

# Asymptotic Tracking applied to the Control of a Turbocharged Diesel Engine

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**Abstract:** In this paper we propose to present Asymptotic Tracking applied to the tracking problem for a Turbocharged Diesel Engine (TDE). Our goal is to track desired values of TDE which are the gas pressure in the intake manifold and the compressor mass flow rate. Nevertheless for this system with its chosen outputs one faces to the known problem of non-minimum phase systems. To avoid this, the problem of tracking of desired values of the original output  $y$  is replaced by that of tracking a suitable constructed modified output  $\tilde{y}$  for which the values to be tracked are specifically chosen: namely, when the modified output approaches them, the original output converges to the desired values. Simulation results are presented to highlight efficiency of the controller.

Keywords: Diesel engine; Exhaust Gas Recirculation (EGR); Variable Geometry Turbocharger (VGT); Asymptotic Tracking.

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## 1. INTRODUCTION

Nowadays, legislative standards as Euro V, Arnold et al. (2006) (Transport and E. E. Federation (2004) and Umweltbundesamt (2003)) impose the reduction of pollution emissions particularly in the field of automobile. This leads to the introduction, in Diesel Engine, of two actuators: Exhaust Gas Recirculation (EGR) valve and Variable Geometry Turbocharger (VGT). The former permits recirculation of exhaust gas into the intake manifold while the latter, the compensation of the amount of fresh air due to the important amount of recirculated exhaust gas into the intake manifold. But some points have to be underlined: an insufficient amount of fresh air leads to an increase in particulate emissions and possibly visible smoke, whereas a low amount of EGR fraction leads to an increase in nitrous oxides ( $NO_x$ ) emissions, Jankovic et al. (2000). For this, a stoichiometric mixture, which contains chemically exact mass of air to burn all the fuel injected, is preferred in the cylinder. For Diesel fuel, the stoichiometric AFR is around 14.6, Jankovic et al. (2000).

To render these two actuators more efficient, during the Diesel Engine combustion, several control design methods have been proposed: Polynomial control, Ayadi et al. (2004), Dynamic feedback linearization, Plianos and Stobart (2007), Optimal nonlinear Control, Plianos et al. (2007), Fuzzy approach, Arnold et al. (2006), Constructive Lyapunov Control Design, Jankovic et al. (2000), Indirect passivation, Larsen et al. (2000), Passivation, Larsen and Kokotovic (1998), Predictive control, Otner and del Re (2007), and Ferreau et al. (2007), Nonlinear Continuous-time Generalized Predictive Control (NCGPC), Dabo et al. (2008).

Continuous-time control design methods have not been enough exploited for the control of diesel engine air path. Starting from this, Asymptotic Tracking control design method is proposed. For the study, two mathematical models of the TDE are presented: the seventh-order and the third-order ones. Because of unstable zero dynamics of the third order model, we resort to an appropriate change of vector output ended with a dynamic extension, Isidori (1995). This permits to avoid the occurrence of zero dynamics.

The paper is organized as following: a description of TDE is given in section 2. Section 3 presents Asymptotic Tracking while section 4 the application of this control design method to the mathematical reduced-order model of TDE. Simulation results are presented in section 5 to show how the controller performed. Section 6 synthesizes the presentation and gives some coming directions of work.

## 2. DIESEL ENGINE DESCRIPTION

Many propositions of mathematical models of Diesel Engine have been proposed since the early 60's (Borman (1964), Ledger et al. (1973)). For sake of simplicity, we prefer the model of Jankovic, Jankovic et al. (2000), validated through experiments by the authors. A schematic diagram of TDE is presented below in Fig. 1.

### 2.1 Full-order TDE model

The full-order TDE model is a seventh-order one described as follows, Jankovic et al. (2000): the change of masses of gas in the intake and exhaust manifolds is derived as:

$$\begin{aligned} \dot{m}_1 &= W_c + W_{egr} - W_e \\ \dot{m}_2 &= W_e - W_{egr} - W_t + W_f. \end{aligned} \quad (1)$$

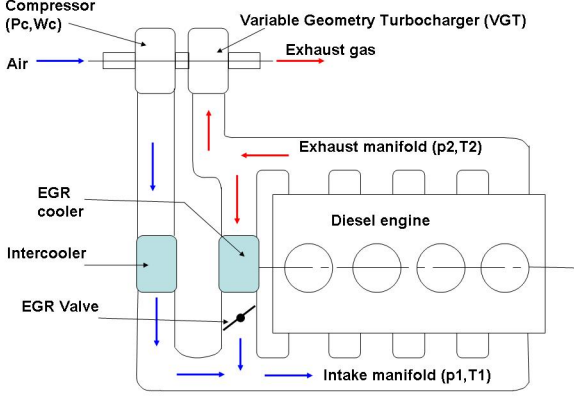


Fig. 1. Diesel Engine.

Similarly, the change of the pressures in the intake and exhaust manifolds is obtained from the first law of thermodynamics. This gives:

$$\dot{p}_1 = \frac{\gamma R}{V_1} (W_c T_c + W_{egr} T_{egr} - W_e T_1) \quad (2)$$

$$\dot{p}_2 = \frac{\gamma R}{V_2} ((W_e + W_f) T_e - W_{egr} T_2 - W_t T_2).$$

Because of a lean combustion, the exhaust gas from the engine is not entirely burnt. Then, the dynamics of fractions of burnt gas  $F_1$  and  $F_2$ , respectively in the intake and exhaust manifolds, are:

$$\dot{F}_1 = \frac{W_{egr}(F_2 - F_1) - W_c F_1}{m_1}$$

$$\dot{F}_2 = \frac{W_e [15.6(1 - F_1) + (AFR + 1)F_1] / (AFR - 1) - W_e F_2}{m_1}. \quad (3)$$

The dynamic  $\dot{\omega}_{tc}$  of the turbocharger is derived from Newton's second law:

$$\dot{\omega}_{tc} = \frac{1}{J_{tc} \omega_{tc}} (\eta_m P_t - P_c). \quad (4)$$

The nomenclature of the considered variables is summarized into the following table.

## 2.2 Reduced-order TDE model

In the sequel, the parameters of the model  $k_1$ ,  $k_2$ ,  $k_c$ ,  $k_e$ ,  $k_t$ ,  $\tau$  and  $\eta_m$  are identified from the seventh-order mean value nonlinear model at a constant speed of 1600 rpm and a fueling rate of 7.2 kg/h, Plianos and Stobart (2007). For a sake of simplicity, the seventh-order TDE model is reduced to a third-order one under specific hypotheses, Jankovic et al. (2000):

- the fractions of intake and exhaust manifolds burnt gas,  $F_1$  and  $F_2$ , are difficult to measure and then they are not considered in the model,
- for the same reasons the intake and exhaust burned gas masses fraction,  $m_1$  and  $m_2$ , are ignored,
- the turbocharger dynamics are modeled as a first-order lag power transfer with a time constant  $\tau$ .

Furthermore, the fuel mass flow rate  $W_f$  is considered as an external disturbance and is not taken into account in the following model. Therefore, the third-order model is:

Nomenclature	
Variable	Description
$EGR$	Exhaust Gas Recirculation
$AFR$	Air Fuel Ratio
$N$	Engine speed
$F_1$	Intake manifold burned gas fraction
$F_2$	Exhaust manifold burned gas fraction
$m_1$	Mass of gas in the intake manifold
$m_2$	Mass of gas in the exhaust manifold
$p_1$	Gas pressure in the intake manifold
$p_2$	Gas pressure in the exhaust manifold
$P_c$	Compressor power
$P_t$	Turbine power
$W_e$	Total mass flow rate into the engine
$W_c$	Compressor mass flow rate
$W_t$	Turbine mass flow rate
$W_f$	Fuel mass flow rate
$W_{egr}$	EGR mass flow rate
$V_1$	Intake manifold volume
$V_2$	Exhaust manifold volume
$T_1$	Intake manifold temperature
$T_2$	Exhaust manifold temperature
$T_c$	Compressor temperature
$T_e$	Temperature of the exhaust from the engine
$T_{egr}$	EGR temperature
$\omega_{tc}$	Turbocharger speed
$J_{tc}$	Turbocharger moment of inertia
$\eta_c$	Compressor isentropic efficiency
$\eta_t$	Turbine isentropic efficiency
$\eta_m$	Turbocharger mechanical efficiency
$\gamma$	Specific heat ratio
$R$	Specific gas constant

$$\dot{p}_1 = k_1 (W_c + u_1 - k_e p_1) + \frac{\dot{T}_1}{T_1} p_1$$

$$\dot{p}_2 = k_2 (k_e p_1 - u_1 - u_2) + \frac{\dot{T}_2}{T_2} p_2 \quad (5)$$

$$\dot{P}_c = \frac{1}{\tau} (\eta_m P_t - P_c),$$

where  $W_c$  and  $W_t$  are related to  $P_c$  and  $P_t$  through the following equations (6) and (7) respectively:

$$W_c = P_c \frac{k_c}{p_1^\mu - 1} \quad (6)$$

and

$$P_t = k_t (1 - p_2^{-\mu}) u_2. \quad (7)$$

Despite of the fact that the real inputs are EGR valve and VGT openings, the considered inputs are  $u_1 = W_{egr}$  and  $u_2 = W_t$ , Jankovic et al. (2000).

In the sequel,  $\dot{T}_1$  and  $\dot{T}_2$  are assumed to vanish because their corresponding measured signals  $T_1$  and  $T_2$  have very slow variations. This yields the following system, Jankovic et al. (2000):

$$\dot{p}_1 = k_1 (W_c + u_1 - k_e p_1)$$

$$\dot{p}_2 = k_2 (k_e p_1 - u_1 - u_2)$$

$$\dot{P}_c = \frac{1}{\tau} (\eta_m P_t - P_c). \quad (8)$$

Replacing  $W_c$  and  $P_t$  by their expressions (6) and (7), respectively, gives:

$$\dot{x} = f(x) + g_1(x)u_1 + g_2(x)u_2 \quad (9)$$

with

$$f(x) = \begin{bmatrix} k_1 k_c \frac{P_c}{p_1^\mu - 1} - k_1 k_e p_1 \\ k_2 k_e p_1 \\ -\frac{P_c}{\tau} \end{bmatrix}, \quad (10)$$

$$g_1(x) = \begin{bmatrix} k_1 \\ -k_2 \\ 0 \end{bmatrix} \text{ and } g_2(x) = \begin{bmatrix} 0 \\ -k_2 \\ K_0 (1 - p_2^{-\mu}) \end{bmatrix}, \quad (11)$$

and where  $K_0 = \frac{\eta_m}{\tau} k_t$ .

### 2.3 Vector output choice

The output of to-be-controlled variables are the input manifold pressure  $p_1$  and the compressor mass flow rate  $W_c$  instead of the Air Fuel Ratio (AFR) and the EGR fraction because the latter are not accessible for measurements in a vehicle, Jankovic et al. (2000). This choice is motivated by the following reasons:

- if one controls the amount of fresh air in order to have a stoichiometric mixture with the exhaust recirculated gas, the control of EGR fraction is consequently done,
- AFR can be deduced from the following relation  $AFR = (1 - F_1)(W_c + W_{egr})/W_f$ , Jankovic et al. (2000).

We then have the nonlinear system (8) (equivalently (9) to (11)) with the vector output:

$$y(t) = \begin{bmatrix} h_1(x(t)) \\ h_2(x(t)) \end{bmatrix}$$

and more precisely without writing the time-parameter  $t$  for sake of simplicity

$$y = \begin{bmatrix} p_1 \\ W_c \end{bmatrix}. \quad (12)$$

In the sequel, all the components  $(p_1, p_2, P_c)$  of the state  $x$  are supposed to be accessible for measurements. The state's components belong to the set  $\Omega$  defined by Jankovic, Jankovic et al. (2000):

$$\Omega = \{(p_1, p_2, P_c) : 1 < p_1 < p_1^{max}, 1 < p_2 < p_2^{max}, 0 < P_c < P_c^{max}\}. \quad (13)$$

The maximum values  $(p_1^{max}, p_2^{max}, P_c^{max})$  are considered because of the physical limits of the TDE.

## 3. ASYMPTOTIC TRACKING

In this section, our goal is to derive the vector control law in the case of Asymptotic Tracking. For this, the following definitions are given.

Consider a square-MIMO ( $m \times m$ ) nonlinear system given by :

$$\begin{aligned} \dot{x} &= f(x) + \sum_{j=1}^m g_j(x) u_j \\ y &= (h_1(x), \dots, h_m(x))^t, \end{aligned} \quad (14)$$

where  $x \in R^n$ ,  $u \in R^m$  and  $y \in R^m$  are, respectively, the state, control and output vectors.

To simplify the exposition, the standard geometric notation for Lie derivatives is used in this paper. For a real-valued function  $h(x)$  on  $R^n$  and a vector field  $f$  on  $R^n$ , the Lie derivative of  $h(x)$  along  $f$  at  $x \in R^n$  is denoted by:

$$L_f h(x) = \sum_{i=1}^n \frac{\partial h(x)}{\partial x_i} f_i(x).$$

Inductively, we define

$$L_f^k h(x) = L_f L_f^{k-1} h(x) = \frac{\partial L_f^{k-1} h(x)}{\partial x} f(x)$$

with  $L_f^0 h(x) = h(x)$ . Finally, we denote

$$L_g L_f^k h(x) = [L_{g_1} L_f^k h(x) \cdots L_{g_m} L_f^k h(x)],$$

where  $g_j$ 's are the vectors field related to the controls  $u_j$  (14).

### 3.1 Vector relative degree

A system of the form (14) has a vector relative degree  $(\rho_1, \dots, \rho_m)$  at all  $x \in R^n$  if:

(i)

$$L_{g_j} L_f^k h_i(x(t)) = 0 \quad (15)$$

for all  $1 \leq i \leq m$ , all  $1 \leq j \leq m$ , all  $0 \leq k < \rho_i - 1$ , and for any  $x \in R^n$

(ii) the  $(m \times m)$  matrix called decoupling matrix

$$A(x) = \begin{bmatrix} L_{g_1} L_f^{\rho_1-1} h_1(x) & \cdots & L_{g_m} L_f^{\rho_1-1} h_1(x) \\ \vdots & & \vdots \\ L_{g_1} L_f^{\rho_m-1} h_m(x) & \cdots & L_{g_m} L_f^{\rho_m-1} h_m(x) \end{bmatrix} \quad (16)$$

is nonsingular for all  $x \in R^n$ , Isidori (1995).

### 3.2 Feedback linearization with Asymptotic Tracking

In the continuous-time domain and supposing zero dynamics stable, the control law derived from a feedback linearization with Asymptotic Tracking is given by:

$$u(x) = A(x)^{-1} \left[ \begin{pmatrix} v_1 \\ \vdots \\ v_m \end{pmatrix} - \begin{pmatrix} L_f^{\rho_1} h_1(x) \\ \vdots \\ L_f^{\rho_m} h_m(x) \end{pmatrix} \right]. \quad (17)$$

In (17)  $A(x)$  is supposed to be invertible. In the case of Asymptotic Tracking, the  $i$ -th component of the vector  $v = [v_1, \dots, v_m]^t$  is:

$$v_i = \omega_i^{(\rho_i)} - \sum_{k=1}^{\rho_i} c_{i(k-1)} (z_k^i - \omega_i^{(k-1)}),$$

where the terms  $\omega_i^{(j)}$  are the  $j$ -th derivative of the  $i$ -th component of the vector reference  $\omega = [\omega_1, \dots, \omega_m]^t$ ; where the coefficients  $c_{i(k-1)}$  are such that the polynomial

$$s^{\rho_i} + c_{i(\rho_i-1)} s^{\rho_i-1} + \cdots + c_{i1} s + c_{i0} = 0 \quad (18)$$

is Hurwitz, Isidori (1995), and the  $z_k^i$  are such that  $z_k^i = L_f^{k-1} h_i$ . The following scheme, Fig. 2, resumes the different steps before the application of the vector control law.

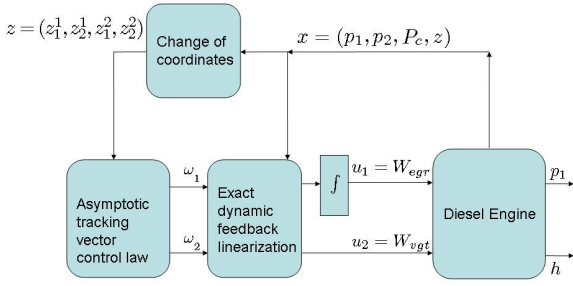


Fig. 2. Dynamic Asymptotic Tracking with dynamic feedback linearization.

#### 4. APPLICATION TO TURBOCHARGED DIESEL ENGINE MODEL

Before applying this control law on TDE, we propose to study the internal dynamics of the system. In order to do this, the vector relative degree is first derived and then zero dynamics studied.

##### 4.1 Vector relative degree

The vector relative degree,  $(\rho_1, \rho_2) = (1, 1)$ , of the third-order model, (8) and (12), is defined in all the set  $\Omega$ , (13), Dabo et al. (2008). Its decoupling matrix is nonsingular and equal to:

$$A(x) = \begin{bmatrix} L_{g_1} h_1 & L_{g_2} h_1 \\ L_{g_1} h_2 & L_{g_2} h_2 \end{bmatrix} \quad (19)$$

and more explicitly equal to

$$A(x) = \begin{bmatrix} k_1 & 0 \\ -\frac{\mu k_c k_1 P_c p_1^{\mu-1}}{(p_1^\mu - 1)^2} & -K_0 k_c \frac{p_2^{-\mu} - 1}{p_1^\mu - 1} \end{bmatrix}. \quad (20)$$

The sum of the vector relative degree's components,  $\rho_1 + \rho_2 = 1 + 1 = 2$ , is less than 3, the dimension of the system. Therefore a one-dimension zero dynamics exists, Dabo et al. (2008).

##### 4.2 Zero dynamics

Zero dynamics is obtained by identically annihilating the vector output  $y(t)$ , (12). But in our case, due to the non-vanishing property of the state vector (because it belongs to the set  $\Omega$ , (13)), we add constant terms to the output's components corresponding to the desired values  $p_{1d}$  of the intake manifold pressure and  $W_{cd}$  of the compressor mass flow rate. Hence:

$$y_t = \begin{bmatrix} y_{t1} \\ y_{t2} \end{bmatrix} = \begin{bmatrix} p_1 - p_{1d} \\ W_c - W_{cd} \end{bmatrix}. \quad (21)$$

This yields:

$$p_2 = k_2 W_{cd} \left[ 1 - \frac{(p_{1d}^\mu - 1)}{\eta_m k_t k_c (1 - p_2^{-\mu})} \right], \quad (22)$$

which is unstable, Fig. 3 and Fig. 4.

The instability of internal dynamics does not render simple the application of feedback linearization. This, because the internal dynamics represent the core of our system. An important result is the involutivity of the distribution

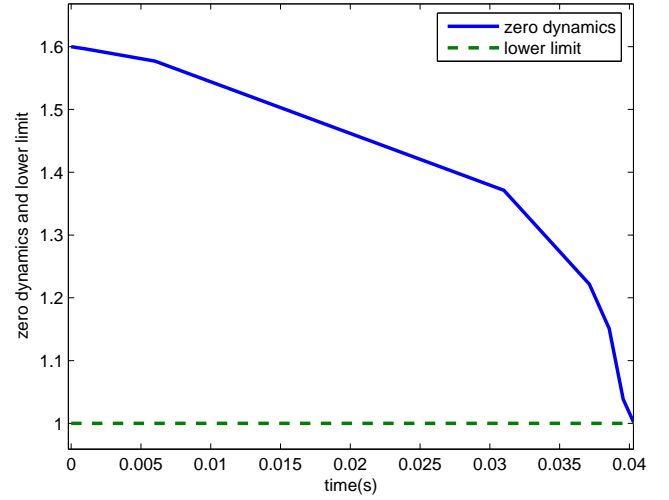


Fig. 3. Third-order TDE with  $p_{20} = 1.6$  bar: unstable zero dynamics.

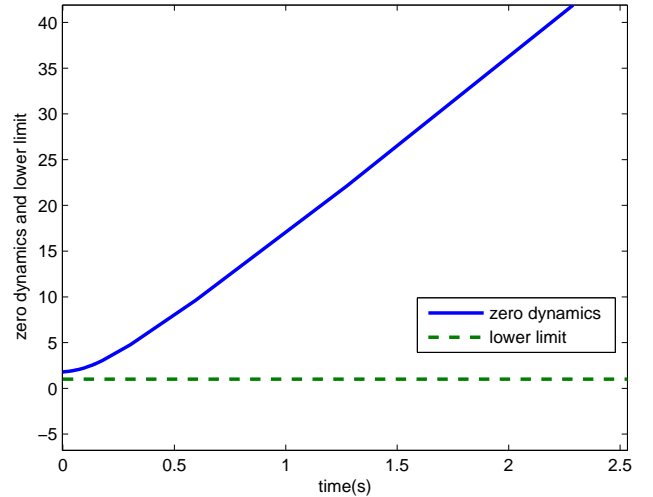


Fig. 4. Third-order TDE with  $p_{20} = 1.8$  bar: unstable zero dynamics.

$D = \text{span}\{g_1, g_2\}$ , spanned by  $g_1$  and  $g_2$ . As this distribution is not involutive, Dabo et al. (2008), this yields a dynamic input-output feedback linearizable system for every chosen vector output. In order to avoid unstable internal dynamics, another vector output  $\tilde{y}$ , is suitably chosen.

##### 4.3 Change of the vector output

The objective is thus to find a new second component  $h$  of the vector output such that  $L_{g_2} h_2 = 0$ , (19). In other words, the vector output is to be chosen such that the appearance of the component  $u_2$  of the vector control is delayed to higher order derivatives of the translated output's components  $y_{t1}$  and  $y_{t2}$ , (21). This can be achieved by keeping the first translated component  $y_{t1}$  and choosing the second output component  $y_{t2} = h$  such that, indeed,  $L_{g_2} h_2 = L_{g_2} h = 0$ . Resolving this equation gives:

$$h = P_c + \frac{K_0}{k_2} \left[ p_2 - \frac{1}{1-\mu} p_2^{1-\mu} \right]. \quad (23)$$

Renaming this new vector output, gives:

$$\tilde{y}(t) = \begin{bmatrix} \tilde{h}_1(x(t)) \\ \tilde{h}_2(x(t)) \end{bmatrix}$$

and, explicitly written without the time-parameter  $t$ ,

$$\tilde{y} = \begin{bmatrix} p_1 \\ P_c + \frac{K_0}{k_2} \left[ p_2 - \frac{1}{1-\mu} p_2^{1-\mu} \right] \end{bmatrix}. \quad (24)$$

We then have a system with the following decoupling matrix:

$$\tilde{A}(x) = \begin{bmatrix} k_1 & 0 \\ -\frac{\mu k_c k_1 P_c p_1^{\mu-1}}{(p_1^\mu - 1)^2} & 0 \end{bmatrix} \quad (25)$$

which is singular. Therefore, the vector relative degree is not defined for (8) and (24), Isidori (1995). To encounter this new problem, because the distribution  $D$  is not involutive, we resort to dynamic extension.

#### 4.4 Dynamic extension

It consists of choosing one or more pre-integrating states to be added to the studied system. Once this done, we obtain an extended system constituted by the original one with the added states. For this study, by putting  $z = u_1$  and  $\dot{z} = v_1$  (see 25), and applying the new control  $v = [v_1, v_2]^t$ , where  $v_2 = u_2$  the following extended system

$$\begin{aligned} \dot{p}_1 &= k_1(W_c + z - k_e p_1) \\ \dot{p}_2 &= k_2(k_e p_1 - z - v_2) \\ \dot{P}_c &= \frac{1}{\tau}(\eta_m P_t - P_c) \\ \dot{z} &= v_1, \end{aligned} \quad (26)$$

is obtained with its vector output (24). It has the vector relative degree  $(\rho_1^e, \rho_2^e) = (2, 2)$ , Dabo et al. (2008) and the following nonsingular decoupling matrix, (27), for all  $(p_1, p_2, P_c) \in \Omega$ :

$$A^e(x) = \begin{bmatrix} k_1 & k_1 k_c K_0 \frac{1 - p_2^{-\mu}}{p_1^\mu - 1} \\ K_0(p_2^{-1} - 1) & \mu k_2 K_0(z - k_e p_1) p_2^{-\mu-1} + \frac{K_0}{\tau}(p_2^{-\mu} - 1) \end{bmatrix} \quad (27)$$

The third-order model has been extended by stating  $z = u_1$  and the resulting extended system is considered with the output (24). Its vector relative degree is equal to  $(\rho_1^e, \rho_2^e) = (2, 2)$ . The sum of its components,  $\rho_1^e + \rho_2^e = 2 + 2 = 4$ , is equal to the dimension of the extended system (26). Therefore this system, (26), has no zero dynamics. The problem of non-minimum phase system is then definitively avoided. At this level and because of the dynamic extension, a Dynamic Asymptotic Tracking can be applied to (26) without any other difficulties.

#### 4.5 Vector control law

In this subsection the application of vector control law is given. In order to do this, let us define some specific terms related to the appropriate change of coordinates

necessary for input-output feedback linearization a step for the Asymptotic Tracking:

$$\begin{aligned} z_1^1 &= L_f^0 h_1 = h_1 = p_1 \\ z_2^1 &= L_f^0 h_2 = h_2 = P_c + \frac{K_0}{k_2} \left( p_2 - \frac{1}{1-\mu} \right) p_2^{1-\mu} \\ z_1^2 &= L_f^1 h_1 = k_1 \left( z - k_e p_1 + P_c \frac{k_c}{p_1^\mu - 1} \right) \\ z_2^2 &= L_f^1 h_2 = -\frac{P_c}{\tau} + K_0(k_e p_1 - z)(1 - p_2^{-\mu}). \end{aligned}$$

This yields to the following linear, controllable and decoupled system:

$$\begin{aligned} \dot{z}_1^1 &= z_2^1 \\ \dot{z}_2^1 &= v_1 \\ \dot{z}_1^2 &= z_2^2 \\ \dot{z}_2^2 &= v_2. \end{aligned}$$

where  $v_1$  and  $v_2$  are the new controls. As our objective is to track desired outputs:  $p_{1d}$ , the desired value of the intake manifold pressure and  $h_d$  the desired value of the new second vector output's component (24), with specific chosen dynamics, the vector control law is chosen equal to

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} \ddot{\omega}_1 - c_{10} [z_1^1 - \omega_1] - c_{11} [z_2^1 - \dot{\omega}_1] \\ \ddot{\omega}_2 - c_{20} [z_1^2 - \omega_2] - c_{21} [z_2^2 - \dot{\omega}_2] \end{bmatrix},$$

where the terms  $\omega_i^{(j)}$  correspond to the  $j$ -th derivative of the  $i$ -th component of the vector  $\omega = [\omega_1, \dots, \omega_m]^t$  and the constants, chosen for the dynamics of the outputs, are  $c_{10} = 16$ ,  $c_{11} = 8$ ,  $c_{20} = 4$  and  $c_{21} = 5$ . This yields the following decoupled sub-systems:

$$\begin{aligned} \dot{z}_1^1 &= z_2^1 \\ \dot{z}_2^1 &= \ddot{\omega}_1 - c_{10} [z_1^1 - \omega_1] - c_{11} [z_2^1 - \dot{\omega}_1] \end{aligned} \quad (28)$$

and

$$\begin{aligned} \dot{z}_1^2 &= z_2^2 \\ \dot{z}_2^2 &= \ddot{\omega}_2 - c_{20} [z_1^2 - \omega_2] - c_{21} [z_2^2 - \dot{\omega}_2]. \end{aligned} \quad (29)$$

Written in a matrix form, (28) and (29) become, respectively, (30) and (31) as following:

$$\begin{bmatrix} \dot{z}_1^1 \\ \dot{z}_2^1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -c_{10} & -c_{11} \end{bmatrix} \begin{bmatrix} z_1^1 \\ z_2^1 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ c_{10} & c_{11} & 1 \end{bmatrix} \begin{bmatrix} \omega_1 \\ \dot{\omega}_1 \\ \ddot{\omega}_1 \end{bmatrix} \quad (30)$$

and

$$\begin{bmatrix} \dot{z}_1^2 \\ \dot{z}_2^2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -c_{20} & -c_{21} \end{bmatrix} \begin{bmatrix} z_1^2 \\ z_2^2 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ c_{20} & c_{21} & 1 \end{bmatrix} \begin{bmatrix} \omega_2 \\ \dot{\omega}_2 \\ \ddot{\omega}_2 \end{bmatrix}. \quad (31)$$

## 5. SIMULATION RESULTS

Simulation results are carried out via Simulink with the version V 7.0 of Matlab and Fig. 5, Fig. 6 and Fig. 7 show that if the output  $h$  matches its reference, hence the original output component  $W_c$  matches also its desired value. A corresponding table can be established between the desired values of  $W_c$  and that of  $h$ . This permits to corroborate that at each desired value of  $W_c$  it corresponds one and only one desired value of  $h$ .

## 6. CONCLUSION

Asymptotic Tracking is proposed in this paper for the tracking problem of Turbocharged Diesel Engine (TDE). The problem of non-minimum phase that we face to is

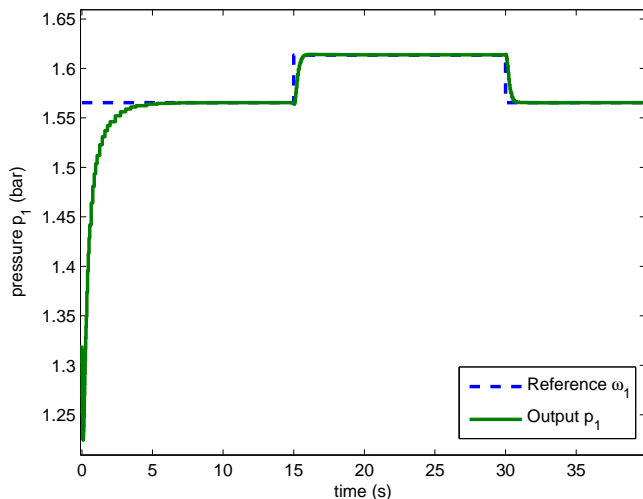


Fig. 5. Output  $p_1$  of the extended TDE model.

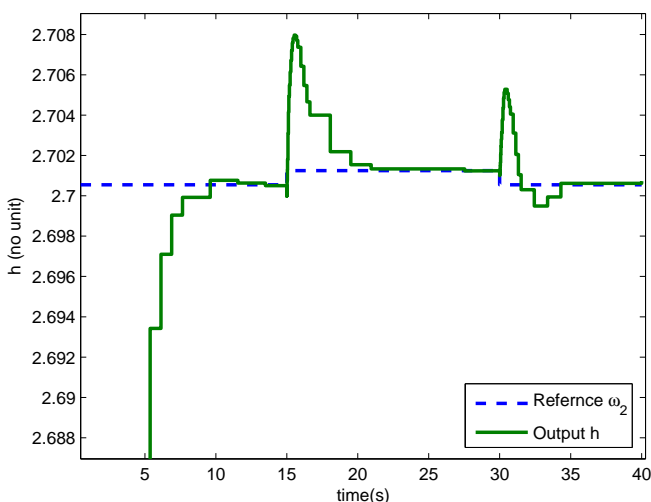


Fig. 6. Output  $h$  of the extended TDE model.

resolved by: first, proposing a suitable change of the vector output (12) and then applying a dynamic extension. Future works will consist on designing a nonlinear observer because, in reality, all the components of the vector state are not accessible for measurements.

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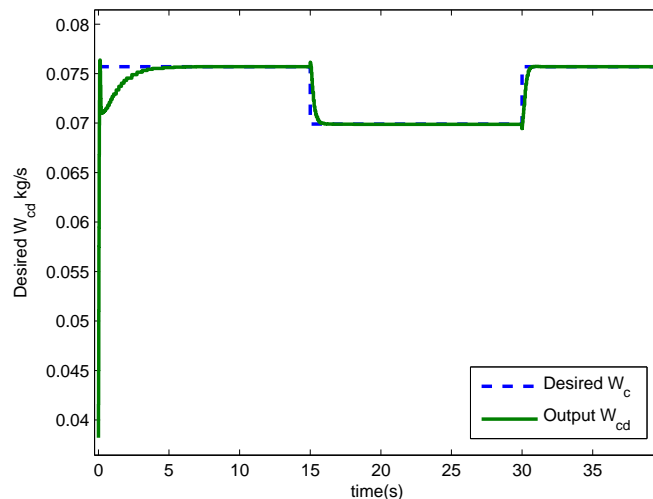


Fig. 7. Desired output  $W_{cd}$  of the third-order TDE model.

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