{F}_s(t) \\ \dot{\tilde{F}}_s(t) &= (\Lambda_1 - \Lambda_2\Gamma_3)\tilde{F}_s(t) - \Lambda_2\Gamma_1 e(t) \\ &\quad - \Lambda_2\Gamma_2[h(x(t)) - h(\hat{x}(t))] - \Lambda_1 F_s(t) + \dot{F}_s(t) - \Lambda_2\Delta(t)\end{aligned} \tag{21}$$

Similar to section 3, the closed loop system comprised of FIF and the stochastic plant must be internally stable.

Theorem 1. If for given parameters $\alpha_i(i = 1,2,3)$ and $\lambda_i(i = 1,2)$, (22) is solvable for $P > 0$, R, and $\Lambda_i(i = 1,2)$,

$$\Upsilon_1 = \begin{bmatrix} \psi_{11} & \psi_{12} & \psi_2 & \psi_3 \\ \psi_{21}^T & \psi_{22} & 0 & 0 \\ \psi_2^T & 0 & -I & 0 \\ \psi_3^T & 0 & 0 & -I \end{bmatrix} < 0 \tag{22}$$

where

$$\psi_{11} = sym(PA - R\Gamma_1) + \tfrac{1}{\lambda_1^2}E^T U_1^T U_1 E + \tfrac{1}{\lambda_2}U_2^T U_2$$

$$\psi_{12} = -R\Gamma_3 - \Gamma_1^T\Lambda_2^T, \psi_{13} = sym(\Lambda_1 - \Lambda_2\Gamma_3)$$

$$\psi_2 = \begin{bmatrix} \lambda_1 R\Gamma_2 & \lambda_2 PG & \alpha_1 R & 0 & 0 \\ 0 & 0 & 0 & \alpha_2\Lambda_2 & \alpha_3\Lambda_2\Gamma_2 \end{bmatrix},$$

$$\psi_3 = \begin{bmatrix} \alpha_3^{-1}E^T U_1^T \\ 0 \end{bmatrix}$$

Then the FIF can estimate the sensor fault value and the resulting closed loop system will be internally stable. In this case, the filter gain can be calculated using $L = P^{-1}R$.

Proof: Considering the following Lyapunov candidate,

$$\Theta_2(e,x,\hat{x},t) = \Theta_1(e,x,\hat{x},t) + \tilde{F}_s^T(t)\tilde{F}_s(t) \tag{23}$$

then it can be verified that

$$\dot{\Theta}_2 \leq \begin{bmatrix} e^T(t) & \tilde{F}_s^T(t) \end{bmatrix} \bar{\psi}_0 \begin{bmatrix} e(t) \\ \tilde{F}_s(t) \end{bmatrix} + (\alpha_1^{-2} + \alpha_2^{-2})\delta^2 - 2\tilde{F}_s^T(t)\Lambda_1 F_s(t) + 2\tilde{F}_s^T(t)\dot{F}_s(t) \tag{24}$$

where

$$\bar{\psi}_0 = \begin{bmatrix} \bar{\Psi}_{11} & \psi_{12} \\ \psi_{12}^T & \bar{\Psi}_{22} \end{bmatrix}$$

$$\psi_0 = \psi_{11} + \lambda_1^2 R\Gamma_2\Gamma_2^T R^T + \lambda_2^2 PGG^T P$$

$$\bar{\Psi}_{11} = \psi_0 + \alpha_1^2 RR^T + \alpha_3^{-2}E^T U_1^T U_1 E$$

$$\bar{\Psi}_{22} = \psi_{13} + \alpha_2^2\Lambda_2\Lambda_2^T + \alpha_3^2\Lambda_2\Gamma_2\Gamma_2^T\Lambda_2^T$$

Applying Schur complement formula to the latter equality, LMI (22) can be obtained.

5. SIMULATION RESULTS

Suppose $a = 0, b = 1$ and $\mu_1 = 0.25$, $\mu_2 = 0.50$, and $\mu_3 = 0.75$, and $\sigma_1 = \sigma_2 = \sigma_3 = 0.06$. In addition, consider the (7) parameters as

$$A = \begin{bmatrix} 2 & -1 \\ 0 & 0.8 \end{bmatrix},\ B = \begin{bmatrix} 0.8 & 0.3 \\ 0.4 & 1.0 \end{bmatrix}, G = \begin{bmatrix} 0.2 & 0.1 \\ 0.3 & 0.4 \end{bmatrix}$$

and $E = I$. The nonlinear function $g(V(t))$ and also the matrices $U_i(i = 1,2)$ were chosen as follows.

$$g(V(t)) = \begin{bmatrix} 0 \\ \sqrt{v_1^2 + v_2^2} \end{bmatrix},\ U_1 = [0.1\ 0.1], U_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Prior to the occurrence of the fault, the output PDF shape under PID control law has the shape illustrated in Fig.2.

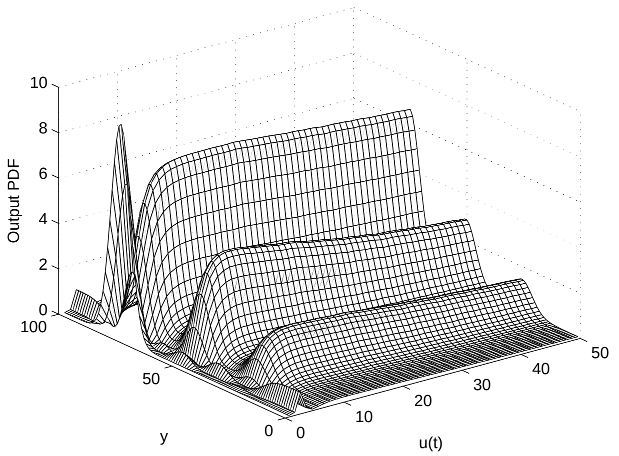

Fig. 2. The controlled shape of output PDF while sensors are healthy

Also, before the fault happens the residual signal shows minor fluctuations caused by the model uncertainty as shown in Fig. 3.

5.1 Abrupt Fault Detection and Isolation

We assume that an abrupt sensor fault introduced in (9) is occurred with the characteristics listed in table 1.

Table 1. The Parameters of the abrupt fault

Parameter	Value
Fault Value (f_{0s})	3.85
Occurrence instant (T)	10 sec
Speed Parameter (Λ)	2

Table 1 would mean that the fault signal will be

$$F_s(t) = 3.85 \times (1 - e^{-2(t-10)})$$

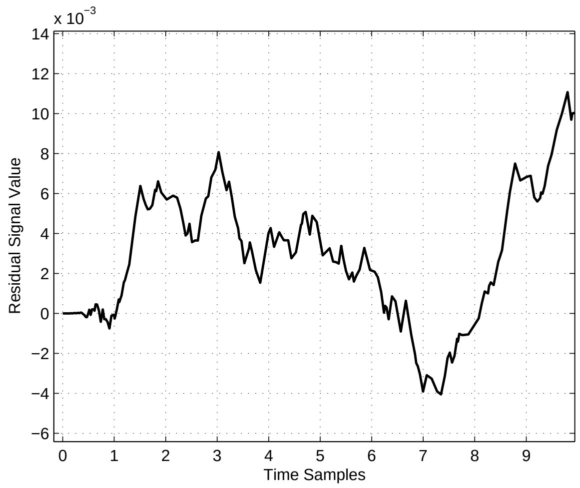

Fig. 3. The residual signal prior to fault

The abrupt fault mentioned, affects the output PDF measurement, which will lead to unwanted control signal calculation, if not diagnosed. The observed shape of the faulty output PDF is shown in Fig. 4

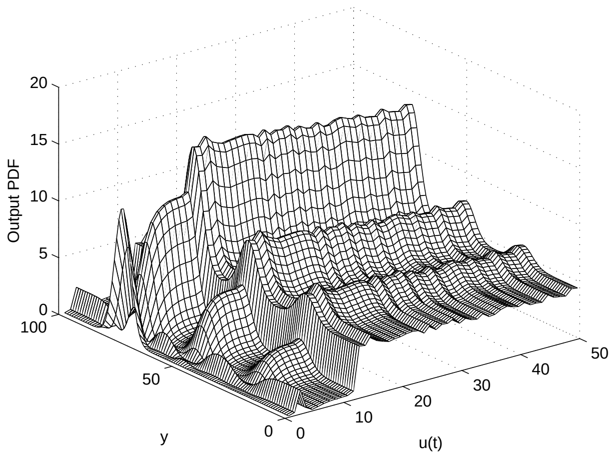

Fig. 4. The effect of abrupt fault in the output PDF shape

Before performing the FDO design Γ_1 and Γ_2 parameters are determined based on notations introduced in 12 to give

$$\Gamma_1 = [0.0429\ 0.1086]\,, \Gamma_2 = 0.1214$$

For solving the feasibility problem introduced in (16), MAtlab LMI toolbox is employed. The resulting values are

$$P = \begin{bmatrix} 3.1555 & 1.4318 \\ 1.4318 & 9.2504 \end{bmatrix}$$

and

$$L = \begin{bmatrix} 3.5781 \\ 1.8499 \end{bmatrix}$$

As the residual signal illustrated in Fig. 5 confirms, having the calculated gain applied, the FDO can detect the fault after 2.5 seconds from its occurrence.

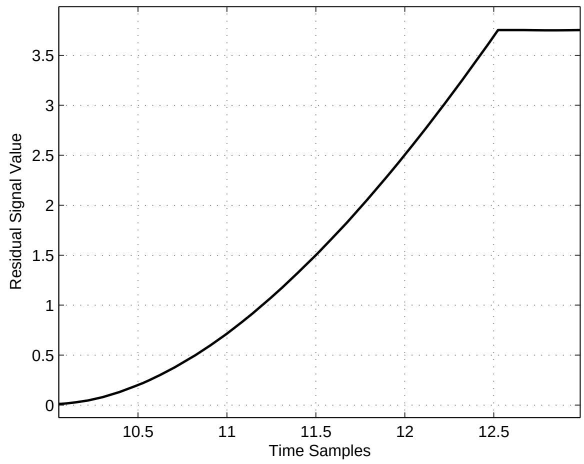

Fig. 5. The value of the residual signal after the fault moment

After the fault is detected, FIF starts estimating the value of the sensor fault occurred. With the initial values $\lambda_1 = \lambda_2 = 1$ and $\alpha_1 = \alpha_2 = \alpha_3 = 2$ the Matlab LMI Toolbox results in the following.

$$P = \begin{bmatrix} 309.2 & 210.9 \\ 210.9 & 181.1 \end{bmatrix}, R = \begin{bmatrix} 16.2280 \\ 32.1275 \end{bmatrix},$$

and finally

$$L = \begin{bmatrix} 0.0386 \\ 0.0203 \end{bmatrix}$$

The toolbox also yields the following values for the non-linear filter gains.

$$\Lambda_1 = 37.1318, \Lambda_2 = -26.0667$$

Applying the above calculated gains to (18), the nonlinear FIF should track the abrupt time profile as close as possible. Fig.6 suggests that such a filter can effectively diagnose the sensor faults in a stochastic distribution control.

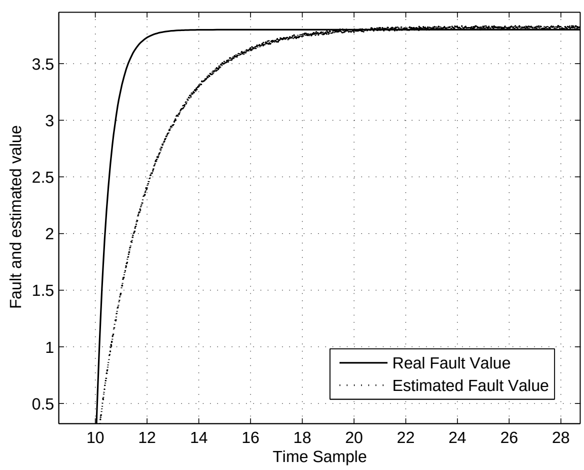

Fig. 6. Performance of the FIF and the fault signal with abrupt faults

5.2 Incipient Fault Isolation

In addition to abrupt faults, the ability of FIF to diagnose incipient faults, i.e., ramp-alike faults, is investigated in this section. The fault is supposed to happen at $T = 20$. The same procedure is applied to detect and isolate the fault. Fig. 7 shows how the FIF tracks the fault value.

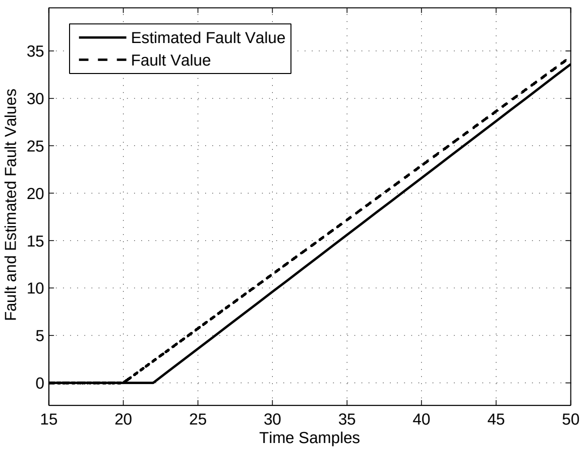

Fig. 7. Performance of the FIF and the fault signal with incipient faults

6. CONCLUSIONS

A method for detection and isolation of both abrupt and incipient faults in non-Gaussian stochastic distribution control systems has been proposed. It has been assumed that a PID controller considering all nonlinearities and uncertainties in the general stochastic system has been already available for fault-free conditions Guo and Wang (2005b). The method consists of two stages in which the sensor fault is firstly detected using a nonlinear observer (FDO) and then is diagnosed through a nonlinear filter (FIF). Both observers are designed using LMI-based convex optimization method which guarantees the internal stability of the system formed by the stochastic plant, FDO, and FIF. The proposed method has the ability to deal with both abrupt and incipient faults occurred in the output PDF measurement sensors. In addition, applying a new windowing function to the residual used to deter the fault, the method decreases the rate of false alarms. Furthermore, employing RBF neural networks in the output PDF estimation has the advantage over B-Spline expansions that enables the method to provide a more general expression of the output PDF in terms of the RBF parameters, i.e., RBF centers and widths. Other interesting related studies can be FDI in presence of multiplicative systematic and sensor faults which will be studied in the future works.

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