L_2 gain analysis for linear discrete switched delay systems ⋆

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Abstract: The problem of L_2 gain for a class of linear discrete switched systems with disturbance input is considered in this paper. It is assumed that not all of the subsystems have normal L_2 gain. Based on the average dwell time method, we search for switching signals to make the switched systems achieve the weighted L_2 gain. As a special case, the criterion of normal L_2 gain under arbitrary switching is also developed. Without considering delays, the proposed result degenerates to existing one.

Keywords: switched systems, average dwell time, delay, weighted L_2 gain.

1. INTRODUCTION

Switched systems belong to a special class of hybrid control systems, which consist of a family of continuous-time or discrete-time subsystems and a switching law that orchestrates the switching between them. In recent years, great development of the switched systems has been made in the control community (see, for example, Branicky [1998], Liberzon [2003], Zhao and Spong [2001], Hespanha et al. [2005]). When system does not satisfy stability under arbitrary switching, it is necessary to search for some effective tools to guarantee the stability of the systems, in which average dwell time technique is one of important methods (Hespanha and Morse [1999], Zhai et al. [2001], Zhai and Lin [2004], Lin et al. [2003]). In Hespanha and Morse [1999], the average dwell time method is first introduced. Then, in Zhai et al. [2001], average dwell time method is extended to the case where switched systems contain unstable subsystems. Lu et al. [2006] gives the applications of this method in F-16 Aircraft control.

As is well known, time delays are very common phenomena in many real control systems and are often main sources of instability and performance deterioration. Thus during the past decades, the problems of stability and stabilizability have received much attention Gu et al. [2003], Fridman and Shaked [2003], He et al. [2004]. Switched systems with time delays are referred to as switched delay systems Xie and Wang [2004], Sun et al. [2006b], Sun et al. [2006a], Kim et al. [2006]. Roughly speaking, a switched delay system appears if switching and time delay co-exist in either system modeling or signal transmission. Due to the interaction between continuous dynamics and discrete dynamics and because of the impact of time delays, the behavior of switched delay systems is usually much more complicated than that of switched systems or delay systems. Our previous papers Sun et al. [2006b] and Sun et al. [2006a] are concerned with the problems of L_2 gain and exponential stability based on average dwell time methods. But these two papers do not consider the case of unstable subsystems as well as discrete case. Although the case containing unstable subsystems is difficult to deal with but seems more practical and thus more desirable. For example, Lin et al. [2003] models a class of NCSs with packet dropout as switched systems with unstable systems; Zhai and Lin in Zhai and Lin [2004] generalize the problem of controller failure into the problem of this case. In addition, for discrete delay case, few papers have appeared. Xie and Wang [2004] only considers the conditions of asymptotical stability under arbitrating switching signals for discrete switched delay systems.

It is necessary to note that the results on exponential stability and L_2 gain for linear discrete switched systems without delay based on average dwell time method have already appeared Zhai et al. [2002], Lin et al. [2003]. But it has been difficult to develop the corresponding criteria for delay case. In this paper, two lemmas reflecting the change of a selected special Lyapunov functional candidate are first developed. Then, based on average dwell time method, sufficient conditions to guarantee weighted L_2 gain of the considered system are presented in the form of linear matrix inequalities.

This paper is organized as follows. In Section II, preliminaries and problem formulations are introduced. In Section III, two lemmas are first given and then the problem of weighted L_2 gain is discussed. The conclusions are summarized in Section VI.

2. PRELIMINARIES AND PROBLEM FORMULATION

This paper uses $P > 0$ ($\geq, <, \leq 0$) to denote a positive definite (positive-semidefinite, negative definite, negative-semidefinite) matrix P. $\lambda_{\max}(P)$ and $\lambda_{\min}(P)$ denote the maximum and minimum eigenvalues of P.

Consider the following discrete switched delay system of the form

$$\begin{aligned} x(k+1) &= A_{\sigma(k)}x(t) + E_{\sigma(k)}x(k-\tau) + F_{\sigma(k)}w(k) \\ z(k) &= D_{\sigma(k)}x(k) \\ x_k &= \varphi(k), k \in \bar{M} = \{-\tau, \ldots, -1, 0\}, \end{aligned} \tag{1}$$

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where $x(k) \in R^{n_1}$ and $z(k) \in R^{n_2}$ denote the state vector and controlled output respectively; $w(t) \in L_2(0,+\infty)$ is disturbance input; $\sigma(t) : [0,\infty) \to M = \{1,2,\cdots,m\}$ is the switching signal which depends on time t or state $x(t)$; A_i, E_i, F_i and D_i are known constant matrices with appropriate dimensions for $i \in M$; $\varphi(k)$ denotes the initial function, $\tau > 0$ is the constant time delay.

When we only consider the stability of system (1), disturbance input is often deleted, and thus system (1) reduces to the following form

$$\begin{aligned} x(k+1) &= A_{\sigma(k)}x(t) + E_{\sigma(k)}x(k-\tau) \\ x_k &= \varphi(k), k \in \bar{M} = \{-\tau,\dots,-1,0\}. \end{aligned} \tag{2}$$

In this paper, we study the case where stable and unstable subsystems coexist in system (2). without loss of generality, suppose for system (2), the ith subsystem is stable for $i \in \{1,\dots,r\}$ and the other subsystems, i.e. $i \in \{r+1,\dots,m\}$ are unstable. Correspondingly, for system (1), we also suppose the ith subsystem can achieve normal L_2 gain for $i \in \{1,\dots,r\}$, while the other subsystems cannot. We will use $K^+(t_1,t_2)$ to denote the total activation time of unstable subsystems (or unstable in the sense of L_2 gain) in time interval $[t_1,t_2)$, and $K^-(t_1,t_2)$ to denote the total activation time of stable systems (or stable in the sense of L_2 gain) in time interval $[t_1,t_2)$. The following definitions are needed.

Definition 1. The equilibrium $x^* = 0$ of the system (2) is said to be exponentially stable under $\sigma(k)$ if the solution $x(k)$ of the system (2) satisfies

$$\|x(k)\| \le \kappa \left\|x_{k_0}\right\| e^{-\lambda(k-k_0)}, \forall k \ge k_0 \ge 0$$

for constants $\kappa \ge 1$ and $\lambda > 0$, where $\|x(k)\|$ denotes the Euclidean norm, and $\|x_k\| = \sup_{\theta\in\bar{M}} \|x(k+\theta)\|$. λ is called stability degree.

Definition 2 (Liberzon [2003]). For any $T_2 > T_1 \ge 0$, let $N_\sigma(T_1,T_2)$ denote the number of switching of $\sigma(t)$ over (T_1,T_2). If $N_\sigma(T_1,T_2) \le N_0 + \frac{T_2-T_1}{T_a}$ holds for $T_a > 0, N_0 \ge 0$, then T_a is called average dwell time and N_0 is called chattering bound. As commonly used in the literature, we choose $N_0 = 0$.

Definition 3 (Zhai et al. [2001]). For $\alpha > 0$ and $\gamma > 0$, the system (1) is said to have (or achieve) weighted L_2-gain , if under zero initial condition, i.e., $\varphi(k) = 0$ for any $k \in \bar{M}$, it holds that

$$\sum_{k=0}^{+\infty} e^{-\alpha k} z^T(k)z(k) \le \gamma^2 \sum_{k=0}^{+\infty} w^T(k)w(k). \tag{3}$$

Problem formulation: Based on average dwell time method, search for switching signals such that system (1) achieves a weighted L_2 gain.

3. ANALYSIS OF DISTURBANCE ATTENUATION

In this section, we will study the disturbance attenuation property of the system (1). First, we consider delay system without switching

$$\begin{aligned} x(k+1) &= Ax(k) + Ex(k-\tau) + Fw(k) \\ z(k) &= Dx(k) \\ x_k &= \varphi(k), k \in \bar{M}, \end{aligned} \tag{4}$$

Choose the following Lyapunov functional candidate,

$$V(k) = x^T(k)Px(k) + \sum_{l=k-\tau}^{l=k-1} e^{\alpha(l-k+1)} x^T(l)Qx(l). \tag{5}$$

Along the trajectory of the system (4), for Lyapunov function candidate (5), we have the following result.

Lemma 1. Given $\alpha > 0, \gamma_0 > 0$, and $\tau > 0$, if there exist matrix $P > 0$, $Q > 0$ such that

$$\begin{bmatrix} A^TPA - e^{-\alpha}P \\ \quad +Q + D^TD & A^TPE & A^TPF \\ * & E^TPE - e^{-\alpha\tau}Q & E^TPF \\ * & * & F^TF - \gamma_0^2 I \end{bmatrix} < 0, \tag{6}$$

then along the trajectory of system (4), for Lyapunov function candidate (5), it holds that

$$V(k+1) \le e^{-\alpha}V(k) - \Gamma(k),$$

where $\Gamma(k) = z(k)^T z(k) - \gamma_0^2 \omega^T(k)\omega(k)$.

Proof. See the appendix.

Remark 1. Lemma 1 also implies that the system (4) achieves normal L_2 gain, as can be seen in the following. First,

$$\begin{aligned} V(k+1) &\le e^{-\alpha}V(k) - \Gamma(K) \le \dots \\ &\le -e^{-\alpha(k+1)}V(0) - \sum_{l=0}^{k} e^{-\alpha(k-l)}\Gamma(l), \end{aligned}$$

then, under zero initial condition, we have

$$0 \le -\sum_{l=0}^{k} e^{-\alpha(k-l)}\Gamma(l),$$

that is

$$\sum_{l=0}^{k} e^{-\alpha(k-l)} z^T(l)z(l) \le \sum_{l=0}^{k} e^{-\alpha(k-l)} w^T(l)w(l).$$

Summing this inequality from $k = 0$ to $+\infty$ leads to normal L_2 gain

$$\sum_{l=0}^{+\infty} z^T(l)z(l) \le \sum_{l=0}^{+\infty} w^T(l)w(l).$$

Lemma 2. Given $\alpha_1 > 0$, $\alpha_2 > 0, \gamma_0 > 0$ and $\tau > 0$, if there exist matrix $P > 0$, $Q > 0$ such that LMI

$$\begin{bmatrix} A^TPA - e^{-\alpha_2}P \\ \quad +Q + D^TD & A^TPE & A^TPF \\ * & \begin{matrix} E^TPE \\ -e^{\alpha_1(-\tau+1)+\alpha_2}Q \end{matrix} & E^TPF \\ * & * & F^TF - \gamma_0^2 I \end{bmatrix} < 0, \tag{7}$$

holds, then along the trajectory of system (4), for Lyapunov function candidate (5), it holds that

$$V(k+1) \le e^{\alpha_2}V(k) - \Gamma(k).$$

Proof. See the appendix.

Now, we give the result on weighted L_2 gain of the system (1).

Theorem 1. Given $\alpha_1 > 0$, $\alpha_2 > 0, \gamma_0 > 0$, and $\tau > 0$, if there exist matrix $P_i > 0$, $Q_i > 0$ such that LMIs

$$\begin{bmatrix} A_i^TP_iA_i - e^{-\alpha_1}P_i \\ \quad +Q_i + D_i^TD_i & A_i^TP_iE_i & A_i^TP_iF_i \\ * & \begin{matrix} E_i^TP_iE_i \\ -e^{-\alpha_1\tau}Q_i \end{matrix} & E_i^TP_iF_i \\ * & * & F_i^TF_i - \gamma_0^2 I \end{bmatrix} < 0, \tag{8}$$

hold for for $\forall i \in \{1,2,\dots,r\}$, and such that LMIs

$$\begin{bmatrix} A_i^T P_i A_i - e^{\alpha_2} P_i + Q_i + D_i^T D_i & A_i^T P_i E_i & A_i^T P_i F_i \\ * & E_i^T P_i E_i - e^{\alpha_1(-\tau+1)+\alpha_2} Q_i & E_i^T P_i F_i \\ * & * & F_i^T F_i - \gamma_0^2 I \end{bmatrix} < 0, \quad (9)$$

hold for $\forall i \in \{r+1,\ldots,m\}$, then the switched delay system (1) achieves a weighted L_2 gain under switching law $\Phi_{[\tau_a]}$ satisfying the following two conditions,

$Cond_1$ Suppose $0 = t^0 < t^1 < \cdots (\lim_{j\to\infty} t^j = \infty)$ be a specified sequence of time instants satisfying $\sup_j (t^{j+1} - t^j) = T < \infty$, under this sequence, the switching signal satisfies

$$\frac{K^-(t^j, t^{j+1})}{K^+(t^j, t^{j+1})} \geq \frac{\alpha_2 + \alpha^*}{\alpha_1 - \alpha^*}$$

for every time interval $[t^j, t^{j+1})$ $(j = 0, 1, \cdots)$, where $\alpha^* \in (0, \alpha_1)$.

$Cond_2$ The average dwell time satisfies $T_a \geq T_a^* = \frac{ln\mu}{\alpha}$, where $\alpha \in (0, \alpha^*)$, and $\mu \geq 1$ satisfies

$$\begin{aligned} &P_i \leq \mu P_j, \\ &Q_i \leq \mu Q_j, \quad \forall i, j \in M. \end{aligned} \quad (10)$$

Proof. Choose the piecewise Lyapunov function candidate

$$V(k) = x^T(k) P_{\sigma(k)} x(k) + \sum_{l=k-\tau}^{l=k-1} e^{\alpha(l-k+1)} x^T(l) Q_{\sigma(k)} x(l), \quad (11)$$

where $\sigma(k) \in M$, $P_i > 0, Q_i > 0$ are positive definite matrix satisfying LMI (8) for $i \in \{1,\ldots,r\}$ and otherwise satisfying (9) for $i \in \{r+1,\ldots,m\}$. Along the solutions of the corresponding subsystems, from LMIs (8), (9), Lemma 1 and 2, we obtain that

$$V_i(k+1) \leq \begin{cases} e^{-\alpha_1} V_i(k) - \Gamma(k), & \sigma(k) \in \{1,\ldots,r\}; \\ e^{\alpha_2} V_i(k) - \Gamma(k), & \sigma(k) \in \{r+1,\ldots,m\}. \end{cases}$$

For any time interval $[0,k)$, $k_1, k_2, \ldots$ is used to denote the switching points. suppose $k_0 = 0 < \ldots < k_q < k < k_{q+1}$.
Thus for time interval $[k_q, k]$, it holds that

$$V(k) \leq \begin{cases} e^{-\alpha_1(k-k_q)} V(k_q) - \sum_{l=k_q}^{k-1} e^{-\alpha_1(k-l-1)} \Gamma(l), \\ \qquad \sigma(k_q) \in \{1,\ldots,r\}; \\ e^{\alpha_2(k-k_q)} V(k_q) - \sum_{l=k_q}^{k-1} e^{\alpha_2(k-l-1)} \Gamma(l), \\ \qquad \sigma(k_q) \in \{r+1,\ldots,m\}. \end{cases}$$

Introduce a sign to denote

$$\alpha(i) = \begin{cases} -\alpha_1, & \sigma(k_i) \in \{1,\ldots,r\} \\ \alpha_2, & \sigma(k_i) \in \{r,\ldots,m\} \end{cases}.$$

Since $V(k_i) \leq \mu V(k_i^-)$ holds on every switching points k_i from (10), we obtain by induction that

$$\begin{aligned} V(k) &\leq e^{\alpha(q)(k-k_q)} V(k_q) - \sum_{l=k_q}^{k-1} e^{\alpha(q)(k-l-1)} \Gamma(l) \\ &\leq e^{\alpha(q)(k-k_q)} \mu V(k_q^-) - \sum_{l=k_q}^{k-1} e^{\alpha(q)(k-l-1)} \Gamma(l) \\ &\leq \mu e^{\alpha(q)(k-k_q)} [e^{\alpha(q-1)(k_q-k_{q-1})} V(k_{q-1}) \\ &\quad - \sum_{l=k_{q-1}}^{k_q-1} e^{\alpha(q-1)(k_q-l-1)} \Gamma(l)] \\ &\quad - \sum_{l=k_q}^{k-1} e^{\alpha(q)(k-l-1)} \Gamma(l) \\ &= \mu e^{\alpha(q)(k-k_q)+\alpha(q-1)(k_q-k_{q-1})} V(k_{q-1}) \\ &\quad - \mu \sum_{l=k_{q-1}}^{k_q-1} e^{\alpha(q)(k-k_q)+\alpha(q-1)(k_q-l-1)} \Gamma(l) \\ &\quad - \sum_{l=k_q}^{k-1} e^{\alpha(q)(k-l-1)} \Gamma(l) \\ &\leq \cdots \\ &\leq \mu^q e^{\alpha(q)(k-k_q)+\sum_{i=1}^{q} \alpha(i-1)(k_i-k_{i-1})} V(0) \\ &\quad - \mu^q \sum_{l=0}^{k_1-1} e^{\alpha(q)(k-k_q)+\sum_{i=2}^{q} \alpha(i-1)(k_i-k_{i-1})+\alpha(0)(k_1-l-1)} \Gamma(l) \\ &\quad - \mu^{q-1} \sum_{l=k_1}^{k_2-1} e^{\alpha(q)(k-k_q)+\sum_{i=3}^{q} \alpha(i-1)(k_i-k_{i-1})+\alpha(1)(k_2-l-1)} \Gamma(l) \\ &\quad - \cdots - \sum_{l=k_q}^{k-1} e^{\alpha(q)(k-l-1)} \Gamma(l) \\ &= \mu^q e^{-\alpha_1 K^-(0,k)+\alpha_2 K^+(0,k)} V(0) \\ &\quad - \mu^q \sum_{l=0}^{k_1-1} e^{-\alpha_1 K^-(l+1,k)+\alpha_2 K^+(l+1,k)} \Gamma(l) \\ &\quad - \mu^{q-1} \sum_{l=k_1}^{k_2-1} e^{-\alpha_1 K^-(l+1,k)+\alpha_2 K^+(l+1,k)} \Gamma(l) - \ldots \\ &\quad - \sum_{l=k_q}^{k-1} e^{-\alpha_1 K^-(l+1,k)+\alpha_2 K^+(l+1,k)} \Gamma(l) \\ &= -\mu^{N_\sigma(l+1,k)} \sum_{l=0}^{k-1} e^{-\alpha_1 K^-(l+1,k)+\alpha_2 K^+(l+1,k)} \Gamma(l) \end{aligned}$$

In the last step, notice $V(0) = 0$ for zero initial condition.
Notice that

$$\begin{aligned} V(k) &\leq -\mu^{N_\sigma(l+1,k)} \sum_{l=0}^{k-1} e^{-\alpha_1 K^-(l+1,k)+\alpha_2 K^+(l+1,k)} \Gamma(l) \\ &= -\sum_{l=1}^{k} \mu^{N_\sigma(l,k)} e^{-\alpha_1 K^-(l,k)+\alpha_2 K^+(l,k)} \Gamma(l-1). \end{aligned} \quad (12)$$

Next, we will show

$$-\alpha_1 K^-(l,k) + \alpha_2 K^+(l,k) \leq -\alpha^*(k-l) + C, \quad (13)$$

where $C = 2(\alpha_2 + \alpha^*) \frac{\alpha_1 - \alpha^*}{\alpha_1 + \alpha_2} T$.
We see the following three cases for l and k.

Case I. $l \leq t^j < t^{j+1} < \ldots < t^p \leq k$ for $j = 1, 2 \ldots,$
$p \geq j+1$;
Case II. $t^j \leq l < k \leq t^{j+1}$, for $j = 1, 2 \ldots$;
Case III. $t^j \leq l < t^{j+1} < k \leq t^{j+2}$, for $j = 1, 2 \ldots$.

For this case, we have

$$\begin{aligned}
&-\alpha_1 K^-(l,k) + \alpha_2 K^+(l,k) \\
&= -\alpha^*(k-l) - (\alpha_1 - \alpha^*)K^-(l,k) \\
&\quad +(\alpha^* + \alpha_2)K^+(l,k). \qquad (14)
\end{aligned}$$

It holds that

$$\begin{aligned}
&-(\alpha_1 - \alpha^*)K^-(l,k) + (\alpha^* + \alpha_2)K^+(l,k) \\
&= -(\alpha_1 - \alpha^*)[K^-(l,t^j) + K^-(t^P,k)] \\
&\quad +(\alpha^* + \alpha_2)[K^+(l,t^j) + K^+(t^p,k)] \\
&\quad + \sum_{i=j}^{p-1} [-(\alpha_1 - \alpha^*)K^-(i,i+1) \\
&\quad +(\alpha^* + \alpha_2)K^+(i,i+1)] \\
&\leq (\alpha_2 + \alpha^*)[K^+(t^{j-1},t^j) + K^+(t^P,k)]. \qquad (15)
\end{aligned}$$

In last inequality, we use

$$\sum_{i=j}^{p-1} -(\alpha_1 - \alpha^*)K^-(i,i+1) + (\alpha^* + \alpha_2)K^+(i,i+1) \leq 0$$

by $Cond_1$.
Also, notice that from $Cond_1$

$$\begin{aligned}
K^+(t^j,t^{j+1}) &\leq \frac{\alpha_1 - \alpha^*}{\alpha_2 + \alpha_1}(t^{j+1} - t^j) \\
&\leq \frac{\alpha_1 - \alpha^*}{\alpha_2 + \alpha_1} T, j = 0, 1 \ldots. \qquad (16)
\end{aligned}$$

From (14) to (16), we get (13).

For Case (II) and (III), it can be shown that (13) always holds.

Multiplying $\mu^{-N_\sigma(0,l)}$ on both sides of the inequality (12) yields

$$0 \leq -\sum_{l=1}^{k} \mu^{-N_\sigma(0,l)} e^{-\alpha_1 K^-(l,k) + \alpha_2 K^+(l,k)} \Gamma(l-1). \qquad (17)$$

That is

$$\begin{aligned}
&\sum_{l=1}^{k} \mu^{-N_\sigma(0,l)} e^{-\alpha_1 K^-(l,k) + \alpha_2 K^+(l,k)} z^T(l-1)z(l-1) \\
&\leq \sum_{l=1}^{k} \mu^{-N_\sigma(0,l)} e^{-\alpha_1 K^-(l,k) + \alpha_2 K^+(l,k)} w^T(l-1)w(l-1).
\end{aligned}$$

Noting that $N_\sigma(0,l) \leq \frac{l}{\Gamma_a}$, we have from $Cond_2$

$$\mu^{-N_\sigma(0,l)} = e^{-ln\mu N_\sigma(0,l)} \geq e^{-\alpha l}.$$

From (13),

$$\begin{aligned}
&\sum_{l=1}^{k} e^{-\alpha l - \alpha_1(k-l)} z^T(l-1)z(l-1) \\
&\leq \sum_{l=1}^{k} e^{-\alpha^*(k-l)+c} w^T(l-1)w(l-1).
\end{aligned}$$

Summing k from 1 to $+\infty$, we get

$$\begin{aligned}
&\sum_{l=1}^{\infty} e^{-\alpha l} z^T(l-1)z(l-1) \sum_{k=l}^{\infty} e^{-\alpha_1(k-l)} \\
&\leq \gamma_0^2 \sum_{l=1}^{\infty} w^T(l-1)w(l-1) \sum_{k=l}^{\infty} e^{-\alpha^*(k-l)+c}.
\end{aligned}$$

Thus

$$\begin{aligned}
&\frac{1}{1 - e^{-\alpha_1}} \sum_{l=1}^{\infty} e^{-\alpha l} z^T(l-1)z(l-1) \\
&\leq \frac{e^c \gamma_0^2}{1 - e^{-\alpha^*}} \sum_{l=1}^{\infty} w^T(l-1)w(l-1),
\end{aligned}$$

that is

$$\sum_{l=0}^{\infty} e^{-\alpha l} z^T(l)z(l) \leq \frac{e^{(\alpha+c)}(1 - e^{-\alpha_1})}{1 - e^{-\alpha^*}} \gamma_0^2 \sum_{l=0}^{\infty} w^T(l)w(l).$$

The proof is completed.

Remark 2. Without considering disturbance input, it is not difficult to see that the LMIs' condition in Theorem 1 also guarantees exponential stability of the switched system (2).

Suppose that (8) hold for any $i \in M$, which means each subsystem achieves normal L_2-gain. And suppose $\mu = 1$, which means $P_i = P_j, Q_i = Q_j$ hold for any $i, j \in M$ and thus a common quadratic Lyapunov function exists for all subsystems. Then from (17), we get

$$0 \leq -\sum_{l=1}^{k} e^{-\alpha_1 K^-(l,k)} \Gamma(l-1).$$

That is

$$\begin{aligned}
&\sum_{l=1}^{k} e^{-\alpha_1(k-l)} z^T(l-1)z(l-1) \\
&\leq \sum_{l=1}^{k} e^{-\alpha_1(k-l)} w^T(l-1)w(l-1).
\end{aligned}$$

Summing k from 1 to $+\infty$, we get

$$\begin{aligned}
&\sum_{l=1}^{\infty} z^T(l-1)z(l-1) \sum_{k=l}^{\infty} e^{-\alpha_1(k-l)} \\
&\leq \gamma_0^2 \sum_{l=1}^{\infty} w^T(l-1)w(l-1) \sum_{k=l}^{\infty} e^{-\alpha_1(k-l)}.
\end{aligned}$$

Thus

$$\begin{aligned}
&\frac{1}{1 - e^{-\alpha_1}} \sum_{l=1}^{\infty} z^T(l-1)z(l-1) \\
&\leq \frac{\gamma_0^2}{1 - e^{-\alpha_1}} \sum_{l=1}^{\infty} w^T(l-1)w(l-1).
\end{aligned}$$

That is

$$\sum_{l=0}^{\infty} z^T(l)z(l) \leq \gamma_0^2 \sum_{l=0}^{\infty} w^T(l)w(l).$$

Therefore, L_2 gain is achieved for the switched system (1) under arbitrary switching. We state the fact in the following corollary.

Corollary 1. Given $\alpha_1 > 0, \gamma_0 > 0$ and $\tau > 0$, if there exist matrix $P_i = P > 0,\ Q_i = Q > 0$ such that LMIs (8) hold for $\forall i \in M$, then the switched delay system (1) achieves the L_2 gain under arbitrary switching signals.

Remark 3. When $\tau = 0$, Corollary 1 degenerates into Theorem 4 in Zhai et al. [2002]. The weighted L_2 gain for discrete switched systems is introduced in Zhai et al. [2002] and Lin et al. [2003], which reflects one of the features of discrete switched systems under the average dwell time scheme. The weighted L_2 gain approaches to the normal L_2 gain if the average dwell time is chosen sufficiently large. Here, we extend the concept of the weighted L_2 gain to discrete switched systems with delay. And we also notice from Corollary 1 that under the special case, discrete delay systems achieves normal L_2 gain under arbitrary switching.

4. CONCLUSIONS

This paper has dealt with the problem of L_2 gain for a class of discrete switched systems with constant delays. The system contains unstable subsystems. Under the help of two lemmas relating to the change estimation of Lyapovnov function candidate for usual delay systems, the conditions of weighted L_2 gain are developed. Deleting the disturbance input, the given LMIs' conditions can also guarantee the exponential stability of the considered system. The results obtained in this paper is expected to solve stability and L_2 gain problem for the networked control systems with delays and package dropout. It will be our next goal.

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