The Optimisation of Stator Vane Settings in Multi-stage Axial Compressors Using A Particle Swarm Optimization Method

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Abstract: Axial flow compressors are required to operate over a wide range of mass flow rate and rotational speeds at high efficiency in industrial gas turbines. However, the useful range of operation of the axial compressor is limited by the onset of two instabilities known as surge and rotating stall. To resolve these problems, variable stator blades or VGVs are considered by optimising the blade setting in order to avoid the stall and subsequent surge. To investigate performance, particularly obtaining acceptable convergence time for practical purposes, a steady state model of a 15 stage multi-axial compressor is utilised. For the effective search for an optimum setting, the variation in VGVs with respect to a different combination of objective functions is considered. In this paper, a particle swarm optimisation method with time-varying inertia weight factor was proposed and utilised to obtain the best value for a normalised objective function. The results of PSO demonstrate the effectiveness and the suitability of its use in this proposed application.

1. INTRODUCTION

Generally, efficient performance is an essential requirement for most engineering systems. Especially, for high performance axial flow compressors that are required to operate over a wide range of mass flow rate and rotational speed at high efficiency in industrial gas turbines. However, there is a certain limitation for the useful operation range of an axial compressor by choking at high mass flows, when the sonic velocity is reached in some component, and at low mass flows, by the onset of two instabilities known as surge and rotating stall (Gravdahl and Egeland, 1999).

Both rotating stall and surge have been a problem for as long as turbomachinery compressors have been built. Gu, Banda & Sparks 1995, provide a comprehensive explanation of instabilities in that rotating stall is a severely non axisymmetric distribution of axial flow velocity around the annulus of the compressor. Steadily, it takes the form of a wave or ‘stall cell’ that propagates in the circumferential direction at a fraction of the rotor speed. One of the first studies about propagation of rotating stall in axial compressors was by Emmons *et al* 1995. Cumpsty and Greitzer 1982, derived a model for stall cell speed, showing that stall cell speed increases with increasing number of compressor stages.

Surge was described as an axisymmetric oscillation of the mass flow along the axial length of the compressor. Normally, surge oscillations are in most unwanted applications so that can even damage the compressor in extreme cases and also induced the vibrations in other components of the compression system, such as e.g. connected piping as discussed by Erskine & Hensman 1975 and Greitzer 1989. According to Cumpsty 1981, one of the most damaging effects of surge in high-pressure ratio axial compressors is the very large transverse load placed on the rotor and the casing because of non-axisymmetric surge. Finally, this may lead to severe blade rubbing and then a range of further damage.

These two instabilities’ behaviours are often coupled, although each can occur without the other in the case of classic surge. Rotating stall and surge are mostly caused by disturbances. Eventually these can not only cause severe damage to the compression system that ultimately leads to a critical failure of component within the machine but also restrict the performance and efficiency of the compressor. In practice, surge and rotating stall have been avoided by using control systems that prevent the operating point of the compression system moving into the unstable domain to the left to the surge line, which is the stability boundary. In the axial compressor, instabilities are caused by high incidence followed by stalling of stages that occurs due to different phenomena at part and full speed operation. The part-speed problem often occurs at the front stages and rear stages, which are operating close to choke and stalling, at high speeds. To reduce the low-speed problems due to this part-speed problem, numerous solutions have been suggested. These are casing treatments, bleed off-take, variable IGVs & stator blades and twin spool arrangements. In this paper, the variable stator blades or VGVs are considered by modifying the compressor characteristics. Sun & Elder 1998, reported that the technology with using VGVs can not only alleviate the low-speed problem but also offer performance benefits at high speed. The redirected flow at the inlet can reduce the inlet flow angle onto the blade so as to prevent stall at the trailing edge for low speed operation. The magnitude of re-stagger decreases as the rotational speed is increased from the front to the last VGV row due to the structure of variable blade row in common axial compressors. White 2002, investigated that the optimisation of the design to full load conditions may provide part-speed problems. Thus, variable geometry over the front region of the compressor is sometimes used to modify the flow angles and avoid stage stall and subsequent surge to achieve the acceptable performance.

The main purpose of this research is to develop an optimisation model for stator blade settings in multi stage axial flow compressors of an industrial gas turbine engine by applying particle swarm optimisation. White 2002, previously developed the numerical model for multi-stage axial flow compressors and coupled this to multi-objective evolutionary algorithm (MOEA). However, the optimisation model for an industrial compressor was not well suited to on-line implementation due to the significant long convergence time of MOEA. The proposed optimisation strategy here can be considered to be practical and effective in reducing the optimisation time. Therefore PSO is capable of searching for an optimum VGV reset angle efficiently with a combination of specified objective functions such as overall pressure ratio, efficiency and surge point at a single speed.

2. PARTICLE SWARM OPTIMISATION

Kennedy and Eberhart 1995, introduced the Particle swarm optimisation (PSO), which is a population-based and self-adaptive search algorithm that is initialised with a population of random solutions, in terms of particles. In PSO, each particle is associated with a velocity. So these particles fly through the search space with a velocity, which are dynamically adjusted using the historical behaviour. This algorithm is motivated from the simulation of simplified animal social behaviours such as bird flocking, fish schooling, etc.

PSO has many similarities with other population–based optimisation methods such as genetic algorithms in that the algorithm initialises a population of particles randomly in the search space and searches for the optimum solutions by updating generations. However, there is no evolutionary process such as crossover and mutation during the search in PSO unlike in other evolutionary optimisation methods. The advantage of PSO algorithm is to implement in applications easily with few parameters to manipulate and therefore, it has been successfully applied to many areas and applications due to its simplicity of utilisation and quick convergence relative to the Genetic Algorithm.

The PSO algorithm is based on the social behaviour of particles in the swarm. Therefore it finds the global best solution by simply adjusting the trajectory of each individual towards its own best location and towards the best particle of the entire swarm at each time step generation as reported by Kennedy and Eberhart 1995.

The i^{th} particle in the d-dimensional search space can be represented as $X_i = (x_{i1}, x_{i2}, \ldots, x_{id})$ and the velocity vector, rate of the position change, for the particle i is $V_i = (v_{i1}, v_{i2}, v_{i3}, \ldots, v_{id})$. The best position of the each particle (the best previous position) corresponds the best fitness value gained by the particle at time t is $P_i = \left(p_{i1}, p_{i2}, p_{i3}, \ldots, p_{id}\right)$ and according to a user defined fitness function, best particle among all the particles in the population can be represent as $P_g = \left(p_{g1}, p_{g2}, \ldots, p_{gd}\right)$. A typical equation for PSO algorithm is described as:

$$\begin{aligned} v_{id} &= v_{id} + c_1 \times rand(\bullet) \times \left(p_{id} - x_{id}\right) \\ &\quad + c_2 \times Rand(\bullet) \times \left(p_{gd} - x_{id}\right) \end{aligned} \tag{1}$$

$$x_{id} = x_{id} + v_{id} \tag{2}$$

where c_1 and c_2 are positive constants known as acceleration coefficients, and $rand(\bullet)$, $Rand(\bullet)$ are two uniformly distributed random functions in the range [0,1]. The previous velocity can be represented as the first part of equation (1) and provides the necessary momentum for particles to roam across the search space while the second part, treated as the "cognitive" component, represents the personal thinking of each particle. The cognitive component encourages the particles to shift to their own best position found so far. Last part of equation (1) is known as the "social" component; produce the collaborative effect of the particles to find the global optimal solution. This social component pulls the particles toward the global best particle found so far all the time.

At each generation, each particle is updated by the equation (1) and the position for the next function evaluation is calculated according to equation (2). In any case, each time a particle finds a better position than the previously found best position then its location is stored in the memory. As stated by Kennedy and Eberhart 2001, changes in the velocity are stochastic, and an undesirable result of this that the particle's trajectory, uncontrolled, can expand into wider cycles through the problem space, eventually approaching infinity. So the maximum velocity (V_{max}) for each modulus of the velocity vector of the particles must be defined in order to control excessive roaming of particles outside the user-defined search boundary. Therefore, every time the velocity vector of the particles exceeds the defined limit, the velocity returns to its maximum setting.

In order to control the global search and convergence during optimization process, Shi and Eberhart 1998, introduced the version of the PSO that incorporates with an inertia weight factor. PSO with inertia weight factor form is given as:

$$\begin{aligned} v_{id} &= \omega \times v_{id} + c_1 \times rand(\bullet) \times \left(p_{id} - x_{id}\right) \\ &\quad + c_2 \times Rand(\bullet) \times \left(p_{gd} - x_{id}\right) \end{aligned} \tag{3}$$

where ω is the value of the inertia weight, respectively. By Eberhart and Shi 1998, implementing the inertia weight has the benefit of decreasing over time typically from the range of approximately from 0.9 to 0.4 and also can neglect the requirement of setting the maximum velocity. However, inclusion of V_{max} can provide more effective performance in computation considering utilisation of constriction factor. In this study, modified form of PSO, equation (3), was implemented for the optimisation for the stator vane setting of axial-compressor.

3. TARGET COMPRESSOR

For this study, an Alstom Power 15-Stage Tornado Plant Compressor (White, 2002) was chosen as an application to implement the PSO algorithm. Fig. 1 shows a detailed scale diagram of the cross section for the whole 6MW Tornado gas turbine engine. The design of targeted 15-stage subsonic compressor is 50% reaction at each stage to divide the overall temperature rise evenly. Each blade is a C4 airfoil and the annulus is of a constant outside diameter for the first seven stages and constant inner diameter for the remainder. The IGV and the first four stators are variable stagger design to allow low speed operation without surging the compressor. A bleed valve is also included over the eighth stage, which enables to bleed extraction up to 20% of the mass flow.

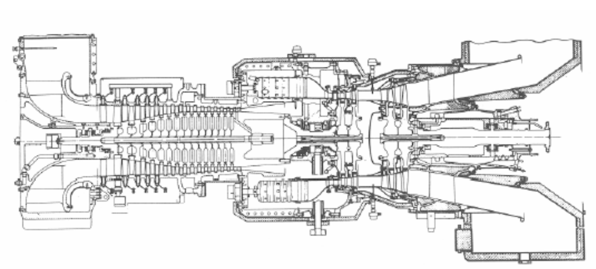


Fig.1. Schematic view of Tornado gas turbine engine

3.1. Compressor Modelling

To obtain an appropriate objective function, a numerical one dimensional (1-D) prediction technique was employed for modelling the compressor. The 1-D performance prediction for compressor was previously developed by White 2002 under the FORTRAN environment using the Tornado compressor stage and overall experimental data. A theoretical basis of meanline prediction model was proposed by Wright & Miller 2004, except for the throat area calculation method and the subsonic-supercritical shock loss estimation method. The benefit through a 1-D performance prediction program is that this requires less computation to reach convergence thus the results are produced within an acceptable time frame. Moreover, the 1-D models are widely employed within industrial organisations, often as a precursor to more detailed analyses or to confirm the designer's intuition. Then, this model was coupled to a surge prediction method to produce stage characteristics and overall performance. Through the MEX function of MATLAB, all the results are passed to an optimisation program. Eventually, three objective functions, surge mass flow rate, polytrophic efficiency and pressure ratio were to be obtained.

4. CONTROL STRATEGY

The general optimisation process for VGV reset angles combined with PSO algorithm is described as Figure 2. Although, the conventional global version of PSO with appropriate time varying inertia weigh values require any V_{max} parameter, it is possible that set the constraints to V_{max} have more advantage in convergence and reducing the computational cost. Kennedy and Ebehart 2001, suggested that limitation of v_{id} is necessary not to approach too closely to 0.0 or 1.0 due to the possibility of moving toward infinity and overloading the exponential function. Therefore, V_{max} can be set at the start of a trial to limit the range of v_{id} and a role of constant parameter V_{max} is so much similar to mutation rate in the genetic algorithm. In this study, 0.1 was set for the constraints to all PSO process during VGV optimisation. In addition, another constraint was set for the particle position due to the limitation in VGV blade set to prevent the optimisation point from placing outside flow range. All the parameters are initialised before evaluating the initial swarm, which has $n \times N$ dimensional population of real numbers. n is the number of dimensions, can be expand by the dependent upon and N denotes number of individuals as its each row represents the particle. All the process related with the PSO optimisation was encoded in MATLAB M-files. First stage of the optimisation initialises input parameters, includes calling the initial objective function data with respect to first VGV blade angle. The position of initial particle is randomly defined with the manually added initial input blade angle degree.

In order to find the optimum VGV reset angles corresponds to objective function, two methods were considered for the optimisation. The first one was to change the single VGV angle within the flow range at pre-defined VGV setting and fixed remain blade angles. To optimise the first VGV angle, the position of initial particles is determined before the swarm is evaluated. Then following values of P_g are decided by comparing the performance of all the members of objective function is the data resulted from normalising three objective functions-surge mass flow, polytropic efficiency and pressure ratio as described at Fig 2. At the end of iteration of each individual optimisation process, the position of particle best is recorded and stored in MATLAB workspace and then moved on to the next iteration.

The second approach is variation of first and second VGV blade angles then find the peak point of normalised data, at which the objective function was given in the hyper surface space. This hyper surface was comprised of the variation of normalised objective functions, combining the three objective functions mentioned above, and corresponds to different VGV reset angle settings at each speed. The VGV bounds were necessary to adjust blade angles in a small range to prevent the optimisation point from moving outside of the operational flow range.

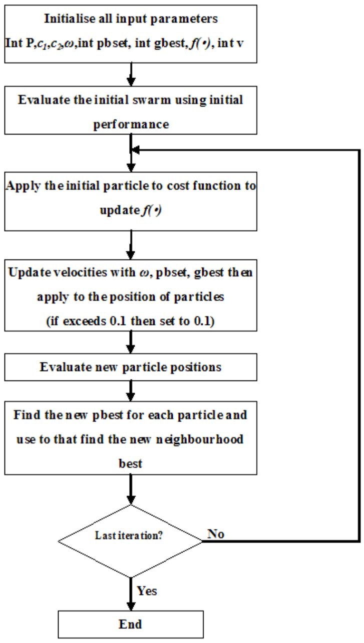


Fig. 2. Flow chart of PSO

4.1. Normalisation of objective function

Normalising the objective functions provides not only a meaningful combination for diverse physical parameters but also reduces the computational effort significantly. Moreover, it also enables the optimisation process to be simply plotted. Normalising the value of each of three objective functions: surge mass flow, polytrophic efficiency and pressure ratio. The normalisation rule for three objective functions is given as:

1. Surge Mass Flow

$$f_{wsg}(x) = \frac{1}{f(x_n)} = \frac{f(x_n)}{f(x_{\min})} \qquad n = 1,\ldots,n\ (n = number\ of\ \ f(\cdot))$$

2. Polytrophic Efficiency

$$f_{eff}(x) = \frac{f(x_n)}{f(x_{\max})} \qquad n = 1,\ldots,n\ (n = number\ of\ \ f(\cdot))$$

3. Pressure Ratio

$$f_{pr}(x) = \frac{f(x_n)}{f(x_{\max})} \qquad n = 1,\ldots,n\ (n = number\ of\ \ f(\cdot))$$

Thus, combined three objective functions can be expressed as follows:

$$J_{comb} = \frac{w_1 f_{wsg}(x) + w_2 f_{eff}(x) + w_3 f_{pr}(x)}{3} \qquad (4)$$

where w_1, w_2 and w_3 are weight vectors for each objective function. In order to accelerate the optimisation process and to normalise effectively, the value of all three weight vectors were set to unity in the sequel; this also reflects the equal importance of each objective function.

4.2. VGV constraints and requirements

Certain constraints are required for the VGV reset angle due to the mechanical limitation within the compressor. These constraints prevent large variations during the optimisation process, which may exceed the capacity of the modelling routine, particularly the surge prediction routine used, Sun & Elder 1998. The surge point was evaluated at each function call to ensure the optimisation procedure can continue until convergence is met. Also, the movement of the overall characteristics was considered for the relative optimisation point. Once the flow range falls below this point, at which state the choke mass flow is less than the optimisation point mass flow, then no function objectives can be found and the optimisation will terminate immediately. Thus, linear constraints for the VGVs are the upper and lower bounds of each angle set.

In the optimisation of single or double VGV blades, it is important to find out whether the evaluation of particles are capable of reaching the peak point on the plot trajectory. Moreover, this way enables an observation of the evolution of optimised points by the PSO method and provides further possibilities to extend the number of control blades. Among the input options for the optimisation analysis, the number of particles, *N*, is determined by the speed of axial compressors and the range of numbers of VGV due to the bound on VGV reset angle. Initial particle positions are decided randomly by inputting the value of the VGV and multiplying by random numbers. For the individuality constant, it was observed by Ratnaweera *et al* 2004, that an improved optimum solution for the range of c1 is from 2.5 to 0.5 and c2 is from 0.5 to 2.5 over the full range. The constraint for the particle range varies and is relatively dependent upon the upper and lower bound of VGV angles.

5. RESULTS

The optimisation analysis was carried out at the speeds of 11085 rpm and 8000rpm. It is noted that all VGV reset angle sets were selected from the results of MOEA and Goal attainment method, utilised by White 2002, for a comparison with the results of PSO for validation purposes. The maximum iteration for all the runs was set at 1000 times respectively.

5.1. Variation with single VGV blade

For the single VGV blade variation, three different representative VGV reset angles are shown in Table 1. The dimension of all the blades is in degree. The upper and lower boundary of first blade can be varied relative to the other four blade angles. The number of particles for this case is 25 with a single variable element meaning the optimisation is one dimensional. The simulation with the first VGV angle set was run from the nominal blade setting where there is no re-stagger occurring within the blade stator. The second and third VGV angle sets were selected from the previous results of MOEA (Multi objective evolutionary algorithm) and the Goal attainment method. As shown in table 1, only small VGV reset angles are allowed at 11085 rpm due to the narrow flow range while the VGV angle degree has more freedom in flow range at 8000rpm.

Table 1. VGV angle set for single blade variation

RPM	VGVs angle	1	2	3	4	5
11805	1st set	-2 ~ 0.4	0	0	0	0
	2nd set	-3 ~ 0.2	0	-0.355	0.732	0.927
	3rd set	-3 ~ 0.4	0	0	0.106	0.029
8000	1st set	-60 ~ 20	0	0	0	0
	2nd set	-40 ~ 13	8	7.896	7.909	5.591
	3rd set	-40 ~ 14	6	8.838	6.796	4.723
	4th set	-40 ~ 14	6	9.999	6.912	2.479

All the results of PSO evolution with single VGV blade variation are shown in Fig. 3, Fig. 4 and Fig. 5. X_1 denotes the first VGV angle variation as indicated in Table 1. The plotted curves shown below indicate the normalised objective function data variation from combining, surge mass flow, efficiency and pressure ratio objectives. The normalised curves in Fig.4 indicate the non-convex nature of the optimisation resulting from the complexity of the objectives and the compressor performance data.

It is observed that initial particles are generated throughout the entire search space thereby avoiding problems associated with local minima. After the 50^{th} generation, particles have moved closer to the maximum point of each plotted curve. The maximum particle converged on the peak area of the curves after the final iteration. Thus, this validates that the optimisation procedure using PSO method can successfully locate the best VGV reset angle. However, the position of initial particles must be within the VGV angle boundary otherwise the best particle will not be updated due to the missing value from the objective function.

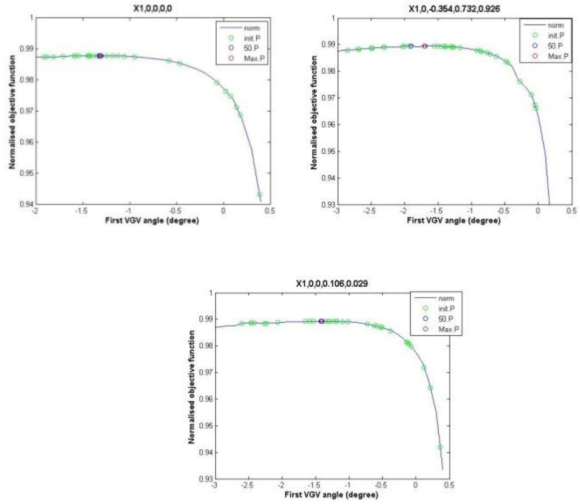


Fig.3. Single VGV variation at 11085 rpm

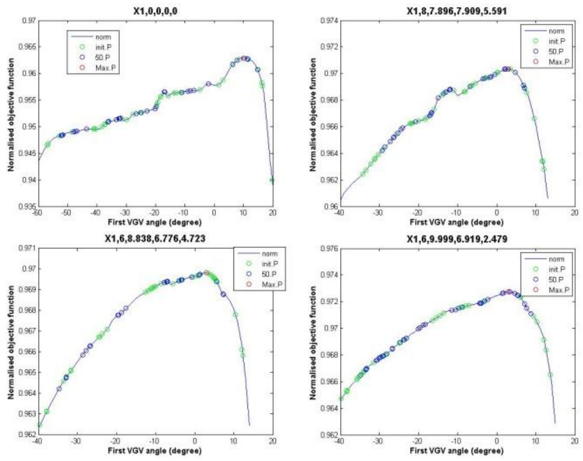


Fig. 4. Single VGV variation at 8000 rpm

5.2. Variation with two VGV blades

In a similar manner to the variation of a single row of VGV blades, the analysis was also done for VGV reset angle as given in Table 2. However, now variation of two different VGV reset angles, the first and second rows, were taken into account for this optimisation case study. Each test case at different speeds was carried out with various different initial VGV settings for the parametric experiment. Thus, an optimised point corresponds to these VGV settings located within the hyper fitness space.

Table 2. VGV angle set for double blade variation

RPM	VGVs angle	1	2	3	4	5
11805	1st set	-2~0	-2~0	-0.355	0.732	0.927
	2nd set	-2~0	-2~0	0	0.106	0.029
8000	1st set	-20~14	-20~14	0	0.439	0.19
	2nd set	-20~13	-20~13	7.896	7.909	5.591
	3rd set	-20~13	-20~13	8.838	6.796	4.723
	4th set	-20~13	-20~13	9.999	6.912	2.479

Similar to the single VGV analysis, *N* is dependent upon the speed of the axial compressor and the upper and lower bounds of VGV reset angle. Initial particle positions are chosen randomly but scaled to lie within the VGV displacement bounds.

All the results, subject to the VGV angle set in Table 2 are plotted in Fig. 5 and Fig. 6. It can be seen that particles were initially populated on the 3-D surface and evolved to the peak area of the normalised objective function surface. In order to validate the result of PSO, the normalised objective function values were used as mentioned previously. At the end of evolution, these particles finally converged on the peak point of the surface as for the results of a single blade variation. All the results of PSO showed a better performance than the result obtained by the MOEA or the Goal attainment method for both VGV angle sets.

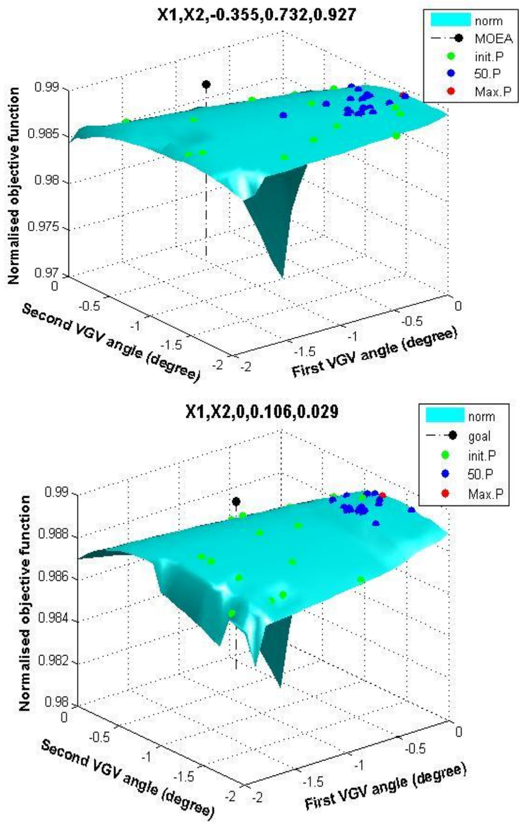


Fig. 5. 2-D VGV variation at 11085 rpm

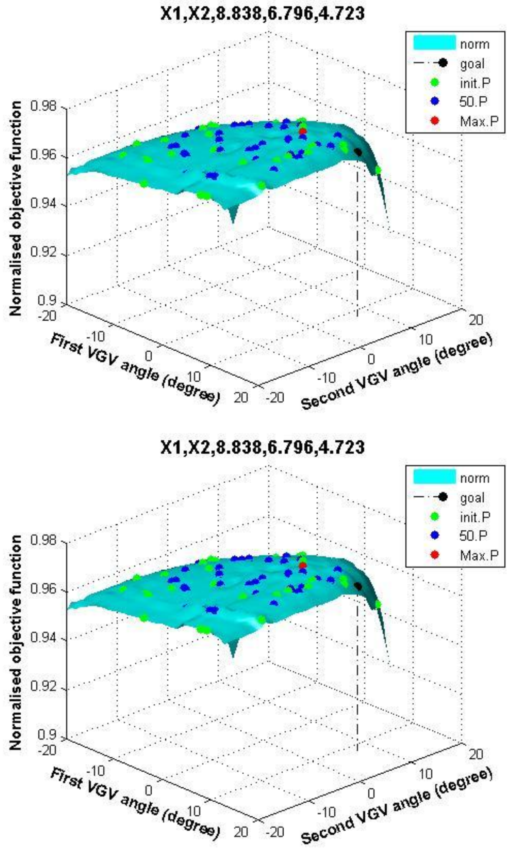
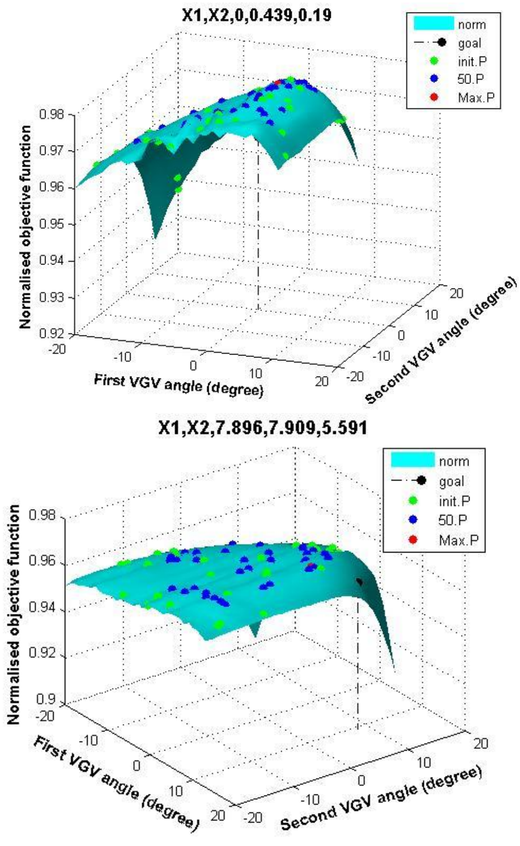


Fig. 6. 2-D VGV variation at 8000 rpm



6. CONCLUSION

An optimisation analysis for VGV reset angle in a 15 stage axial compressor using the PSO algorithm with time-varying inertia weight factor has been presented. The aim is to avoid stall and subsequent surge and to achieve better performance with acceptable convergence time to enable practical implementation. Prior to the optimisation, a numerical 1-D prediction compressor model was utilised to obtain appropriate objective functions. In order to search for an optimum VGV setting effectively, three normalised objective functions; surge mass flow rate, polytrophic efficiency and pressure ratio, were combined. To optimise the VGVs and obtain the best global value of the normalised objective function, the variation of single and double blade rows were carried out at the speeds of 11085 rpm and 8000 rpm. For the validation of the PSO results, the optimum value of MOEA or Goal attainment method, produced in an earlier study, was used as a basis for comparison. All the results of PSO showed a better optimal point within a shorter convergence time. Therefore, this optimisation tool has the potential to be effective for on-line optimisation of practical industrial multi-stage axial compressors. Moreover, due to its simplicity, it is possible for this optimisation method to be extended to enable implementation for the variation of the whole set of VGV blade rows.

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