ROBUST CONTROLLER TUNING BASED ON COEFFICIENT DIAGRAM METHOD

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Abstract: In this paper, Coefficient Diagram Method (CDM), which is a controller design method that provides remarkable time-domain characteristics, is combined with a PI controller in order to design robust controllers. In particular, it has been demonstrated that it is possible to provide robust tuning rules for first order plus time delay (FOPTD) systems. Here, PI controller is used for improving the steady-state response of the system and for providing an extra parameter for tuning robustness. Pole Colouring method is used for measuring robustness. Calculation of robust tuning rules is computationally expensive, since it is required to find the best values of the free parameter of the PI controller for different plants. However, after using a curve fitting algorithm it is possible to obtain simple tuning rules to determine robust controllers.

Keywords: Coefficient Diagram Method, CDM, CDM-PI, Robust Control, Pole Placement, Pole Colouring, FOPTD systems

1. INTRODUCTION

Coefficient Diagram Method (CDM) is now a well established approach to design controllers that provide outstanding time domain characteristics in closed-loop (Manabe, 1994; Manabe, 1998; Budiyono and Sutarto, 2004; Cahyadi, et al, 2004). Basically CDM is based on pole assignment, where the locations of the closed-loop system are obtained using predetermined templates. Although it has been demonstrated that the designs based on CDM has some robustness, it is possible to show that some of the nice characteristics of the design can be lost if large perturbations in the model of the system exist. To this extent, Öcal et al (2005) have shown that a more robust design can be obtained using a first order pre-compensator with CDM. A particular disadvantage related with CDM and the use of a first order compensator with CDM is the possibility of having steady state error under model uncertainties.

Therefore a PI type pre-filter is proposed to be used with CDM in this study. Although several other results are reported in the literature for mixing PI(D) type controllers with CDM (Khuakoonratt, et al, 2003; Kumpanya, et al, 2000), it should be stressed that the approach given here is new to the best of the authors knowledge.

The method proposed utilizes pole colouring approach (Söylemez and Munro, 1997), which in essence determines the maximum distance between desired and perturbed closed-loop system poles under all possible perturbations, to assess the robustness of the resulting compensators. Even though the pole-colouring approach provides a direct measure for robustness, it usually brings a computationally complexity that is not suitable for online tuning. Nevertheless, it has been shown in this study that it is possible to do pole-colouring optimizations off-line to provide robust tuning rules. First order plus time delay (FOPTD) systems are considered as an application. In particular, a robust tuning rule for CDM-PI controllers is obtained for a class of FOPTD systems. It is believed that more general robust tuning formulas can be obtained using the ideas presented in this introductory paper.

This paper is organized as follows: the coefficient diagram method is briefly introduced in Section 2. Section 3 presents a robust CDM-PI design method that utilizes the pole colouring approach. A robust tuning method for FOPTD systems is given in Section 4. Finally, conclusions and suggestions for future research are given in the last section.

2. COEFFICIENT DIAGRAM METHOD

2.1 Theory

The standard block diagram of the CDM for SISO systems is shown in Figure 1. Here, R(s), Y(s), U(s), Q(s), E(s) and M(s) represent reference input, system output, control signal, disturbance signal, error, and measurement disturbance, respectively.

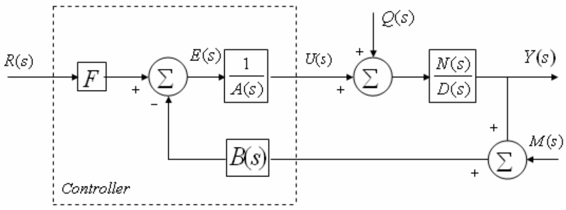

Fig. 1. The standard block diagram of CDM.

In Figure 1, A(s), B(s), and F are controller polynomials. Finally, G(s) is the system to be controlled.

$$G(s) = \frac{N(s)}{D(s)} \tag{1}$$

It is worth to mention that although the block shown by B(s) is improper, it is always possible to obtain an equivalent block diagram as shown in Figure 2 where all blocks are proper.

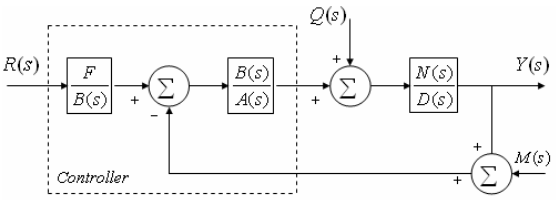

Fig. 2. Equivalent block diagram of CDM

In the following it is assumed that G(s) is a strictly proper system (that is $\deg\{D(s)\} \geq \deg\{N(s)\}$). It is possible to show that

$$Y(s) = \frac{A(s)N(s)}{P(s)}Q(s) + \frac{F.N(s)}{P(s)}R(s) - \frac{B(s)N(s)}{P(s)}M(s)$$

and

$$U(s) = \frac{F.D(s)}{P(s)}R(s) - \frac{B(s)N(s)}{P(s)}Q(s) - \frac{B(s)D(s)}{P(s)}M(s)$$

where P(s) is the characteristic polynomial of the closed-loop system given by

$$P(s) = \mathrm{A(s)D(s)} + \mathrm{B(s)N(s)} = \sum_{i=0}^{n} a_i s^i$$

2.2 Performance Parameters and the Target Characteristic Polynomial

CDM needs some design parameters with respect to the characteristic polynomial coefficients which are τ the equivalent time constant, γ_i the stability indices, and γ_i^* the stability limits. The relations between these parameters and the coefficients of the characteristic polynomial (a_i) are shown in (2).

$$\gamma_i = \frac{a_i^2}{a_{i+1}a_{i-1}} \qquad i \in [1, n-1] \qquad \gamma_0 = \gamma_n = \infty \tag{2a}$$

$$\tau = \frac{a_1}{a_0} \tag{2b}$$

$$\gamma_i^* = \frac{1}{\gamma_{i-1}} + \frac{1}{\gamma_{i+1}} \quad i \in [1, n-1] \tag{2c}$$

Using the relations in equation (2), it is possible to formulate the characteristic polynomial P(s) in terms of the design parameters τ and γ_i as follows:

$$P_T(s) = a_0 \left[\left\{ \sum_{i=2}^{n} \left(\prod_{j=1}^{i-1} \frac{1}{\gamma_{i-j}^{j}} \right) (\tau s)^i \right\} + \tau s + 1 \right] \tag{3}$$

This polynomial ($P_T(s)$) is called as the target characteristic polynomial.

It is worth to remark that there is a relation between equivalent time constant τ and the settling time T_s as $\tau = T_s / \alpha$, where $\alpha \in [2.5, 3]$.

2.3 The Stability Indices and Stability

In addition to Routh-Hurwitz criterion, CDM inserts the Lipatov – Sokolov criterion (Manabe, 1994; Lipatov and Sokolov, 1972; Lipatov, 1976).

Manabe proposed a standard form for choosing the stability indices for the CDM (Manabe, 1994; Manabe and Kim, 2000).

$$\gamma_{n-1} \sim \gamma_2 = 2\,, \qquad \gamma_1 = 2.5 \tag{8}$$

2.4 Design Procedure

Equating the closed-loop system characteristic polynomial found in section 2.1 to the target characteristic polynomial given in section 2.2, the following Diaphontine equation is obtained

$$\begin{aligned} P(s) &= \mathrm{A(s)D(s)} + \mathrm{B(s)N(s)} = \sum_{i=0}^{n} a_i s^i \\ &= a_0 \left[\left\{ \sum_{i=2}^{n} \left(\prod_{j=1}^{i-1} \frac{1}{\gamma_{i-j}^{j}} \right) (\tau s)^i \right\} + \tau s + 1 \right] \end{aligned} \tag{9}$$

where

$$A(s) = \sum_{i=0}^{p} l_i s^i \text{ and } B(s) = \sum_{i=0}^{q} k_i s^i$$

Then, equation (9) is converted to Sylvester Form as shown below

$$[C]_{rxr}\begin{bmatrix} l_i \\ k_i \end{bmatrix}_{rx1} = [a_i]_{rx1} \tag{10}$$

Here, matrix C and values of a_i are known. Therefore the coefficients of the controller polynomials A(s) and B(s) can be calculated easily.

Finally, F can be obtained from

$$F = \left(\frac{P(s)}{N(s)}\right)\Bigg|_{s=0} \tag{11}$$

While choosing the values of stability indices γ_i, the standard Manabe Form mentioned in Section 2.3 can be used. Note that there also exist other ways mentioned in the literature to determine these indices (Öcal, 2004).

3. ROBUST POLE ASSIGNMENT

As can be observed from above development, the Coefficient Diagram Method (CDM) is essentially a pole assignment method where suitable locations of the closed-loop system are automatically determined through equation (3) by selecting just a few design parameters. When the system defined by equation (1) involves parametric uncertainties, the closed-loop system poles perturb from the desired pole locations, which may lead to a performance degradation or even instability (Söylemez and Munro, 1995; Söylemez and Munro, 1999).
In order to minimize perturbations from the desired pole locations, a PI controller as defined by equation (12) is proposed to be used (see Figure 3).

$$K(s) = k_p \frac{s+a}{s} \tag{12}$$

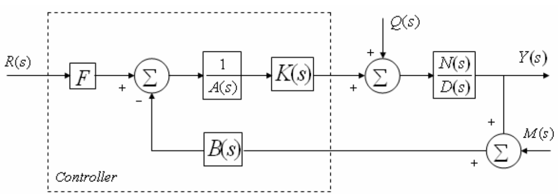

Fig. 3. CDM with PI controller K(s).

It should be noted that the proportional term of the PI controller can always be prefixed (say $k_p = 1$), since the gain of the controller is determined by CDM. The PI controller introduces a new design parameter, namely a, which will be used to restrict the perturbations from the desired pole locations. The main idea of the design is then to find suitable values of a such that the standard Coefficient Diagram Method (applied to the system with the PI controller) results in a system insensitive to large parameter variations.

Let us assume that the numerator and denominator polynomials of the system G(s) has parametric uncertainties such that

$$G(s) = \frac{N(s,q)}{D(s,q)} \tag{13}$$

where **q** represents the uncertainty vector

$$q^T = [q_1 \quad q_2 \quad \cdots \quad q_i] \tag{14}$$

in which, q_i are interval parameters $q_i \in [q_i^-, q_i^+]$. Then the characteristic polynomial of the closed-loop system can be expressed as

$$p(s,\mathbf{q},a) = \sum c_i(\mathbf{q},a)s^i \tag{15}$$

A robustness measure (i.e. a cost function J_k) is used to determine how robust the result of pole-assignment procedure defined in the previous section. This cost function is minimized over the design parameter a to achieve robust pole assignment.

The cost function J_k can be defined as follows

$$J_k = \max_q(J_q) \tag{16}$$

where J_q is the maximum distance between the nominal poles and the resulting poles for a given q. It should be remarked that usually using a fixed number of points of the parameter box ($Q \triangleq \{\mathrm{q} \mid q_i \in (q_i^-, q_i^+)\}$) is enough in the optimization problem defined by equation (16). Actually, it is possible to use the following algorithm in design:

Algorithm:

1. Define an initial test set that consists of only one of the vertices of the perturbation ball (e.g. $T = \{\mathbf{q}_{\min}\}$, where $\mathbf{q}_{\min}$ is obtained by setting all uncertain parameters to their minimum values).
2. Minimize the cost function J_k for possible values of a. In calculating J_k, use only the uncertainty vectors in the test set T.
3. Analyze the solution found in step 2, this time considering all possible perturbations, and find the perturbation $\mathbf{q} = \mathbf{q}^*$, where the perturbation from desired poles is maximum.
4. If $\mathbf{q}^* \in T$ then a solution to the design problem has been found. Otherwise add $\mathbf{q}^*$ to T and go back to step 2.

Note that finding J_q in equation 16 is not a trivial problem. To find J_q, a pole colouring technique, explained in the next section, can be used.

3.1 *Pole Colouring*

Consider the simple case of a third-order system where the nominal poles and perturbed poles for a fixed q is given in Figure 4. Here, assume that big points represent perturbed poles and small points represent nominal poles. The process of determining which of the nominal poles corresponds to which of the perturbed poles is called "pole colouring" (Soylemez and Munro, 1997).

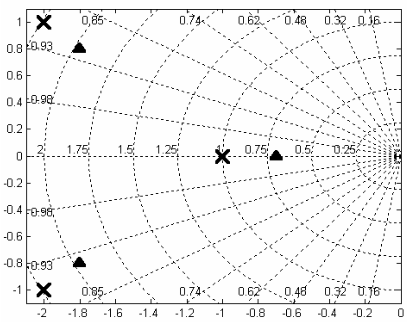

Fig. 4. The nominal poles and perturbed poles. X and Δ points represent the perturbed poles and the nominal poles, respectively

Note that even for this trivial case there are 6 possible permutations (different possible colouring). If for each perturbation (p) the maximum distance between any nominal-perturbed pole pairs is shown by J_{qp}, it is then possible to show that the cost function J_q can be given as

$$J_q = \min_p (J_{qp}) \tag{17}$$

In general, there are n! permutations, where n is the number of poles. Fortunately, it is not required to try all possible permutations to find J_q as explained in (Söylemez and Munro, 1997), and the problem is equivalent to a linear bottleneck assignment problem (Burkard, and Derigs, 1980).

4. A TUNING METHOD FOR CONTROLLING FOPTD SYSTEMS

Let us consider a control system as shown in Figure 5. Assume that a FOPTD system is given as follows:

$$G(s) = \frac{N(s)}{D(s)} = \frac{K}{Ts+1} e^{-Ls} \tag{18}$$

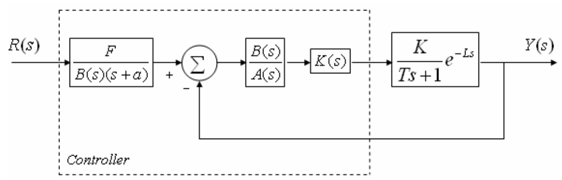

Fig. 5. The block diagram of the control system controlling FOPTD systems.

For small values of L, a first order Padé approximation, ($e^{-Ls} \cong \frac{2-Ls}{2+Ls}$), can be safely used to obtain an approximate system as follows:

$$G(s) = \frac{K(2-Ls)}{(2+Ls)(1+sT)} \tag{19}$$

Considering the fact that the delay time (L) can be measured quickly and accurately for many practical systems, it is assumed that L is certain and does not change over time. Nevertheless, it is also assumed that the gain (K) and the time constant (T) of the system contains 5% uncertainty. That is, it is possible to write $0.95K < K_a < 1.05K$ and $0.95T < T_a < 1.05T$, where K_a and T_a are the actual values of the gain and time constant, respectively. We remark that in determining a tuning rule for FOPTD systems, it is possible to take $K = 1$ without losing generality. For systems with gain $K \neq 1$, the proportional term of the PI controller is taken as $k_p = 1/K$ and calculations for CDM is done accordingly.

For the examples considered in this paper, a settling time between 4 and 5 seconds is aimed. Hence, the equivalent time constant is chosen as $\tau = 2$. Firstly using standard CDM it is possible to show that the controller parameters can be found as follows:

$$A(s) = l_0 + sl_1, \qquad B(s) = s + k_0 \tag{20}$$

where

$$k_0 = \frac{25TL^3 + 100(T-1)TL^2 - 32(5T-4)L + 128T}{25T^2L^3 + 4(4-5T)^2L^2 - 32T(5T-4)L + 128T^2}$$

$$l_0 = \frac{25TL^3 + 100TL^2 + 32(5T-4)L - 128T}{25T^2L^3 + 4(4-5T)^2L^2 - 32T(5T-4)L + 128T^2}$$

$$l_1 = \frac{64L(L+2T)}{25T^2L^3 + 4(4-5T)^2L^2 - 32T(5T-4)L + 128T^2}$$

Note that such calculations can be carried out using symbolic algebra (Söylemez and Üstoğlu, 2006) and furthermore $Q(s) = 0$ and $M(s) = 0$ are assumed. For T = 2 and L = 0.5, the pole spread and the unit step response of the closed-loop system are given in Figure 6 and Figure 7, respectively.

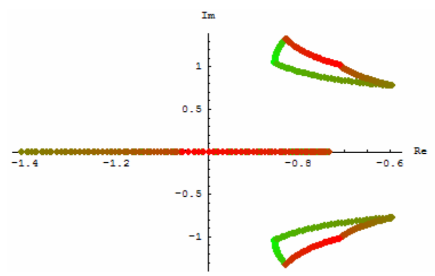

Fig. 6. Pole spread of the closed-loop system using standard CDM (T=2, L=0.5)

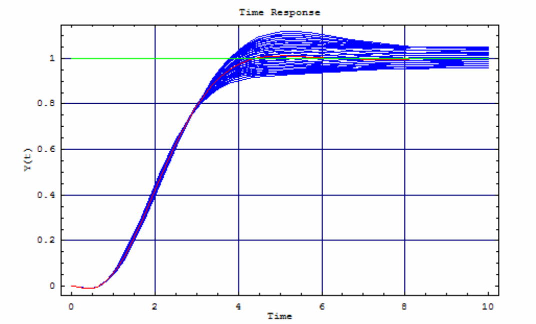

Fig. 7. Unit step response of the closed loop system using standard CDM (T=2, L=0.5)

As seen in Figure 7, although the system that is controlled with CDM is stable, it is observed that a steady state error occurs due to uncertain parameters. In order to increase the robustness of the system and improve steady state response a CDM-PI controller as described in the previous section is designed. The pole spread of the closed loop system and the unit step response are given in Figure 8 and Figure 9, respectively, after selecting the free parameter a = 1. As seen from these figures, the suggested design gives a much more robust solution especially in terms of the performance. We also remark that the control signal is also within acceptable ranges (between 0 and 1.6) for this example.

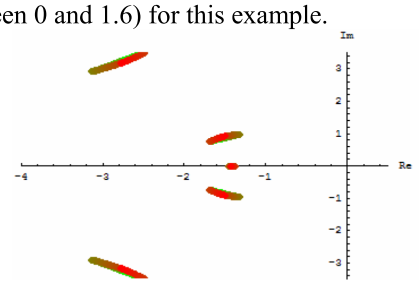

Fig. 8. CDM-PI pole spread (T=2, L=0.5, a=1)

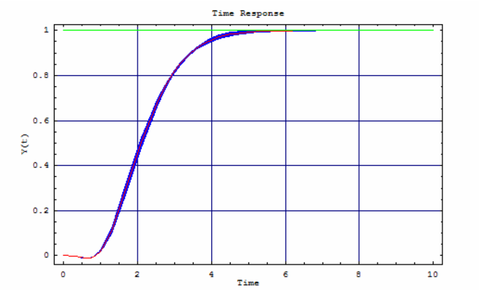

Fig. 9. for CDM-PI controller, the unit response spread of the closed loop system (T=2, L=0.5, a=1)

Here, an important question is that how to select a suitable value for the free parameter a, such that the closed-loop system is robust. It is obvious that if a changes, the robustness of the system also changes. For example, if the value of a is taken as 0.25, the pole spread of the closed loop system will pass through to the right half plane (Figure 10), and hence, the system will not be robustly stable.

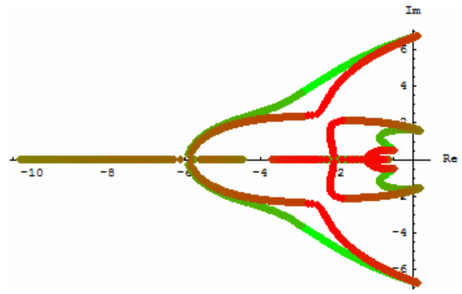

Fig. 10. CDM-PI pole spread (T=2, L=0.5, a=0.25)

The change of the robustness (cost) function (J), which is described in Section 3, with respect to the value of a is shown in Figure 11. It is possible to observe from this figure that a can be chosen as a=0.9 or a=2.5, which provide near optimal solutions. However, it is possible to discuss that choosing a=2.5 is safer due to a steep hill near a=0.9.

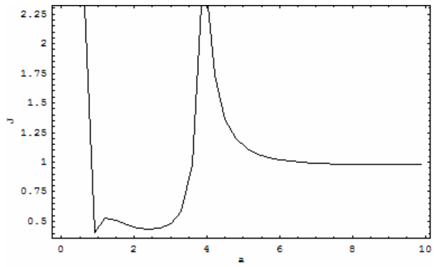

Fig. 11. For 0.4< a <10 the change of the pole colouring cost function (T=2, L=0.5)

Above calculations have been repeated and suitable values of a have been calculated by changing the value of T in region 0.1< T < 20. The change of optimal values of a with respect to change of T are given in Figure 12.

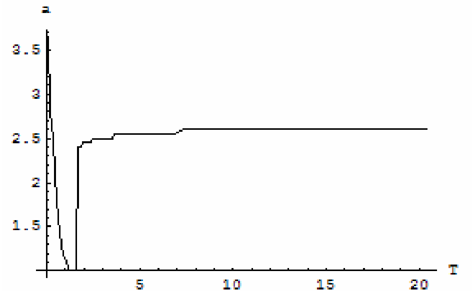

Fig. 12. The values of a parameters which make the system most robust for 0.1< T <20. (L=0.5)

It is possible to employ a curve fitting algorithm to fit a polynomial for the curve obtained in Figure 13. Since there is a sharp change around T=1.7 in the figure a piecewise-defined function is used in the curve fitting as follows:

$$a(T)=\begin{cases}1.822T^2-4.556T+3.77 & T<1.7\\ -0.001T^2+0.029T+2.414 & T\geq 1.7\end{cases} \quad (21)$$

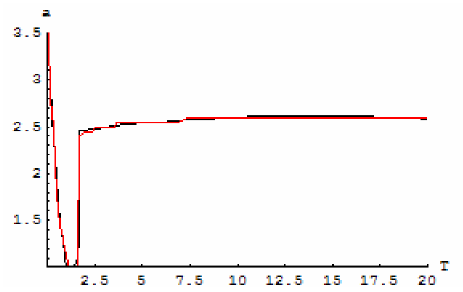

Fig. 13. Change of a (black) for 0.1< T <20 and its approximation (red) a(T) as described in (21). (L=0.5)

As can be observed from Figure 13, the approximation provided by (21) is very satisfactory. It is worth to remark that finding optimal values of the free parameter a using pole colouring is computationally expensive. It takes around 5 hours to obtain Figure 12 using a personal computer with Intel Pentium II (1.7 GHz) processor. Whereas, the expression provided by (21) can be calculated in a mater of milliseconds. Therefore, the following algorithm is proposed to design a robust CDM-PI controller for FOPTD systems with the transfer function

$$G(s) = \frac{Ke^{-0.5s}}{Ts+1} \tag{22}$$

Algorithm:

1. Choose K(s) as $K(s) = \frac{s+a}{Ks}$
2. Find the value of a using equation (21).
3. Use CDM to find A(s) and B(s). (Note that F=1, since a PI controller is used in the loop).

4. CONCLUSION

In this paper, a method called CDM-PI is proposed by combining Coefficient Diagram Method (CDM) with a PI controller. It has been demonstrated that the proposed method can be used to increase robustness of the closed-loop system.

As an application, first order systems are considered. It has been shown in particular that it is possible to easily find robust controllers using a rather simple computation for systems with delay time $L=0.5$ and time constant $0.1<T<20$. It is possible to obtain similar results for different values of L. Continuing research is carried out to find more general tuning formulas that would cover a wider range of FOPTD systems (different values of T and τ), and different types and sizes of uncertainty. Furthermore, employment of this method to real-time applications is also considered as a future work.

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