COMPARISION OF CONTROL DESIGN TECHNIQUES FOR A NUCLEAR REACTOR

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Abstract: This paper presents a comparative study of different control design methods applied to a nuclear reactor model. A nuclear reactor temperature controller is designed using the H-infinity (H∞) control. This advanced controller is compared with a conventional Proportional–Integral–Derivative controller **(**PID) controller and a traditional optimal controller design using Linear Quadratic Gaussian (LQG) method.

1. INTRODUCTION

Many control methods of nuclear power plant control and robustness analysis have been studied over the past two decades, and numerous theoretical and practical works have been proposed in the field of nuclear reactor control. (Akin, and Altin, 1991; Edwards, et al., 1990a,b).

The rule-based fuzzy logic controller for a nuclear power plant for 10% variation of reactor power about nominal power has been proposed in (Akin, and Altin, 1991). The above mentioned is robust under noisy operation conditions, but it had a limited range of operations for reactor control. The State Feedback Assisted Classical (SFAC) control has been designed to improve the thermal response performance of nuclear reactor and to increase the system robustness, by using the concept of state feedback to modify the reference signal of a classical control loop (Edwards, et al., 1990a).

Also a self-tunning regulator in SFAC configuration has been reported to obtain robust optimal self-tuning regulations to nuclear reactor power control problem (Khajavi, et al., 2000). Such a design helps to achieve good performance in wide range of operations. An automatic tuning method of a fuzzy logic controller for nuclear reactors based on a fixed optimal controller has been introduced in (Ramaswamy, et al., 1993).The fuzzy logic controller displays good stability and performance robustness characteristics for a wide range of power variations. In addition a time optimal control law of nuclear reactor power with adaptive PIF gain has been presented in (Park and Cho, 1993). Time optimal control strategy with PIF controller consists of coarse time-optimal control and an adaptive proportional-integral-feedforward controller has been applied to nuclear reactor. Although the proposed controller shows desired performance, the method is applicable to the small applications as pilot plants or spacecraft nuclear reactors.

In this paper, an H∞ controller is designed for a nuclear reactor model, and is compared with conventional optimal controller design of the reactor system using Proportional Integral Derivative and Linear Quadratic Gaussian controllers (Burl and Jeffery, 1998). The comparison criterion is based on the residual distribution of reactor temperature error, the closed loop tracking error entropy, stability, and performance of nominal plant under each of the above mentioned controllers (Ben Abdennour, et al., 1992).

The paper is organized as follows. In section 2, a model for the nuclear reactor is provided. Section 3 introduces the different controllers as mentioned above. Section 4 introduces the basis of comparison. Implementation of controllers is presented in section 5. Finally, the concluding remarks and future work plan are proposed in section 6.

2. MATHEMATICAL MODEL

The actual nuclear reactor is a nonlinear, highly-ordered system. It is very difficult and complicated to study this type of system. Some nonlinear equations can be approximated by linear equations under certain conditions. Therefore, in this paper, the linearized version of a simple fifth-order model is used to design the controller. The model assumes point kinetics equations with one delayed neutron group and temperature feedback from lumped fuel and coolant temperatures. The normalized fifth-order model can be summarized as follows (Ben Abdennour, *et al.*, 1992; Edwards, *et al.*, 1990a).

$$
\begin{aligned}
\frac{dn_r}{dt} &= \frac{\delta\rho-\beta}{\Lambda}n_r+\lambda c_r\\
\frac{dc_r}{dt} &= \frac{\beta}{\Lambda}n_r-\lambda c_r\\
\frac{dT_f}{dt} &= \frac{f_f P_o}{\mu_f}n_r-\frac{\Omega}{\mu_f}T_f+\frac{\Omega}{2\mu_f}T_l+\frac{\Omega}{2\mu_f}T_e\\
\frac{dT_l}{dt} &= \frac{(1-f_f)P_o}{\mu_c}n_r+\frac{\Omega}{\mu_c}T_f-\frac{2M+\Omega}{2\mu_c}T_l+\frac{2M-\Omega}{2\mu_c}T_e\\
\frac{d\delta\rho_r}{dt} &= G_r z_r\\
\delta\rho &= \delta\rho_r+\alpha_f(T_f-T_{fo})+\alpha_c(T_c-T_{co})
\end{aligned}
\tag{1}
$$

where
$c_r = c/c_0$ is the precursor density relative to initial equilibrium density
c is the precursor density (atom/cm^3)
c_0 is the initial equilibrium precursor density
$n_r = n/n_0$ stands for the neutron density relative to initial equilibrium density
n is the neutron density (n/cm^3)
n_0 represents the initial equilibrium neutron density
λ stands for effective precursor radioactive decay constant (s^{-1})
Λ is the effective prompt neutron lifetime (s)
β is the fraction of delayed fission neutrons
Ω is the heat transfer coefficient between fuel and coolant $(\text{MW/}^{o}\text{C})$
M stands for the mass flow rate times heat capacity of the water $(\text{MW/}^{o}\text{C})$
T_f is the average reactor fuel temperature (^{o}C)
T_e is the temperature of the water entering the reactor (^{o}C)
T_l stands for the temperature of the water leaving the reactor (^{o}C)
$T_c = (T_l - T_e)/2$ represents the average reactor coolant temperature (^{o}C)
T_{fo} is the initial equilibrium fuel temperature
T_{co} is the initial equilibrium coolant temperature
f_f is the fraction of reactor deposited in the fuel
μ_f represents the total heat capacity of the fuel and structural material $(\text{MW}\cdot\text{s/}^{o}\text{C})$
μ_c means the total heat capacity of the reactor capacity
α_f is the fuel temperature reactivity coefficient
α_c stands for coolant temperature reactivity coefficient
$\delta\rho_r$ represents the reactivity due to the control rod
$\delta\rho$ is the reactivity.
z_r is the control rod speed
G_r represents the reactivity worth of the rod per unit length

The proposed control system requires the model to be in the form of series of first order linear differential equations which may be written in the standard state vector matrix form. By linearizing (1) around an equilibrium power point, the system can be written in the following standard state space model

$$\dot{x} = A.x + Bu \qquad (2)$$
$$y = Cx$$

where:
x is an n dimensional vector of pant states.
u is an r dimensional vector of pant inputs.
y is an m dimensional vector of pant outputs.
A. is an n×n dimensional system matrix.
B is an n×r dimensional forcing matrix.
C is an m×n dimensional output matrix.

$$A=\begin{bmatrix} -\beta/\Lambda & \beta/\Lambda & n_{ro}\alpha_f/\Lambda & n_{ro}\alpha_c/2\Lambda & n_{ro}/\Lambda \\ \lambda & -\lambda & 0 & 0 & 0 \\ f_f P_o/\mu_f & 0 & -\Omega/\mu_f & \Omega/2\mu_f & 0 \\ (1-f_f)P_o/\mu_c & 0 & \Omega/\mu_c & -(2M+\Omega)/2\mu_c & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$
$$B=\begin{bmatrix} 0 & 0 & 0 & 0 & G_r \end{bmatrix}^T \qquad (3)$$
$$C=\begin{bmatrix} 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$
$$x=\begin{bmatrix} \delta n_r & \delta c_r & \delta T_f & \delta T_l & \delta\rho_r \end{bmatrix}^T$$
$$y=\begin{bmatrix} \delta T_f \end{bmatrix}$$
$$u=\begin{bmatrix} z_r \end{bmatrix}$$

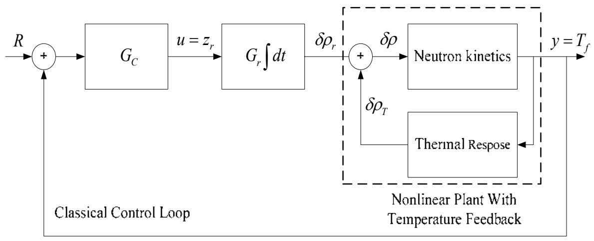

Fig. 1. Conventional output feedback reactor control.

The nominal model states of the diagram in Fig. 1 are relative reactor power δn_r, relative precursor density δc_r, average fuel temperature δT_f, average coolant temperature leaving the reactor δT_l, and control reactivity $\delta\rho_r$. The system output *y* is the average fuel temperature, and the control input is the control rod speed. Also the required constant values used for the modelling are summarized in Table 1. These parameters represent a Three Mile Island Type reactor at the middle of the fuel cycle as in (Edwards, *et al.*, 1990; Ramaswamy, *et al.*, 1993).

Table 1 the parameter values of reactor

β=0.006019	f_f=0.92
Λ=0.00002s	T_c=290°C
P_0=2500MW	$n_{r0}=1$
μ_f=26.3MW.s/°C	λ=0.150s^{-1}
G_r=0.0145Δk/k	M=102MW/°C
μ_c=71.8MW·s/°C	Ω=6.6MW/°C
α_f=-0.0000324 Δk/k·C^{-1}	α_c=-0.000213 Δk/k·C^{-1}

3. CONTROLLER DEISGN

In this section, three types of controller, namely the H∞, Optimal control and PID control, will be designed to control the power of nuclear reactor

3.1 H-infinity Controller

The H∞ controller is generated by combining the H∞ optimal full information controller and H∞ estimation controller. The approach of design is proposed as in (Burl and Jeffery, 1998); Ben Doyle, *et al.*, 1992; Maciejowski, 1989; Skogestad, 1996).The following state model represents the linear model of plant:

$$\dot{x} = Ax(t) + \begin{bmatrix} B_u & B_w \end{bmatrix} \begin{bmatrix} u(t) \\ w(t) \end{bmatrix} \tag{4}$$

$$\begin{bmatrix} m(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} C_m \\ C_y \end{bmatrix} x(t) + \begin{bmatrix} 0 & D_{mw} \\ D_{yu} & 0 \end{bmatrix} \begin{bmatrix} u(t) \\ w(t) \end{bmatrix} \tag{5}$$

where the B_u is the control matrix, B_w is the reference input matrix, u the manipulated variables and w the exogenous input. Also in (5) C_m is the feedback matrix and C_y is the output matrix.

The matrix (D_{yu}, C_y) and (D_{mw}, B_w) should satisfy the following conditions to guarantee the existence of a steady state H∞ controller (Ben Doyle, *et al.*, 1992):

$$D_{yu}^T C_y = 0 \quad , \quad D_{mw} B_w^T = 0$$
$$D_{yu}^T D_{yu} = I \quad , \quad D_{mw} D_{mw}^T = I \tag{6}$$

The control objective in H∞ full information control design is to find a feedback controller for the above plant such that the infinity-norm of the closed-loop system is bounded. This can be expressed as follows

$$\left\| G_{yw} \right\|_{\infty,[0,t_f]} = \sup_{\|w(t)\|_{2,[0,t_f]} \neq 0} \frac{\|y(t)\|_{2,[0,t_f]}}{\|w(t)\|_{2[0,t_f]}} < \gamma \tag{7}$$

Where γ is often called the performance bound

The optimal control law is given as follows. The control law calculations are given in (Burl and Jeffery, 1998)

$$u(t) = -B_u^T P(t) \hat{x}(t) = -K \hat{x}(t) \tag{8}$$

with $P(t)$ being a symmetrical positive semi-definite matrix that satisfies the following algebraic Riccatti equation.

$$PA + A^T P - P(B_u B_u^T - \gamma^{-2} B_w B_w^T) P + C_y^T C_y = 0 \tag{9}$$

An optimal H∞ estimator acts to minimize the worst-case gain between the disturbance input and the estimation error. The bound in (10) can be used to formulate an H∞ output estimation problem:

$$\left\| G_\Delta \right\|_\infty = \sup_{w(t) - \gamma^{-2} B_w^T P(t) x(t) \neq 0} \frac{\left\| u(t) + B_u^T P(t) x(t) \right\|_{2[0\,tf]}}{\left\| w(t) - \gamma^{-2} B_w^T P(t) x(t) \right\|_{2[0\,tf]}} < \gamma \tag{10}$$

The estimator gain shall be designed as in (Burl and Jeffery, 1998) to give

$$G(t) = Q(t) C_m^T \tag{11}$$

where $Q(t)$ is a symmetric positive semi-definite matrix that satisfying the estimator algebraic Riccatti as follows

$$\dot{Q}(t) = Q(t)A^T + AQ(t) + B_w B_w^T - Q(t)(C_m^T C_m - \gamma^{-2} C_y^T C_y) Q(t) \tag{12}$$

The design parameters are the noise amplitude and performance bound γ values to achieve the desired stability and performance.

3.2 Conventional PID Controller

Tuning the PID controller is very sensitive to proportional gain and derivative time constant In this paper, Ziegler - Nichols method is tuning the controller (Astrom and Hagglund, 1995; O'Dwyer, 2003; Gorez, 1997).

3.3 Optimal LQG Controller

The separation principle and full state-feedback properties are used in designing and tuning the LQG controller (Brogan, 1991; Maciejowski, 1989).

4. The BASIS OF COPMARISION

The closed loop tracking error entropy is considered as a criterion for controller performance evaluation, and the following formulation has been employed for entropy calculation

$$H(e) = \frac{1}{1-\alpha} \log\left(\sum_i \gamma_i^\alpha(e) \right) = \frac{1}{1-\alpha} \log\left(V_{Ra}(e) \right) \tag{13}$$

Where α is the order of Renyi's quadratic entropy. Also $V_{Ra}(e)$ is usually called the Information Potential, and $\gamma(e)$ is called the Probability Density Function (PDF) of the closed loop tracking error. The required PDF is estimated by the Kernel estimation method to give

$$\gamma_e \approx \hat{\gamma}_e = \frac{1}{N} \sum_{i=1}^N K_\sigma (e - e_i) \tag{14}$$

where K_σ is a symmetrical Kernel Silverman (1992). The Information Potential can be further expressed as

$$V_{Ra}(e) = \frac{1}{N^\alpha} \sum_{i=1}^N \left[\sum_{j=1}^N K_\sigma (e_i - e_j) \right]^\alpha \tag{15}$$

In addition, the effectiveness of each control strategy is judged based on analysis carried out on step response and comparing the overshoot and settling times.

5. SIMULATIONS RESULTS

5.1 Implementation of H-infinity Controller

Analyzing the closed-loop system by generating a unit step input as shown if Fig.2 depicts that the reactor temperature follows the reference input. Also the settling time is 1.2 sec and no overshoot is observed. In addition, the steady state error is zero.

Fig. 3 and 4 show the reactor temperature and rod reactivity when γ is 1540 and the noise is a white noise with zero mean and 0.1 variance. In this case, the standard deviation of reactor temperature is 3.0327e-5 degC.

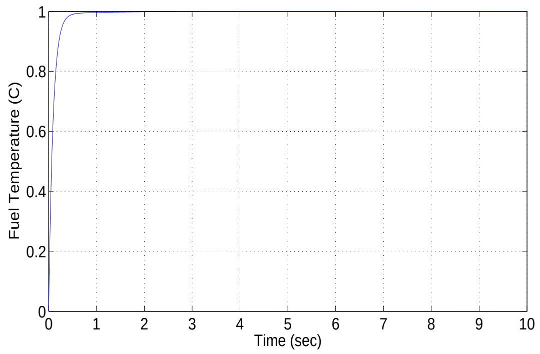

Fig. 2. Output Signal for H infinity Controller

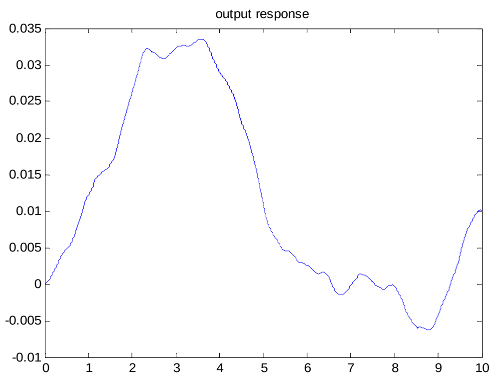

Fig. 3. Output Signal for γ is 1540

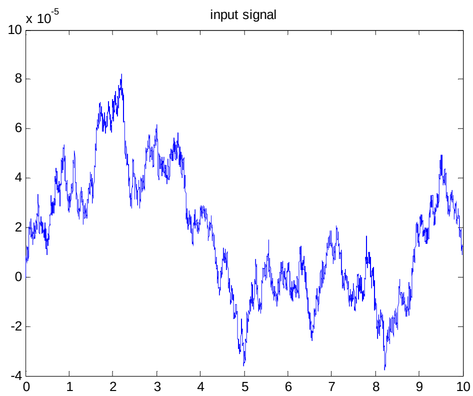

Fig. 4. Input Signal for γ is 1540

The simulation is run with a step size T=0.01 seconds and standard deviations of both reactor temperature and rod reactivity are recorded for different values of γ as shown in Table 2. The recorded data in Table 2 shows that increasing gamma improves the performance of the reactor temperature in general. Although there is no abrupt jump in magnitudes of standard deviation values, but still there a small rise in magnitude value before going down.

The closed loop tracking error signal is shown in Fig. 5. Also, the histogram of the reactor temperature error data is shown in Fig. 6. It is clear that the distribution of the error signal is non-Gaussian.

Table 2 the Simulation Results for Reactor System

γ	Reactor Temperature	Rod Reactivity
1440	0.0111	2.1907e-005
1450	0.0225	2.8938e-005
1460	0.0153	2.3696e-005
1480	0.0215	3.7223e-005
1500	0.0264	3.7200e-005
1520	0.0143	3.1037e-005
1540	0.0210	3.7322e-005
1560	0.0129	3.0327e-005
1580	0.0179	2.1635e-005
1800	0.0057	1.7114e-005
2800	0.0091	1.9403e-005
4000	0.0079	2.0086e-005

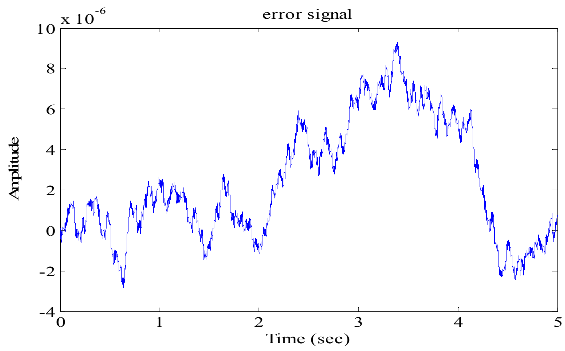

Fig. 5. Error Signal for H∞ Controller.

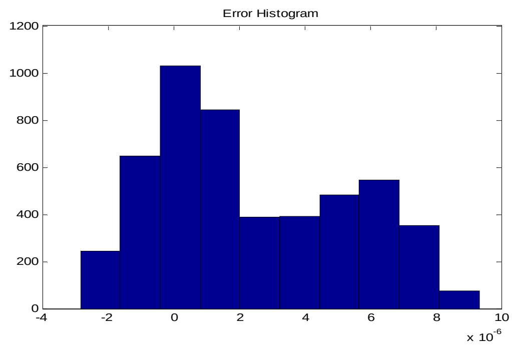

Fig. 6. Error Histogram Plot for H∞ Controller.

Based on simulation results, it can be confirmed that the H∞ control drives the system fairly quickly and smoothly to the desired trajectory and shows short settling time and non-overshoot.

5.2 PID Controller results

In a similar fashion, the closed-loop system was analyzed by generating the time response to a unit step input. The output of reactor temperature follows the reference input, the settling

time is 2secs and the overshoot is 17%, as shown in the output response of the controller Fig. 7.
By considering the effect of disturbance at the input of the reactor plant. The error signal will be as displayed on the plot in Fig. 8. As shown in Fig. 9 the histogram test of the reactor temperature error data confirms that the error distribution is not Gaussian.

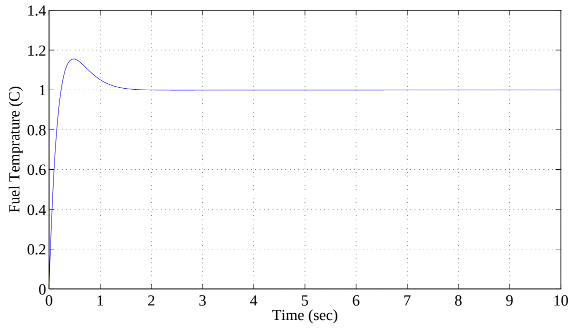

Fig. 7. Output Signal for PID Controller

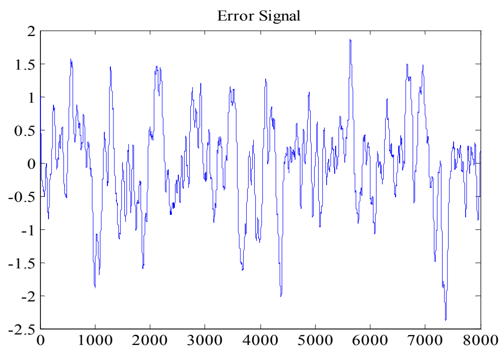

Fig. 8. Error Signal for PID Controller when adding noise

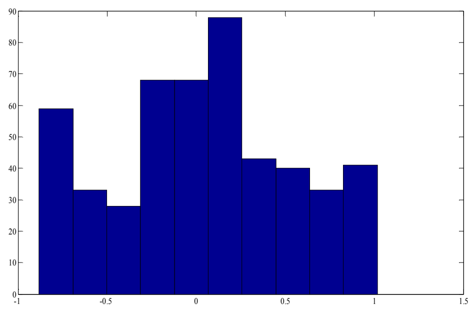

Fig. 9. Error Histogram Plot for PID Controller

The PID controller shows fairly long settling time and big overshoot in the nuclear reactor process. However, the distribution of tracking error remains non-Gaussian.

5.3 Implementation of LQG Controller

Fig 10 shows that the overshoot is 5% and the settling time is 1.75 sec. When noise is added to the plant input, the error signal will be as shown in Fig 11. Also as shown in Fig. 12, the residual distribution of the temperature error is non-Gaussian.

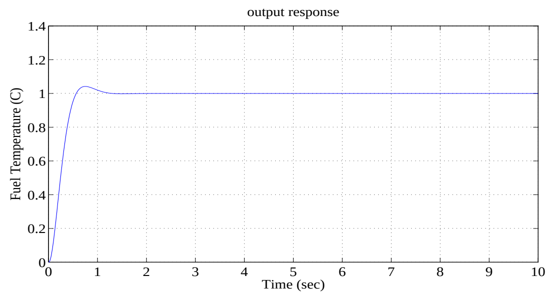

Fig. 10. Output Signal for LQG Controller

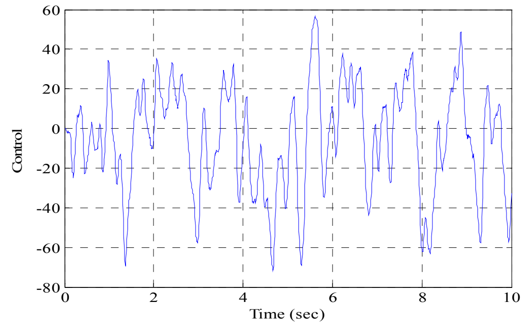

Fig. 11. Control input when adding input noise

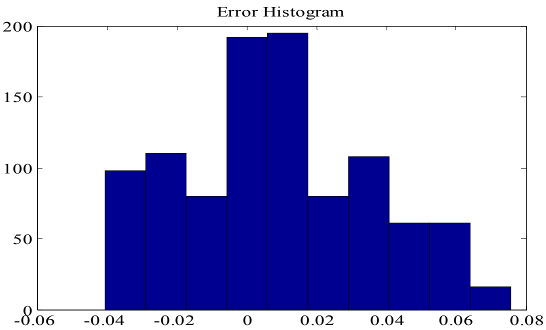

Fig. 12. Error Histogram Plot for LQG Controller

The results of this controller indicate that the steady state LQR for nuclear reactor has phase margin more than 60 deg., and all the design requirements have been achieved with the minimum value of control error. Moreover, the residual distribution of the reactor temperature error is non Gaussian.

5.4 Comparing the results

The effectiveness of each control strategy is judged based on analysis carried out on step response and comparing the overshoot and settling times as summarised in Table 3.
Table 3. shows that the LQG controlled step response had less overshoot and shorter settling time than the PID controlled step response. The H∞ control drives the system faster and more smoothly to the desired trajectory and shows better control performance than PID and LQG controllers. In addition, Table 3 shows that H∞ controller has the largest closed loop stability margin comparing with the traditional controllers.

Table 3. Comparison of controllers performance

Type of controller	H∞	LQG	PID
Time Settling (sec)	1.2	1.75	2
Overshoot %	0	5	17
Phase Margin (degree)	150.6	68.2	57.3

In This paper, the closed loop tracking error entropy is considered as a criterion for controller performance evaluation.

Table 4 shows that the H∞ controller has the minimum relative closed loop tracking error entropy which means the less effort is required to keep the value of closed loop error small.

Table 4. Comparison of controllers' error entropy

Type of controller	H∞	LQG	PID
Error Entropy	1.840	1.85	1.844

A comparison of control methodologies in terms of the error signal has been done when an input noise is added to the reactor plant, as shown in Fig. 4, 7 and 10. Moreover, the histogram test carried out over the reactor temperature error data, shows the distribution for the PID, LQG and H∞ controllers respectively, are shown in Fig. 5, 8 and 11. It is shown that no matter what type of controller is used, the distribution of tracking error is non-Gaussian. However, the standard deviation of H∞ controller is the smallest while that of the PID controller is the largest.

It can be concluded that the H∞ controller is a prefable strategy for reactor as it represents better performance and faster response.

6. CONCLUSION

Three control methodologies, namely PID, LQG and H∞, are applied to a nuclear reactor simplified model and compared from performance, and closed loop tracking error entropy point of view. Simulation results demonstrate that both the phase margin and performance of the H∞ controller are far better than that of the LQG and PID controllers. Also, the above mentioned performance measures as of the LQR controller are considerably better than that of the PID controller.

These results are obtained for a linearized model of a non-linear plant. In future it is planned to use a non-linear model for validation of the control performance and robustness. In addition, design a robust adaptive disturbance attenuating controller for nonlinear system with uncertainty.

REFERENCES

Akin, H. L. and V. Altin (1991). Rule-based fuzzy logic controller for a PWR type nuclear power plant. *IEEE Trans. Nuclear Sci.,* **38(2)**, 883-890.

Astrom, K. J. and T. Hagglund (1995). *PID Controllers: Theory, Design and Tuning.* Instrument Society of America, Research Triangle Park, NC.

Ben Abdennour, A., R.M Edwards and K.Y. Lee (1992). LQR/LTR robust control of nuclear reactors with improved temperature performance, *IEEE Trans. Nuclear Sci.*, **39(6)**, 2286-2294

Brogan, W. L. (1991). *Modern Control Theory, 3rd ed*. Englewood Cliffs, NJ: Prentice Hall.

Burl, Jeff. B. and B. B. Jeffery (1998). *Linear Optimal Control: H(2) and H (Infinity) Methods*. Addison-Wesley, Longman Publishing Co., Inc.

Doyle, J.C., B.A. Francis and A. R. Tannenbaum (1992). *Feedback Control Theory*. MacMillan Publishing Company, New York, USA.

Edwards, R.M., K.Y. Lee and M.A. Schultz (1990a). State feedback assisted classical control: an incremental approach to control modernization of existing and future nuclear reactors and power plants. *Nuclear technology*, **92(2)**, 167–185.

Edwards, R. M., K.Y. Lee and A. Ray(1990b). Robust optimal control of nuclear reactors and power plants. *Nuclear Technology*, **98(2)**, 137–148.

Gorez, R. (1997). A survey of PID auto-tuning methods. *Journal A*, **38**, 3-10

Khajavi, M. N., M. B. Menhaj and M. B. Ghofiani (2000).Robust optimal self-tuning regulator of nuclear reactors, *First conference of applications of physics and nuclear science in medical and industry, Amir Kabir university of technology, Tehran-Iran*.

Maciejowski, J. M. (1989). *Multivariable feedback design.* Addison-Wesley, New York.

O'Dwyer, A. (2003). *Handbook of PI and PID Controller Tuning Rules*. Imperial College Press, London.

Park, M. G and N. Z. Cho (1993). Time-optimal control of a nuclear reactor power with adaptive proportional-integral-feedforward gains. *IEEE Trnns. Nuclearl. Sci.,* **40**, 266–270.

Ramaswamy, P., R.M Edwards and K. Y. Lee (1993) An Automatic tunning Method of fuzzy logic controller for Nuclear Reactors. *IEEE Trans. Nuclear Sci.,* **40(4),** 1253-1262.

Silverman, B. W. (1992). *Density Estimation for Statistics and Data Analysis.* London, Uk, Chapman and Hall.

Skogestad, S. and I. Postlethwaite (1996) *Multivariable feedback control analysis and design*. Wiley, Chichester, UK.