An Arnoldi Based Method to Discrete Time Linear Optimal Multi-periodic Repetitive Control

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Abstract: For LTI plant, a benchmark tracking solution was recently proposed by optimal technique for discrete time linear multi-periodic repetitive control system that gives asymptotic perfect tracking, where the original tracking problem was transfered into a regulator problem by developing a new state-space representation that combines the plant and demand signal. However, in practice, the periods of the demand signals are usually very large, therefore the dimension of this new plant description increases and that naturally leads to high order computations for solving discrete Riccati equation. In order to overcome this problem, an Arnoldi based method is applied in this paper to first reduce the high order state-space representation to a low order counterpart, then a direct method is used to solve its corresponding low order riccati equation. Finally a projection between two riccati solutions is applied to retrieve an approximate high order riccati solution. This result also leads to a new multi-periodic repetitive controller in terms of low order riccati solution. A numerical example is given and asymptotic perfect tracking is guaranteed.

Keywords: multi-periodic, high order computation, Riccati, Arnoldi, projection, approximate

1. INTRODUCTION

Many signals in engineering are periodic, or at least they can be approximated by a periodic signal over a large time interval. This is true, in many tracking problems, the desired output or disturbance input often includes a periodic component with known period. If only applying standard feedback controller design method such as *PID,LQR* or H^∞ algorithms, the result will always have some amount of transient and steady state errors. In order to track or reject such periodic signals, internal model principle (*IMP*) [1] instead allows us to tailor the design by including a model of the reference/disturbance signals in the controller and design the system for closed loop stability. In other words, perfect tracking of any reference input or disturbance rejection in the steady state can be accomplished if a generator of that signal is included in the stable closed-loop system. Such a control method is called repetitive control or RC and the resultant system is called repetitive control system that has been widely used in disc file servo system [2], robot manipulator [3], position control system [4], steel casting process ([5], [6]) magnet power supply [7] and many other areas ([8], [9]).

It is widely acknowledged that the first paper about RC was written by Inoue *et al.* ([10], [11]). In their paper, a RC scheme using a positive feedback around a time delay was proposed to generate an internal model of the periodic disturbance, and a proton synchrotron magnet power supply was well controlled in the end to give a 10^{-4} tracking accuracy.

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Both continuous and discrete RC problem were studied in the last few decades. In continuous time, different design and synthesis methods were proposed for the prototype and modified RC systems with different configurations. Two of these are called standard ([12], [13]) and plug-in ([14], [15]) RC structures. In these works, earlier works such as ([10], [11], [14]) are based in the frequency domain, while later works like [13] and [16] are based on lyapunov analysis for Multi Periodic RC problems. Meanwhile, synthesis procedures such as state-space approach and H^∞ optimal design approach are also widely used. Among these works, Hikita *et. al* [17] proposed a repetitive control system of a model following type where the dynamic system was driven by a periodic disturbance. Owens *et al.* [13] proposed a lyapunov stability analysis for MIMO systems under a positive real condition and exponential stability under a strict positive real condition.

In discrete time, ([2], [18], [12], [19]) and many other papers have contributed to this area. In [2], Tomizuka *et al.* proved the asymptotic convergence properties for a class of discrete-time RC systems. In their paper, a feedforward compensation cascaded with the non-minimum plant was used by applying pole-zero cancellation and/or zero-phase compensation principle and derived the stability conditions according to small gain theorem. More recently, Owens *et al.* [20] have given a benchmark tracking solution for discrete time Linear Optimal Multi-periodic Repetitive Control (MPRC) problem, where a new state-space representation was derived to transform the original tracking problem to a familiar regulator problem. In this paper, our concentration is on discrete time MPRC system.

Meanwhile, from the number of periodic signals contained in the demand signal, studying on Repetitive Control can also be classified into single and multi-periodic RC problem, both of which were carried out by many researchers in the past. In single-period case, the demand signal has only one periodic component, *IMP* suggests that the controller has to contain poles at the frequency of the periodic signal and all its non-zero harmonics (possibly infinitely many), earlier works such as ([18], [21], [2], [6] [22], [19]) are all studyings about RC systems under a single-period reference signal. Among these works, [2] introduced a "plug in" prototype repetitive controller in discrete time and applied it to track following in a disk-file actuator system.

In many practical situations, however, the reference and/or disturbance signals often consist of a superposition of signals of different fundamental periods. In multi-periodic case, past and recent applications of RC systems under a multi-periodic reference signal include ([23], [4], [13], [16]) and most recently in [20], where in [20] the original tracking problem was transfered into a linear quadratic regulator problem that proposed an explicit formula for a global stabilizing tracking controller. In continuous time implementation, the resulting overall signal may not be periodic as the ratio of these frequencies may be irrational. In discrete time application, all the periods are approximated as integers of sampling time, so the ratio of these period is rational. Therefore, for any desired outputs and disturbance inputs consisting of periodic signals $r_i[k], i = 1 \cdots M$ with periods $N_i, \quad i = 1 \cdots n$. The generator $1/(1 - z^{-N_i})$ of the periodic signal with single period is given by

$$N = LCM\{N_i, i = 1 \cdots M\} \in Z_+ \tag{1}$$

where Z_+ denotes the set of positive integers including 0. Furthermore, it could be a sum of periods of periodic components such as explained in [20] with

$$N = \sum_{i=1}^{i=m} N_i \tag{2}$$

In this case, the dead-time length of the generator in the RC controller becomes big, and it becomes very large in multi period RC problems. This is not desirable from the viewpoint of implementation, since the size of memory and computation cost will be increased a lot. In order to overcome this problem, earlier works by ([4], [6], [22]) proposed some prototype RC controllers which require to solve no equation and the design methods are independent of the period and can work on tracking reference with any number of periods. Later, Gunnar *et.al* [24] proposed a lower order model by introducing a lowpass Q-filter in the feedback loop to achieve asymptotic zero error property. However, all their works are for single period repetitive control systems. For multi-periodic case which is more usual in reality, in section 3, we will introduce a new projection method that needs very little memory and computation cost and at the same time gives asymptotic perfect tracking.

This paper is organized as follows: Section 2 defines the problem resulted from an earlier work and motivates the subsequent work carried out in this paper. In section 3, an Arnoldi algorithm is applied to reduce the dimension of high order state space representation into its low order counterpart, then a direct method such as *dare.com* in Matlab© is applied to solve the low order riccati solution. Thereafter, a projection relation is applied to retrieve an approximate high order riccati solution from its low order counterpart and the constant feedback gain is also obtained. In section 4, the controller is given in an explicit form that has a similar structure as the previous result, where "delayed" feedback of inputs and states and a moving window of weighted error data are present. In Section 5, the controller is applied to a numerical example and Section 6 contains conclusions and directions for future research.

2. PROBLEM DEFINITION

Recently, Owens,*et al.* [20] proposed a benchmark tracking solution for discrete time linear optimal Multi-Periodic Repetitive Control that gives asymptotic perfect tracking. In that paper, the reference signal had a finite sum of periodic signals of different frequencies. i.e. multi-periodic. Due to the periodic nature of reference signal, the reference was "plugged in" into system representation by an"annihilating polynomial" that act as an internal model. Under this transformation, the original tracking problem was transformed into a regulation problem by constructing a new augmented state-space representation, mapping a modified input into the tracking error.

We also start with a linear time-invariant discrete SISO system having the following form:

$$x(k+1) = A\ x(k) + B\ u(k), \quad x(0) = x_0 \tag{3}$$
$$y(k) = C\ x(k) \tag{4}$$

where the state $A \in \Re^{n\times n}$, $B \in \Re^{n\times 1}$, $C \in \Re^{1\times n}$, $x(\cdot) \in \Re^n$,output $y(\cdot) \in \Re$ and $u(\cdot) \in \Re$. The reference signal $r(\cdot) \in \Re$ is taken to be a multi-periodic signal with a linear combination of periodic signals with different fundamental frequencies $f_j = \frac{2\pi}{N_j h}$ representing high order periods of N_j times the 'sampling period' h, or it can be expressed as a time series,

$$r(k) = \sum_{j=1}^{M} r_j(t), \quad r_j(k) = r_j(k - N_j) \tag{5}$$
$$1 \le j \le M, k \ge 0$$

the relation between the total period and individual period in each periodic component satisfies (2). Meanwhile, Owens *et. al* [20] also proposed the internal model for a 'typical' discrete time MPRC system in forms of Fig. 1.

Such an "internal model" is incorporated into the original system which bears it's z-transfer function as:

$$\frac{U(z)}{E(z)} = \sum_{i=1}^{M} \frac{\alpha_i}{1 - W(z)z^{-N_i}} \tag{6}$$

where $W(z)$ is a low-pass filter being an exact internal mode by choosing it as 1 or an approximate internal model otherwise. α_j is the coefficient of the "annihilating polynomial" which will be defined next.

By satisfying the following two requirements:

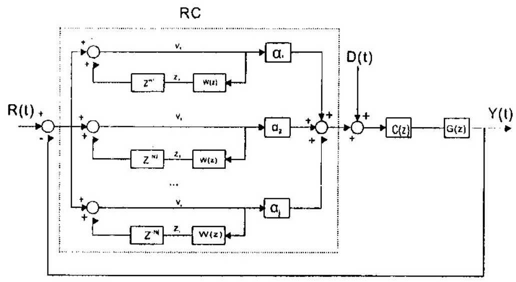

Fig. 1. Internal Model for Discrete MPRC

(1) If the linear forward shift operateor Δ is defined by the operation on the sequence $\{f(k)\}_{-\infty<k<\infty}$ defined by $\Delta f(k) = f(k+1)$ with inverse (backward shift operator) $\Delta^{-1}f(k) = f(k-1)$. Under these conditions, the multi-periodic signals satisfy:
   - $P(\Delta^{-1})r(k) = 0, -\infty < k < \infty$.

(2) the "annihilating polynomial" $P(z^{-1})$ (where z is the complex variable) is defined by:
   - $P(z^{-1}) = \Pi_{j=1}^{M}(1 - z^{-N_j}) = 1 + \sum_{j=1}^{N} \alpha_j z^{-j}$, where $N = \Sigma_{j=1}^{M} N_j$

An augmented state-space representation was derived as follows:

$$\underbrace{\begin{pmatrix} \tilde{x}(k+1) \\ \Psi(k+1) \end{pmatrix}}_{Z(k+1)} = \underbrace{\begin{pmatrix} A & O \\ -F_2C & F_1 \end{pmatrix}}_{\Pi} \underbrace{\begin{pmatrix} \tilde{x}(k) \\ \Psi(k) \end{pmatrix}}_{Z(k)} + \underbrace{\begin{pmatrix} B \\ O \end{pmatrix}}_{\Gamma} \tilde{u}(k)$$

$$e(t) = \underbrace{\begin{pmatrix} -C & F_3 \end{pmatrix}}_{\Omega} \underbrace{\begin{pmatrix} \tilde{x}(k) \\ \Psi(k) \end{pmatrix}}_{Z(k)} \tag{7}$$

or simply as:

$$\begin{aligned} Z(k+1) &= \Pi Z(k) + \Gamma \tilde{u}(t) \\ e(k) &= \Omega Z(k) \end{aligned} \tag{8}$$

where $\Pi \in \Re^{(n+N)\times(n+N)}$, $\Gamma \in \Re^{(n+N)\times 1}$, $\Omega \in \Re^{1\times(n+N)}$, $Z(\cdot) \in \Re^{n+N}$,output $e(\cdot) \in \Re$ and $\tilde{u}(\cdot) \in \Re$.

if we define the parameter α by

$$\alpha = (\alpha_N, \alpha_{N-1}, \ldots, \alpha_2, \alpha_1)^T$$

and F_1,F_2,F_3 in (7) and (8) are identified by:

$$F_2 = (\, 0 \; 0 \; \cdots \; 0 \; 1 \,)^T; \quad F_3 = -\alpha^T;$$

$$F_1 = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & \cdots & 0 & 1 \\ -\alpha_N & \cdots & \cdots & -\alpha_2 & -\alpha_1 \end{pmatrix} = F_0 - F_2\alpha^T$$

$$F_0 = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & \cdots & 0 & 1 \\ 0 & \cdots & \cdots & \cdots & 0 \end{pmatrix}$$

and

$$\Psi(k) = (e(k-N), e(k-N+1), \cdots, e(k-1))^T$$

$\Psi(k)$ is built from the past values of tracking errors and $\tilde{x}(k)$ is constructed through differencing operation on the original plant. i.e. If we denote:

$$\tilde{v}(k) = (P(\Delta^{-1})v)(k) = v(k) + \sum_{j=1}^{N} \alpha_j v(k-j) \tag{9}$$

then

$$\tilde{x}(k) = P(\Delta^{-1})x(k)$$

which states the tracking errors as well as state vectors are both needed for $-N \le k \le -1$, they are arbitrary as far as the control input $u(k)$, $-N \le k \le -1$ is arbitrary.

The state-space representation in (7) has the same structure as that in [20] except for the dimension difference, so the stabilizing control design algorithm for (7) can also be obtained by solving a standard linear quadratic optimal control problem on an infinite time internal $[0, \infty)$. Under controllability and observability condition of (7), the stabilizing feedback solution can be obtained by solving discrete Riccati equation where the solution of which is denoted by X. Given weighting matrices Q,R, the optimal control input $\tilde{u}(k)$ bears exactly similar form as [20]:

$$\tilde{u}(k) = -(\Gamma^T X \Gamma + R)^{-1} \cdot \Gamma^T \cdot X \cdot \Pi \cdot Z(k) \tag{10}$$

However, given SISO system order n, as the individual periods N_i, $i = 1, 2, \ldots$ in the reference $r(k)$ are large, the dimension of (7) will be $n + N$ (where N satisfies (2)). in which case the matrices in the system becomes large and sparse. Take an example, if we have a second order discrete system and are going to track a demand signal formed by a linear combination of two periodic signals with periods 317 and 509 respectively, i.e. $n = 2, N_1 = 317$ and $N_2 = 509$. From (7), the dimension for the new state state-space representation is $n + N_1 + N_2$, which is 828×828. In order find the optimal input $\tilde{u}(k)$, it will inevitably require very large computation cost and memory for solving discrete ricatti equation to obtain X. This is however not desired. In order to reduce memory and computation cost, section 3 will introduce an arnoldi model reduction algorithm to first reduce the representation (8) to its low order counterpart. Under controllability and observability satisfied by this low order description, the low order riccati solution is to be obtained by any direct method such as *dare.com* in Matlab©. Next, from a projection relation satisfied by two riccati solutions, an approximate high order riccati solution can therefore be deduced, and then it comes with the stabilizing feedback gain.

3. AN ARNOLDI BASED MODEL REDUCTION METHOD FOR DISCRETE MPRC

In this section, an alternative method will be proposed to reduce the high computation cost caused by high period application in Section 2. As it is stated before, our purpose is how to find the high order riccati solution X in (10) with low cost. Observe (8), it can be seen that for solving riccati equation only, the solution of (8) is independent of the control input $\tilde{u}(k)$, therefore, we only need to solve the riccati solution to (8) by a method as long as it works for large sparse system.[2] . Next, we apply an Arnoldi based

[2] Note the dimension of matrices Π, Γ and Ω are very large

model reduction method to reduce the dimension of (8) into a low order dimension system where only the choice of m (m is the order of reduced system) is needed. As we have stated before, our concentration in this paper is only on SISO system. Based on Ruhe's version [25], the algorithm can be stated as follows:

Algorithm 1: Arnoldi Model Reduction Method For SISO Discrete MPRC System

(1) Compute $\Omega^T = W_1 \cdot U_1$; QR decomposition
(2) Set $V_1 = W_1(:,1)$ and $v_1 = V_1$
Define an arbitrary value $m, m \geq 1$, which is the dimension for the reduced model we expect.
(3) For $k = 1, \dots m$
$\omega = \Pi^T v_k$,
for $i = 1, \dots, k$,
$h_{i,k} = trace(\omega^T v_i)$,
$\omega = \omega - h_{i,k} v_i$,
end
$h_{k+1,k} = \|\omega\|_2$,
if $h_{k+1,k} \neq 0$, then $v_{k+1} = \Omega / h_{k+1,1}$,
End.
(4) $\Upsilon_m = [v_1, v_2, \dots, v_m]$
(5) $H_m = \Upsilon_m \Pi \Upsilon_m^T, B_m = \Upsilon_m^T \Gamma \quad and \quad C_m = \Omega \cdot \Upsilon_m$

Due to space limitation, more details of the background knowledge of the algorithm will be omitted here. The main contribution of this algorithm is that a matrix $\Upsilon_m = [v_1, v_2, \dots, v_m]$ is obtained which forms an orthonormal basis of the krylov subspace $K_m = span\{V_1, \Pi^T V_1, \dots, \Pi^{m-1^T} V_1\}$.

Therefore, a low order state-space representation can be formed as below:

$$\begin{aligned} x_m(k+1) &= H_m x_m(k) + B_m \tilde{u}_m(t) \\ e_m(k) &= C_m x_m(k) \end{aligned} \tag{11}$$

where $H_m \in \Re^{(m)\times(m)}$, $B_m \in \Re^{(m)\times 1}$, $C_m \in \Re^{1\times(m)}$, $x_m(\cdot) \in \Re^m$, output $e(\cdot) \in \Re$ and $\tilde{u}(\cdot) \in \Re$.

Remark: As stated in [26], increase the value of m, the dimension of (11) is also increasing. When $m = n + N$, representation (8) and (11) equals.

For space limitation, it is taken for granted that (H_m, B_m) is stabilizable and (C_m, H_m) is detectable. These conditions ensure state-space representation of (11) has a unique symmetric positive semidefinite solution Y_m. If these conditions are not satisfied, the implicit restarted strategy explained in ([27], [28]) can be used to remove the unstable eigenvalues to obtain a stabilizable and detectable low order model.

So far, we have found an approximation to the solution of high order riccati equation. Only the low order riccati solution remains to be solved. Generally speaking, since the dimension of (11) is relatively very low now. There are many direct methods such as ([29], [30]) to find the answer. However, for simplicity reason, we use function *dare.m* in Matlab © instead.

Next, as it was suggested in ([31], [32]), the approximate high order riccati solution X and low order solution Y_m has the following projection relation:

$$X = \Upsilon_m Y_m \Upsilon_m^T, \quad m \geq 1 \tag{12}$$

By now, the riccati solution imposed by high order representation of (8) is fully obtained.

4. OPTIMAL REPETITIVE CONTROLLER

Substitute (12) into (10), the discrete multi-periodic repetitive controller for tracking reference with high order periods is formed as:

$$\begin{aligned} \tilde{u}(k) = -(\Gamma^T \Upsilon_m Y_m \Upsilon_m^T \Gamma + R)^{-1} \Gamma^T \times \\ \Upsilon_m Y_m \Upsilon_m^T \cdot \Pi \cdot Z(k) \end{aligned} \tag{13}$$

Denote:

$$S = (\Gamma^T \Upsilon_m Y_m \Upsilon_m^T \Gamma + R)^{-1} \Gamma^T \Upsilon_m Y_m \Upsilon_m^T \cdot \Pi = (S_1 \quad S_2)$$

which is the optimal gain. Since: $Z(k) = \begin{pmatrix} \tilde{x}(k) \\ \Psi(k) \end{pmatrix}$, using definitions of $\tilde{u}(t)$, $\tilde{x}(t)$ in forms of (9) and also that of $\Psi(k)$, the optimal multi periodic repetitive controller can be written in a more explicit way as follows:

$$\begin{aligned} u(k) = -\Sigma_{j=1}^N \alpha_j u(k-j) - S_1 x(k) \\ -S_1 \Sigma_{j=1}^N \alpha_j x(k-j) - S_2 \begin{pmatrix} e(k-N) \\ e(k-N+1) \\ \vdots \\ e(k-1) \end{pmatrix} \end{aligned} \tag{14}$$

This feedback control law is very similar to the one in [20] and has all the same structures as that in the benchmark solution [20], such as the "delayed" feedback of inputs and states; A moving window of weighted error data; An internal model within the control structure. The big advantage of (14) is that the repetitive controller here works for tracking multi-periodic reference signals with large periodic components while the one in the previous work cannot. For the same reason, if we take the Z-Transform of the optimal multi-periodic controller, which has a form:

$$u(z) = -S_1 \cdot x(z) + S(z) e(z) \tag{15}$$

where the forward path transfer function is expressed as:

$$S(z) = \frac{1}{P(z^{-1})} (-1) S_2 \begin{pmatrix} z^{-N} \\ z^{-N+1} \\ \vdots \\ z^{-1} \end{pmatrix} \tag{16}$$

therefore, the internal model of the multi-periodic reference signal is just represented by $\frac{1}{P(z^{-1})}$, which is just what *IMP* asked for.

5. SIMULATION EXAMPLE

5.1 Reference signal containing two periods

Here we choose a discrete system with matrices A, B, C being as follows:

$$A = \begin{pmatrix} 1.5595 & -0.6095 \\ 1 & 0 \end{pmatrix}; B = \begin{pmatrix} 0.5 \\ 0 \end{pmatrix}$$
$$C = (\, 0.1643 \;\; -0.1486 \,);$$

The reference signal is a linear combination of two unit amplitude sinusoidal and cosusoidal signal with periods $N_1 = 473$, $N_2 = 527$ respectively.

By using the background knowledge mentioned in section 2, the new state-space representation in (7) and (8) has a dimension of $N_1 + N_2 + 2 = 1002$ and matrices $\Pi_{1002\times1002}$,$\Gamma_{1002\times1}$ and $\Omega_{1\times1002}$ take the forms as follows:

$$\Pi = \begin{pmatrix} 1.5595 & -0.6095 & 0 & 0 & \dots & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \ddots & \ddots & \ddots & \ddots & \vdots & \vdots \\ -0.1643 & 0.1486 & 0 & 0 & \dots & 0 & 0 & 0 \end{pmatrix}_{1002\times1002}$$

$$\Gamma = \begin{pmatrix} 0.5 & 0 & \dots & \dots & \dots & \dots & \dots & \dots \end{pmatrix}^T_{1\times1002}$$

$$\Omega = \alpha^T = \begin{pmatrix} -0.1643 & 0.1486 & -1 & \dots & 1 & \dots & 1 & \dots \end{pmatrix}_{1\times1002}$$

As we can see, all the matrices have large order and are sparse. e.g. $\Pi \in \Re^{1002\times1002}$ and has many zero elements in the matrix. The weighting matrix R and Q are arbitrarily set as $R = 0.01. * I$ and $Q = 5$ for tracking purpose. Tracking error $e(t)$ and control law $u(t)$ are given as below

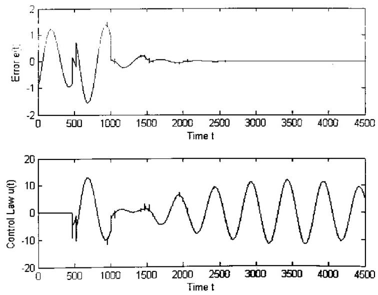

Fig. 2. Convergence of the error $e(t)$ and the control law $u(t)$ for reference signal with two large periods

It is shown in Fig.2 that asymptotic perfect tracking result with $e(t) \to 0$ is obtained as $t \to \infty$. However, since large delay exists in the controller where $\tilde{u}(t) = (P(\Delta^{-1})u)(t)$ defined before, poor performance with slow tracking speed and ultimate instability will be caused, which is always a drawback of delay. This problem can be eased with a careful choice of weighting matrices Q and R and big performance improvement could be achieved.

5.2 Reference signal containing three periods

This time all the parameters are the same as before except the reference signal is composed of three periodic components. Here we use two sinusoidal signals and one cosusoidal signal with unit amplitudes and periods are $N_1 = 53$, $N_2 = 493$ and $N_3 = 673$ respectively. Therefore the new state-space representation in (7) and (8) have a dimension of 1221×1221 ($N{=}n + N_1 + N_2 + N_3 = 1221$). Weighting matrices R and Q are chosen as $R = 0.01. * I$ and $Q = 4$ this time. The result is as follows: As expected, Fig.3 shows that error $e(t)$ converges asymptotically to zero and control law $u(t)$goes to constant. Furthermore, as stated before, choosing different weight matrices Q and R will affect the speed of ultimate performance.

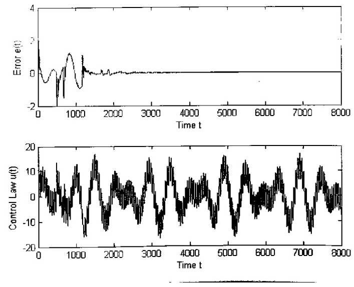

Fig. 3. Convergence of the error $e(t)$ and the control law $u(t)$ for reference signal with three large periods

6. CONCLUSIONS AND FUTURE RESEARCH

In this paper, an Arnoldi algorithm is applied to a previous optimal benmarmark solution on linear discrete MPRC system for tracking multi periodic reference with high periodic components. model reduction technique is applied to reduce the high order state-space representation into its low order description. The stabilizable of (H_m, B_m) and detectable of (H_m, C_m) are assumed to be satisfied, in which case if not they can also be made to be stabilizable and detectable by removing unstable eigenvalues using a restart technique. Thereafter, an existing projection relation between high order and low order riccati solution was applied to obtain an approximate high order riccati solution. By substituting this approximate result into the optimal controller in (10), the optimal discrete Multi periodic Repetitive Controller for tracking high period reference is derived. Finally, this controller is applied to a simulation example and perfect tracking is obtained for both the reference signal containing 2 periodic and 3 periodic signals.

As we have only considered SISO discrete multi-periodic repetitive control system, future work can be tried on MIMO discrete MPRC system and the method should be straightforward by applying krylov subspace methods in [26]. Meanwhile, when solving the low order riccati solution, the Matlab© function *dare.m* is not the best choice of computation cost issue. Future work can be concentrated on: 1. finding the best method which needs the least amount of memory; 2. Spectral analysis of reference can be used as a new method to reduce the computation cost.

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