

# A Disturbance Rejection Supervisor in Multiple-Model Based Control

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**Abstract:** In this paper, a multiple models, switching, and tuning control algorithm based on pole-placement control is studied. Drawbacks of the algorithm in disturbance rejection are discussed, and a novel supervisor to enhance the decision-making procedure is developed. The modified algorithm is evaluated in a simulation study for a nonlinear pH neutralization process. Comparison results are provided to evaluate the performance and robustness characteristics of the proposed algorithm.

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## 1. INTRODUCTION

The model of a process can change due to different operating points, changes in physical structure or chemical components, and environmental changes. In the case of time varying plant parameters, a fixed-parameter controller is not applicable and efficient. Retuning the controller, which leads to adaptive control approaches, is a possible solution. An adaptive controller overcomes parametric uncertainties by updating the process model. However, the speed of model estimation, shortcomings of identifiers, and the lack of use of past experience have restricted the implementation of conventional adaptive controllers to applications in which the initial model is almost precise, and model parameters are slowly varying.

Due to implementation successes of linear control system theory, experts turned to the design of control systems in which the problem breaks down into small linear problems in different working conditions. Then, appropriate pair of model-controller is designed for each case. The control system has a supervisor, which is responsible for choosing or combining appropriate pairs. This strategy is called multiple-model based control or multiple-control method. The idea of multiple models is widely used in modelling, identification and control of nonlinear systems (Johansen and Murray-Smith, 1997).

Generally, in multiple-control methods, a global model of the process is obtained by a combination of models or switching among them. Using the model reference control approach and switching method, (Narendra and Balakrishnan, 1997; Narendra and Xiang, 2000) proposed multiple models, switching and tuning (MMST) method, and proved stability of the closed-loop (CL) system. The MMST approach has been applied in diverse applications and in combination with various control design methods such as model reference controllers or pole-placement controllers (PPC). (Pishvaie

and Shahrokhi, 2000; Karimi and Landau, 2000; Boling et al., 2007).

In most of articles, MMST is assumed robust if each model-controller pair is individually robust to unmodeled dynamics and bounded external disturbances (Narendra and Balakrishnan, 1997). Different approaches to cope with disturbances are proposed in the literature. (Karimi and Landau, 2000) use a robust PPC. (Xiang and Narendra, 2002) recommend a method to reject known disturbances which can be transformed into an integral action to handle step-like disturbances in the simplest form. However, in (Boling et al., 2007) a constant term is added to process input in the model; then, the model bank is extended according to participating this parameter in modelling; that is, it uses the capability of multiple models approach to reject disturbances. It, also, accounts that the proposed control strategy based on MMST is weaker in disturbance rejection than the proposed switching multiple models (SMM) control strategies in disturbance rejection.

In this paper, a regulator and disturbance rejection MMST algorithm is proposed. The algorithm ensures CL stability, and reduces the number of switches. To show the effectiveness of the proposed method, the modified MMST algorithm is applied to a pH plant. Furthermore, the drawbacks of conventional adaptive controllers and MMST controllers in the presence of disturbances, resulting in abrupt changes in process parameters, are addressed. Finally, a new supervisor is designed to improve the performance of MMST algorithm.

This paper is organized as follows. Section 2 is allocated to PPC design in both adaptive and non-adaptive schemes. The MMST structure which is proposed and the supervisor which is designed to reject disturbances are explained in section 3 and 4 respectively. The simulation results are given in section 5. Finally, the paper is concluded in the last section.

## 2. POLE-PLACEMENT CONTROL DESIGN

In this section, both non-adaptive and adaptive pole placement schemes are briefly reviewed (Astrom and Wittenmark, 1995). A pole-placement controller (PPC) uses an ARX model of the process which is described by:

$$A(q)y(t) = B(q)u(t) + v(t) \quad (1)$$

where  $y$  is the output and  $u$  is the input, and  $v$  is white noise all in time instant  $t$ .  $A$  and  $B$  are relatively prime polynomials in forward shift operator  $q$ . Let the model reference be as follows:

$$A_m(q)y_m(t) = B_m(q)u_c(t) \quad (2)$$

where  $u_c$  is the reference signal. The control objective is to design a control law such that the plant output,  $y$ , follows the model reference output,  $y_m$ . Thus, the dynamical output feedback controller is given by:

$$R(q)u(t) = -S(q)y(t) + T(q)u_c(t) \quad (3)$$

Controller parameters,  $R$ ,  $S$ , and  $T$ , are designed to achieve the desired tracking and disturbance rejection behaviour (Astrom and Wittmark, 1995).

In the case of unknown or time varying plant parameters, using an identifier to estimate process model parameters, an indirect adaptive PPC (APPC) is obtained. Recursive least squares identification algorithm (RLS) is the most common identifier in adaptive controllers due to its simplicity and implementability, fast convergence, and robustness (Malik et. al., 1991). Not surprisingly it is vital to enhance the identifier to promote performance of an adaptive controller. See (Malik et. al., 1991).

To improve the overall CL performance of the adaptive controller: White noise with appropriate magnitude is injected into the control signal, and to discard steady-state values from input and output data owing to identification of a nonlinear system by a linear model, a high-pass proper filter is designed which has unity static gain. To prevent parameter drift due to measurement noise the pre-filter is changed to a band-pass filter  $H_f$ . Therefore, the filter

$$H_f(q) = (1 - a_1)^2 (q - 1) / (q - a_1)^2 \quad (4)$$

is proposed, where  $a_1$  is chosen 3 to 5 times faster than the dominant desired CL poles. Moreover, to reduce the harmful effects of high-frequency components of step-like set-points, a low-pass filter with unity static gain is used and its pole is selected fifty times faster than the dominant pole of the CL system. Finally, to prevent covariance matrix blow-up, especially in the presence of measurement noise, the trace of covariance matrix will be held constant (Malik et. al., 1991; Astrom and Wittmark, 1995). Then, we use:

$$P(t) = c_1 \frac{\bar{P}(t)}{tr(\bar{P})} + c_2 I, \quad c_1 / c_2 \approx 10^4, \quad c_1 \varphi^T \varphi \gg 1 \quad (5)$$

where  $\bar{P}$  is the covariance matrix computed in the conventional RLS.

## 3. PRINCIPLES OF CONTROL BASED ON MULTIPLE MODELS, SWITCHING AND TUNING

The basic idea is to choose the best model, describing the current operating condition of the process, in every instant from a pre-designed model set according to a numerical criterion, and locate the corresponding controller in the feedback loop. A fixed model bank is not sufficient to match all possible process behaviour with high accuracy. On the other hand, the use of more fixed models increases the computational complexity. An attempt to overcome this difficulty is to adapt the model bank to the current environment. In summary, like other adaptive control approaches, there are two loops: identification and control. However, there is a two-stage identification structure in which large and abrupt variations in process parameters are observed by switching, and slow variations in the model of an environment are detected by tuning.

The multiple models, switching, and tuning (MMST) strategy, presented in this paper, consists of three major units, as shown in Fig. 1. The first unit is the model bank composed of  $P$  models which are fixed except one that is adaptive. They are the predictors which anticipate one-step-ahead output of the process according to the past input and output data. The adaptive model whose parameters are updated in every instant by the RLS algorithm is a reinitializable model; that is, its parameters at specific times will be reset to the parameters of one of the fixed models. Hence, it can be said that a fixed model which is called the best model is always in the background of the adaptive one. In this strategy we do not use a free-running adaptive model unlike (Narendra and Balakrishnan, 1997; Narendra and Xiang, 2000) in which it is used to guarantee stability.

The second unit is the supervisor, which receives differences between predicted outputs of the models,  $\hat{y}_s$  and  $s \in [1, P]$ , and real output of the process, called the prediction error. It also proposes a switching scheme in accordance with a performance index to pick a fixed model as the best, and orchestrate the control action to switch between the best fixed model and the adaptive one. The performance index is given by:

$$J_s(t) = \alpha e_s^2(t) + \beta \sum_{k=1}^M \lambda^k e_s^2(t-k) \quad (6)$$

$$, \quad 0 < \lambda \leq 1, \quad \alpha, \beta, M > 0, s \in [1, P]$$

in which  $e_s = y_f - \hat{y}_s$ , the prediction error, and  $\alpha, \beta, \lambda$ , and  $M$  are the free-design parameters, which are effective in the control system performance. To constrain switching speed, two hysteresis cycles in the tuning and the switching stages are used; they are positive values  $h_a$  and  $h_f$ . The subscripts  $a, B, i$ , and  $s$  are used to denote the adaptive model, the best-fixed model, all  $(P-1)$  fixed models, and all models in the bank, respectively. The supervisor calculates  $\bar{J} = \min_i \{J_i\}$  and  $l = \arg \min \{J_i\}$ ,  $i \in [1, P-1]$  at each sampling time. To guarantee stability, the adaptive model does not reinitialize immediately after the best background fixed model changes,

so the adaptive model can contribute in the switching stage like a free-running adaptive one. Fig. 2 shows the modified switching scheme that the supervisor does in each sampling time.

The third unit of the strategy is the PPC, which gets the parameters of the models, selected by the supervisor, as the process model parameters, and calculates the control signal that must be applied to the process. In case the supervisor picks the adaptive model, an adaptive controller will control the process.

*Comment 1:* At reinitializing moment, not only the parameters of the adaptive model change to the best ones, but also the memory of performance index of the adaptive model must be replaced by that of the best model. Also, in this strategy the parameters of the adaptation algorithm, for example in the RLS method the covariance matrix, can be easily revised to increase the convergence rate.

*Comment 2:* There is only one controller to which parameters of the model are sent, so unlike other SMM structures it is not necessary to employ a bumpless transfer method.

*Comment 3:* One is able to demonstrate that CL stability can be guaranteed without the free-running RLS estimator and with the modified switching scheme. According to (Ibeas et al., 2004; Narendra and Xiang, 2000), the worst case happens when the fixed model bank is not stabilizing. In this case relating to the magnitude of  $h_f$  which prevents successive switches, the adaptive controller is chosen after a finite time and the switching action stops. Hence, stability will be guaranteed by a stabilizing adaptive controller.

*Comment 4:* Free-design parameters are  $\alpha, \beta, \lambda, M, h_a$  and  $h_f$  which have a direct effect on the control performance. For instance, switching in variations will be too fast, provided  $\alpha/\beta$  is chosen large, while  $M$  and  $\lambda$  is selected small. Thus, these conditions lead to a poor performance, and they make the overall system sensitive to measurement noise. Moreover, decreasing  $h_f$  results in slow switching. It is worth mentioning that the proposed algorithm has two ways to weaken the effect of measurement noise: by integral characteristic of the performance index and by the magnitude of hysteresis cycles.

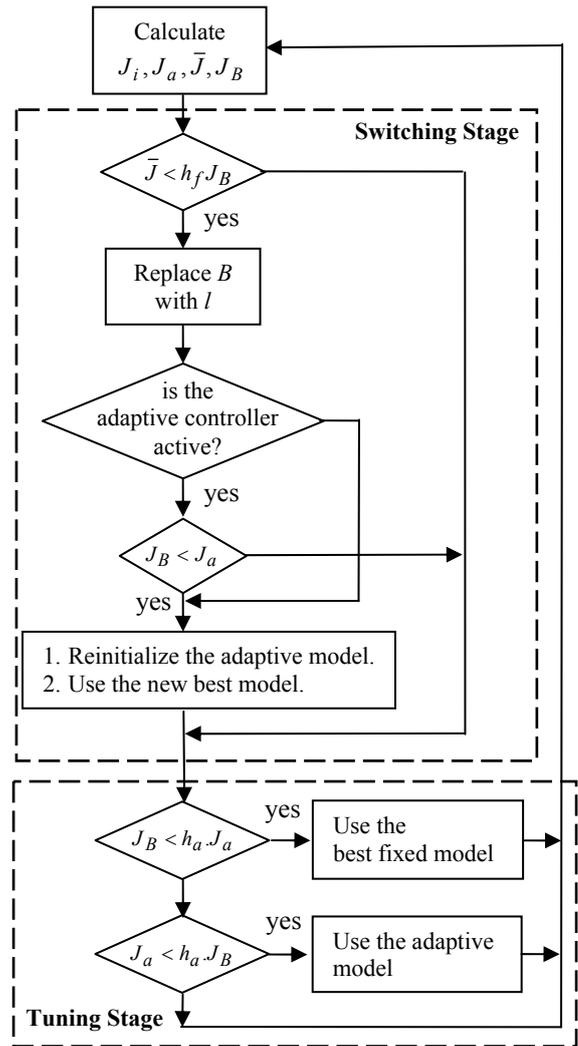


Fig. 2. The flowchart of the proposed switching scheme in the main supervisor that selects the parameters of the best model or the adaptive model as those of the process model.

#### 4. DESIGN A SUPERVISOR FOR DISTURBANCE REJECTION

In process control, rarely does command signal change. However, what usually occur are low-frequency disturbances. Hence, disturbances are a major source of excitation, and if there is supervision, they can be advantageous for convergence of parameters (Hagglund and Astrom, 2000). On the other hand, when an estimator is used in a control system, it is critical to manipulate a supervisor in order for assurance of parameters convergence to real values; thus, conventional adaptive controllers are delicate in the presence of disturbances on the ground that process model parameters may drift to wrong values. Indeed, despite the fact that an integral action can overcome step-like load disturbances, the performance of the controller is intensively poor.

It is evident that a fixed controller can do better disturbance rejection than an adaptive controller can do. Likewise, the performance of a switching multiple models (SMM) based controller is superior to that of a MMST based controller

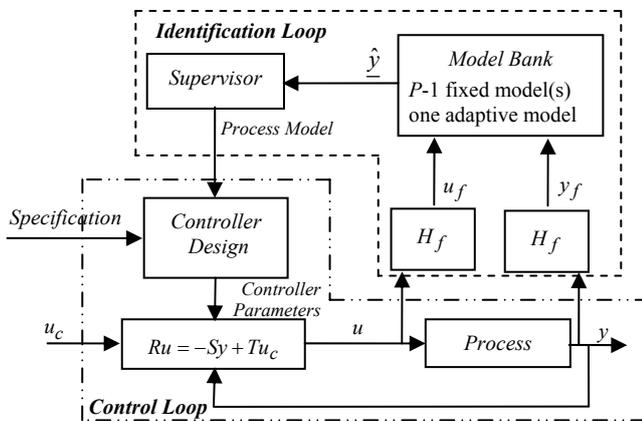


Fig.1 . The block diagram of the MMST strategy.

from the point of view of the disturbance rejection. (Hagglund and Astrom, 2000) introduced a kind of supervisors to increase robustness and reliability of conventional adaptive controllers. It brings in a method to detect when a load disturbance occurs and then the supervisor suspends adaptation so that incorrect excitation does not lead to devastation of the estimation.

Algorithm has a delay in detecting disturbances for nonlinear systems, which results in deviation in the estimates. Then, it is not possible to control the process according to the last model reported by the identifier. However, when we have a stabilizing fixed model bank, it is possible for the supervisor to switch to the best model after detecting a disturbance. As a result, the idea suggests a way to enhance the supervisor of the control strategy.

The disturbance supervisor functions in the following way. Consider a SISO process with a positive gain. First, process input and output are passed through a high-pass filter, or in the presence of measurement noise through a band-pass filter like  $H_f$  in order to generate  $u_f$  and  $y_f$ . If  $abs(u_f) > \bar{u}$  and  $abs(y_f) > \bar{y}$ , in which  $\bar{u}, \bar{y} > 0$  are two pre-defined thresholds, the sign of  $u_f y_f$  will be checked. If it is a positive value, set-point is changed; otherwise, a disturbance has occurred. The latter restricts the use of the tuning mode for a particular period of time. Throughout this period, switching to and adjusting of the adaptive model is not allowed, and the adaptive model is reinitialized at the end of the period. The length of the period is almost equal to time constant of the CL system. Since the adaptation starts in the second phase of disturbance rejection, the excitation is suitable, and the convergence is fast, so the control action switches quickly to the adaptive one.

However, in the event that the deviation from set-point is large with regard to a measure, it is possible to choose poles of the model reference faster than normal case to force the output to settle rapidly. The selected measure is an integrator with a limited memory given by:

$$J_{mon} = \sum_{k=0}^N (y(t-k) - y_m(t-k))^2 \quad (7)$$

in which  $y$ ,  $y_m$ , and  $N$  are output of the process, output of the model reference, and memory of the performance index respectively. Briefly, if  $J_{mon}$  is in a specific range, and the supervisor reports that a disturbance has occurred, the process will be controlled faster than before. This feature comes of decoupling regulation and disturbance rejection problems by the defined detector. Fig. 3 depicts how the supervisor works.

The suggested algorithm without imposing excessive computational burden can improve the ability of the overall control strategy to response to disturbances that result from either load disturbance or abrupt changes in process model parameters.

## 5. SIMULATION RESULTS

A simulated pH neutralization plant is considered to study the ability of the suggested algorithm. Model details are given in (Henson and Seborg; 1994). The process contains a continuous stirred tank reactor that has three inlet streams: base, acid, and buffer. The objective is to control pH value of the outlet stream. The buffer flow rate is the major

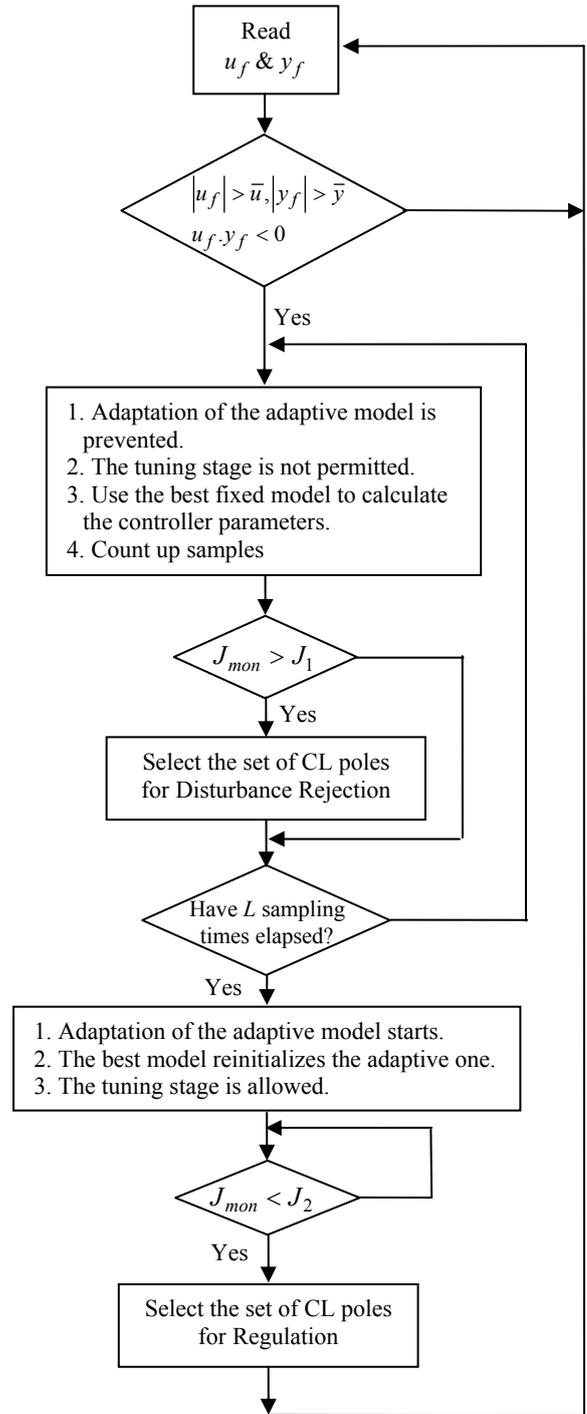


Fig. 3. The flowchart of the proposed supervisor for disturbance rejection. This supervisor is located above the main supervisor of the MMST strategy, and control the tuning identification stage.

unmeasured disturbance; moreover, one can consider the acid flow rate as another disturbance. Sampling time and system time delay are 6 and 18 seconds, respectively. In this section, we evaluate a conventional adaptive pole-placement controller (APPC), a switching multiple-model based controller (SMM), and the proposed algorithm in the pH plant, i.e. multiple models, switching, and tuning (MMST).

For all cases, fixed model bank is composed of simple first order models with a time delay whose parameters are estimated by mobilizing an APPC in various operating points. In all scenarios, we use the same pH set-point profile, which is:

$$\begin{aligned} &5 \rightarrow 6 \rightarrow 7 \rightarrow 8 \rightarrow 9 \rightarrow 10 \rightarrow 9 \rightarrow 8 \rightarrow 7 \rightarrow 6 \\ &\rightarrow 5 \rightarrow 7 \rightarrow 9 \rightarrow 10 \rightarrow 8 \rightarrow 6 \\ &\rightarrow 5 \rightarrow 9 \rightarrow 7 \rightarrow 5 \rightarrow 10 \rightarrow 7 \rightarrow 6 \rightarrow 10 \end{aligned}$$

The profile makes three kinds of changes with different amplitudes. The comparison according to mean square errors (MSE) in these three parts is done. There is, also, a sequence of disturbances on acid and buffer flow rates represented by  $q_1$  and  $q_2$  respectively. We use the sequence in scenario III that is:

$$\begin{cases} q_1 \\ q_2 \end{cases} \rightarrow \begin{cases} 16.6 \\ 0.55 \end{cases} \rightarrow \begin{cases} 14.6 \\ 1.2 \end{cases} \rightarrow \begin{cases} 18.6 \\ 2 \end{cases} \rightarrow \begin{cases} 16.6 \\ 0 \end{cases} \rightarrow \begin{cases} 12.0 \\ 0.55 \end{cases} \rightarrow \begin{cases} 20.0 \\ 0.55 \end{cases}$$

The comparison of disturbance rejection of the control algorithms is done based on MSE and maximum deviation from pH=7 in the worst case. The first scenario is tested in an ideal condition, yet the others are noisy, disturbed by white noise added to pH value with a variance of  $10^{-4}$ ; thus  $H_f$  and a constant trace RLS method are used in these cases.

In regulation problem, two poles of the model reference are selected 3 times faster than the pole of the open-loop model, and the rest are in the origin of the discrete domain.

*Scenario I:* There is a fixed model bank with 4 models which have been estimated and selected in 4 different operating points. The aim is to judge the performance of APP, SMM, and MMST algorithms to track the output of a model reference. The values of  $h_f$  and forgetting factors of RLS in the APPC and MMST controller are 0.8, 0.99, and 0.99 in order. We choose the parameters of the performance index and hysteresis bandwidth as  $\alpha = 3, \beta = 1, \lambda = 0.985$ , and  $M = 30, h_a = 0.5, h_f = 0.8$ . Table 1 summarizes the results of this scenario. Also, Fig. 4. shows the performance of APPC and MMST.

**Table 1 MSE for scenario I**

	Small steps	Medium steps	Large steps	Overall
APPC	0.027	0.414	10.330	3.217
SMM	0.062	0.371	2.985	1.019
MMST	0.009	0.101	1.140	0.372

*Scenario II:* The objective is to assess the ability of the MMST algorithm in the presence of measurement noise with a very imprecise fixed model bank in order to control the pH plant in two highly discrepant situations; one is nominal plant with a predetermined buffer flow rate, and the other is without buffer, considered as disturbed plant. To assure stability in SMM controller it is chosen the model bank as it has 3 models, specifically for nominal case and one another model whose gain is high for the worst case. The trace of covariance matrix of RLS is 20000. The altered parameters of the performance index and hysteresis bandwidth are  $\alpha = 2, M = 20$ , and  $h_f = 0.5$ . The results are given in table 2 and 3.

**Table 2 MSE for scenario II; nominal case**

	small	medium	large	Overall
APPC	0.657	18.707	56.220	21.892
SMM	0.242	1.164	2.113	1.041
MMST	0.142	0.939	2.726	1.122

**Table 3 MSE for scenario II; disturbed case**

	small	medium	large	overall
APPC	0.250	7.065	22.568	8.699
SMM	2.668	1.793	67.178	21.800
MMST	0.118	0.8667	14.3461	4.579

*Scenario III:* We repeat scenario II to compare disturbance rejection of each algorithm. In this scenario, acid and buffer flow rates change in turn, based on the mentioned sequence. Table 3 shows the results. Furthermore, Fig. 5. depicts the performance of SMM, MMST, and MMST with the disturbance rejection supervisor.

**Table 4 the results of scenario III – dist. rejection**

	The worst case		Overall	Usage of adaptive model
	MSE	Max. Dev.	MSE	
APPC	2668	2.60	After the last case, the CL system is saturated.	
SMM	564	2.23	723	-
MMST	990	2.22	1126	6.6%
MMST+Dist Supervisor	311	2.20	600	30%

*Discussion:* From scenario I, it can be deduced that the proposed MMST algorithm has a better transient response than the others. As demonstrated in scenario II, the MMST algorithm in the presence of measurement noise has an acceptable performance in both nominal and disturbed cases. The performance of the SMM based controller deteriorates highly in the event buffer flow rate becomes zero owing to the fact that its model bank selected specifically for the nominal case. However, the suggested algorithm can obtain an acceptable performance in this condition. Regarding table

4, one can easily infer that the SMM based controller is better than the MMST based controller in disturbance rejection. On the other hand, the proposed supervisor is efficient in improving the MMST based controller. Also, this additional supervisor has increased the percentage of the use of the adaptive model, which is beneficial for the control performance.

## 6. CONCLUSION

In this paper, a modified MMST strategy is presented, and its advantageous in tracking and disadvantageous in disturbance rejection are discussed. The proposed strategy has the following key characteristics. i) The presence of a reinitializable adaptive model that can contribute in switching identification stage. ii) The presence of a hysteresis cycle instead of dwell time, which can be used to deal effectively with measurement noise. iii) The consideration of a limited memory in the performance index that can make switching both fast and accurate. iv) The design of a disturbance rejection supervisor. v) The absence of a free-running adaptive model in spite of stability assurance in contrast to (Narendra and Xiang, 2000).

The suggested disturbance rejection supervisor can improve the properties of the multiple models, switching, and tuning based controllers in the presence of unmeasured low-frequency disturbances in a simple and low-computational way. The simulation results with a broad analytical comparison substantiate an improvement in disturbance rejection and tracking of the suggested control algorithm.

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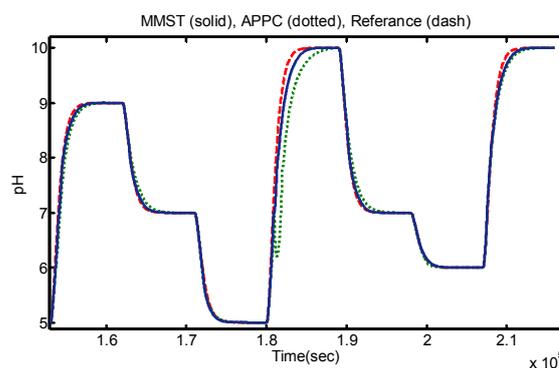


Fig. 4. Scenario I, comparison between APCC and MMST for large step changes.

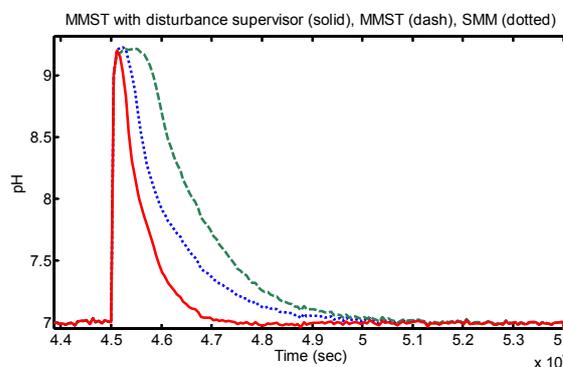


Fig. 5. Scenario III, deviation from pH=7 the worst case in disturbance rejection.