Condition Monitoring Approaches To Estimating Wheel-Rail Profile

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Abstract: The wheel and rail interaction is the main influence on the dynamic response of a rail vehicle. Any changes in the wheel and rail will change the overall response of the vehicle. The condition monitoring challenge is to interpret these changes into useful condition information. This paper presents the results from initial feasibility studies into model-based condition monitoring at the wheel-rail interface applied to estimating the wheel-rail profile estimation.
A number of approaches are presented, based around a Kalman Filter method and least squares methods, applied to a linearised simulation model that included a nonlinear conicity function. The function was successfully estimated using a Kalman Filter that included self-updating information about the shape of the conicity function, and by a piecewise cubic least squares approach.

Keywords: Railways; Vehicle dynamics; Monitoring

1. INTRODUCTION

The most important element in the dynamics of a railway vehicle is the interaction between the wheel and the rail. Typical monitoring approaches will inspect the shape of the wheel and rail separately, however it is the interaction of the two in situ that really influences the system dynamics. Model-based condition monitoring uses knowledge of the system in the form of a mathematical model and the measured vehicle response to track irregularities to perform estimations of the system parameters of interest.

This paper presents the results from feasibility studies carried out into model-based condition monitoring applied at the wheel-rail interface. It is proposed that 'fast' changes would be due to the rail profile and 'slow' changes due to the wheel profile. To the authors' knowledge this work is the first to investigate model-based condition monitoring at the wheel-rail interface. The concept was inspired by previous work which looked into condition monitoring of suspension components (Li et al. [2006]).

A number of approaches are presented here, based around Kalman Filter methods (Kalman [1960]) and Least Squares estimation techniques (Ljung [1999]). Both of these approaches are well understood and frequently used in system parameter estimation work. Nevertheless, this is not a trivial application due to a number of issues. The track irregularities that excite the dynamics of interest are not typically measurable. Certain assumptions can normally be made to overcome this, but here the proximity of the exciting input to the wheel-rail dynamics means that the track irregularities also need to be estimated. Furthermore, the contact dynamics are complex, nonlinear, and often in practice non-symmetric in the profile geometries.

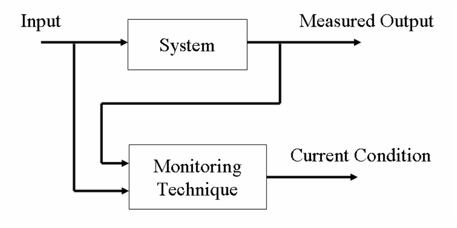
Phase one of the work is simulation based, hence the results shown here are all based on a well understood wheelset and half vehicle models implemented in Simulink. The emphasis is on evaluating the viability of this approach and establishing what useful information can be taken from condition monitoring in this application.

2. CONDITION MONITORING

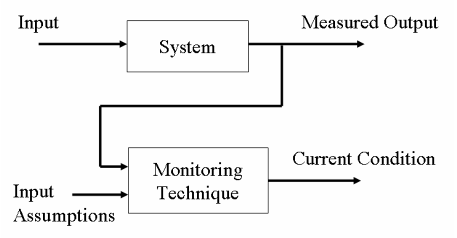
Condition monitoring aims uses some level of knowledge of the system of interest to establish it's current condition. This knowledge may be in the form of a model, expert system, experience, learnt behaviour, etc. Figure 1(a) shows the block diagram for a generic condition monitoring system. The controlled input and measured output for the system is given to the condition monitoring strategy. In this work we are using a Kalman Filter to produce an estimate of system parameters. These direct results can then be taken into further processing algorithms to establish a condition or provide fault detection.

In a railway vehicle the dynamic response of the system is driven by the irregularities in the track. This input is not the usual controlled input seen in feedback control systems, and is in practice unmeasurable. This means that the condition monitoring process has to be based entirely on the measured output. This alone is not sufficient for the Kalman Filter, however the track irregularities are close enough to the required ideal noise to allow the condition monitoring to be carried out with effectively zero control input and assuming a normal noise to excite the system dynamics (figure 1(b)).

The previous condition monitoring work mentioned in the introduction used Kalman Filter and Particle Filter approaches to estimate components in the secondary suspension (Li et al. [2006]). Data was taken from track



(a) Generic condition monitoring



(b) Railway vehicle condition monitoring

Fig. 1. Block diagrams for generic condition monitoring schemes.

tests on the Tyne and Wear metro and on the Chester to Holyhead line. A set of sensors housed in a single unit were attached to the bogie frame, and the estimated parameters converged well to the designed values.

3. RAIL VEHICLE MODEL

A linearised quarter vehicle model model is used to simulate the basic wheel-rail dynamics, consisting of a single wheelset and suspended mass (Garg and Dukkipati [1984]). The plan view dynamics (yaw and lateral displacement) are sufficient to describe the stability and guidance response to lateral track irregularities.

The system equations are given by:

$$\ddot{y} = \frac{1}{m}\left\{-\frac{2f_{22}}{v}\left(\dot{y} + \frac{r_0\lambda}{l}\dot{y} - v\psi\right) - \frac{2f_{23}}{v}\dot{\psi} - \frac{W\lambda}{l}(y-d) + F_{sy}\right\} \quad (1)$$

$$\ddot{\psi} = \frac{1}{I}\left\{-2f_{11}\left(\frac{l\lambda}{r_0}(y-d) + \frac{l^2}{v}\dot{\psi}\right) - \frac{2f_{33}}{v}\dot{\psi} - \frac{I_{wy}v\lambda}{lr_0}\dot{y} + \frac{f_{23}}{v}\left(\dot{y} + \frac{r_0\lambda}{l}\dot{y} - v\psi\right) + W\lambda l\psi + M_{s\psi}\right\} \quad (2)$$

$$\ddot{y}_m = \frac{1}{m_m}\{-F_{sy}\} \quad (3)$$

where y is the lateral position of the wheelset, y_m is the lateral position of the suspended mass, ψ is the yaw angle, W is the wheel load, d is the lateral track irregularities, and λ is the nonlinear conicity function of the relative wheel-rail position. F_{sy} and $M_{s\psi}$ are the suspension lateral force and yaw moment respectively, given by:

$$F_{sy} = k_y(y_m - y) + f_y(\dot{y}_m - \dot{y}) \quad (4)$$

$$M_{s\psi} = -k_\psi\psi - f_\psi\dot{\psi} \quad (5)$$

All other terms and their values used in this work are given in table 1.

f_{11}	longitudinal creep coefficient	7.44e6	N
f_{22}	lateral creep coefficient	6.79e6	N
f_{23}	spin creep coefficient	13.7e3	N
f_{33}	spin creep coefficient	0	N
f_y	lateral damper coefficient	50e3	Ns/m
f_ψ	yaw damper coefficient	0	Ns/rad
I	wheelset yaw inertia	700	kgm^2
l	half wheelset width	0.7452	m
m	wheelset mass	1250	kg
m_m	suspended mass	8000	kg
k_y	lateral suspension stiffness	0.23e6	N/m
k_ψ	yaw suspension stiffness	2.5e6	N/rad
r_0	rolling radius at zero lateral position	0.45	m
v	velocity	20	m/s

Table 1. Parameter values used in the wheel-rail profile estimation model.

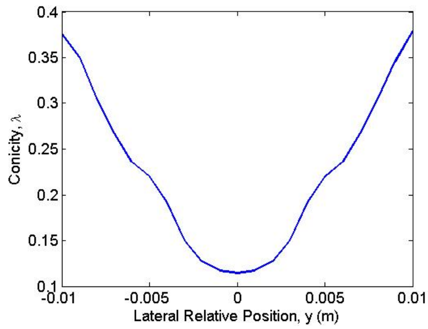


Fig. 2. Effective conicity for the single wheelset and lateral mass model vs relative lateral wheel-rail position

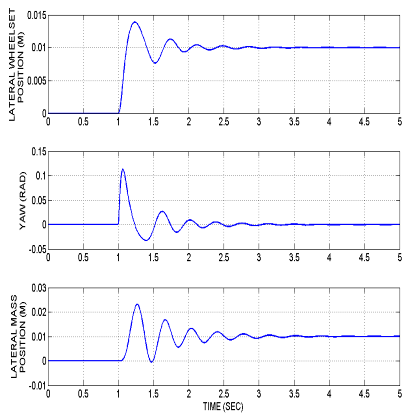


Fig. 3. Single wheelset and lateral mass model response to a 10mm lateral track step.

Conicity is a linearised term to describe the nonlinear wheel-rail contact shape. It is effectively a secant gradient of the wheel rolling radius difference, given by:

$$\lambda(y,d) = \frac{r_L - r_R}{2(y-d)} \quad (6)$$

where r_L and r_R are the rolling radius of the left and right wheels respectively. Typically it is taken as a constant for analysis, but here we have created a nonlinear function of the relative lateral wheel-rail position to include some degree of the nonlinear contact profile. A generic wheel-rail geometry has been used in this work, and the effective

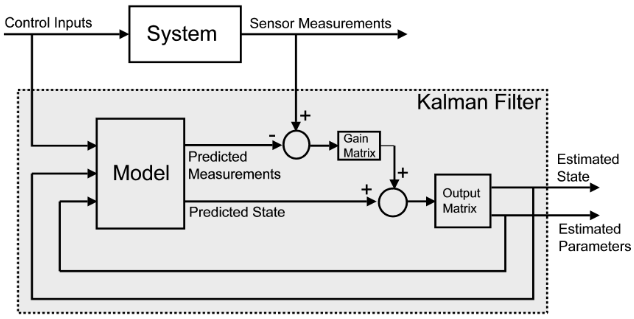


Fig. 4. Block diagram for the Kalman Filter

conicity is shown in figure 2. Figure 3 shows the response of the single wheelset to a step input.

4. KALMAN FILTER

In 1960, R.E.Kalman published his famous paper describing the recursive solution to the discrete linear filtering problem (Kalman [1960]). The Kalman Filter addresses the general problem of trying to estimate the state of a system given a set of measurements. The knowledge of the system is included in the form of a model, which has to be sufficiently detailed to include the dynamics of the parts of the system that are to be estimated.

Figure 4 shows a block diagram of the Kalman Filter. Here it can be seen how the mathematical model is used to predict the system state and the sensor inputs from the measured control inputs. The gain matrix is calculated based on the statistical covariance of the system process noise and measurement noise in order to correct the model predictions. The Kalman Filter can also be used to estimate parameters within the system itself by extending the estimated state to include the unknown parameters, however this moves the problem from a linear to a nonlinear one. The Extended Kalman Filter (EKF) accommodates the nonlinear problem (Grewal and Andrews [1993], Welch and Bishop [1995]).

4.1 Nonlinear Conicity Estimation

The Kalman Filter estimation model uses the same linearised single wheelset and lateral mass model as given in the modelling section, with the addition of an extra estimation state given by:

$$\dot{\lambda} = 0 \tag{7}$$

Effectively the Kalman Filter is asked to consider the conicity as an additional state to estimate. The common assumption is to assume that the parameter is a constant, i.e. the derivative w.r.t. time is zero. Figure 5 shows the response of the Kalman Filter to estimating the nonlinear conicity term. The Kalman Filter does not converge and has a widely varying estimation of the conicity. This is because the single wheelset model is driven by unknown lateral track irregularities. The conicity is a function of the wheel to track displacement rather than the absolute wheelset displacement, so the basic assumption that the track irregularity can be treated as a statistical noise input is not sufficient. Here we need to know the track irregularity in order to establish the lateral relative wheel-track position.

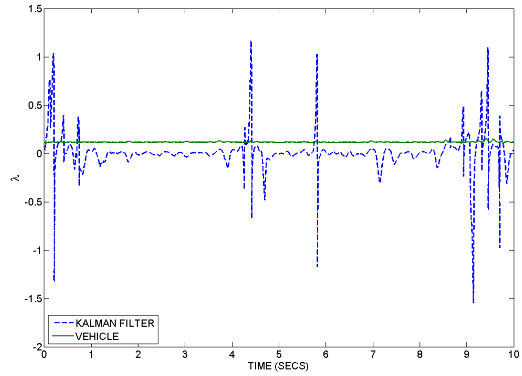


Fig. 5. Estimation of nonlinear conicity function using the Kalman Filter estimation states given by 7.

To aid the Kalman Filter, the lateral track irregularity, d, can be estimated alongside the conicity, λ. As before, the standard linearised mathematical model for the wheelset and lateral mass is given to the Kalman Filter, but now the additional parameter states are given as:

$$\dot{\lambda} = 0 \tag{8}$$
$$\dot{d} = 0 \tag{9}$$

Figure 6 shows the Kalman Filter response to estimating both the lateral track irregularity and the nonlinear conicity function. The estimation tracks the parameters well, but there is some uncertainty, particularly at the lower conicity values. This corresponds to the points where the excitation by the track irregularities is relatively low and less energy is being put into the system. Therefore there is less information for the Kalman Filter to use in its estimation of the conicity. The performance of the Kalman Filter is largely set by tuning the noise covariance. The estimation model implies that the expected conicity term is a constant as the derivative is set to zero (equation 8). This is typically true in Kalman filter applications, or at worst a slowly varying parameter. Here however, conicity is varying rapidly as the relative wheel-rail displacement changes with the track irregularities.

The estimation of the conicity function can be improved by including some knowledge of the conicity function and it's variation with the relative track-wheel position. The conicity is assumed to be a function of the relative lateral wheel-rail position, allowing us to add the following states to the Kalman Filter:

$$\dot{\lambda} = \frac{d\lambda}{dy}\dot{y} \tag{10}$$
$$\dot{d} = 0 \tag{11}$$

Practically, the Kalman Filter stored a look up table corresponding to the variation of the estimated conicity with lateral wheel-rail position. As the wheelset moved laterally, the Kalman Filter updated its conicity values in the table. This table could then be used to give the Kalman Filter some knowledge of the variability of the conicity.

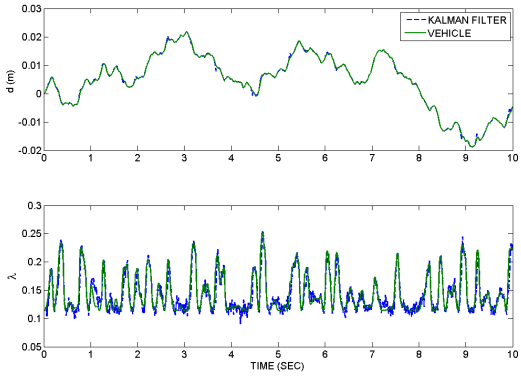


Fig. 6. Estimation of nonlinear conicity function using the Kalman Filter estimation states given by 8 and 9.

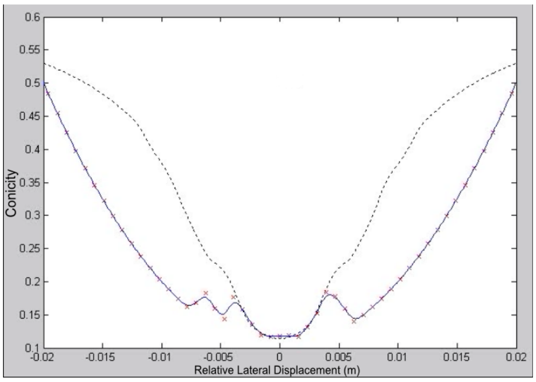


Fig. 7. Estimation of nonlinear conicity function using the Kalman Filter estimation states given by 10 and 11. The dotted line is the function used in the simulation. The crosses are the Kalman Filter stored values, and the solid line is the spline interpolated curve between the stored values.

Figure 7 illustrates the updating nature of the conicity function estimation. The track irregularities estimation is still being used to provide the Kalman Filter with the knowledge of where on the look up table the system is operating. As the track irregularities drive the wheelset across the rail, the Kalman Filter updates the function. In practice this is achieved by updating a series of spline nodes to make up the curve (the crosses) so that the derivative of the function remains smooth.

5. LEAST-SQUARES PARAMETER ESTIMATION

A complete background on the least squares estimation and regression methods can be found in Aström [1989] or Ljung [1999]. The basis of the Least Squares method is to optimise the set of system parameters in such a way as to minimise the error between the measured output of the system and the regression model of the system given by:

$$\hat{y}(i) = \phi_1(i)\theta_1 + \phi_2(i)\theta_2 + ... + \phi_n(i)\theta_n \quad (12)$$
$$= \phi^T(i)\theta \quad (13)$$

where $\hat{y}$ is the estimated observed variable, ϕ is the vector of regressors, θ is the vector of unknown parameters, and i is the discrete sample number $(1 \leq i \leq t)$. Nonlinear systems may be formed into this linear regression structure by using nonlinear regressors in the model.

The problem is to optimise the parameter vector, θ, in such a way that the output from the regression model given in 12 will agree as closely as possible over the sample set of observations and regressors, $\{y(i), \phi(i)\}$. The error at sample i is given by

$$\epsilon(i) = y(i) - \hat{y}(i) = y(i) - \phi^T(i)\theta \quad (14)$$

Introducing the notation for the complete sample set

$$Y(t) = [y(1)\, y(2)\, ...\, y(t)]^T \quad (15)$$
$$E(t) = [\epsilon(1)\, \epsilon(2)\, ...\, \epsilon(t)]^T \quad (16)$$
$$\Phi(t) = [\phi^T(1)\, \phi^T(2)\, ...\, \phi^T(t)]^T \quad (17)$$

allows the error to be written as:

$$E = Y - \hat{Y} = Y - \Phi\theta \quad (18)$$

The least-squares error can then be written as

$$V(\theta, t) = \frac{1}{2}\sum_{i=1}^{t}\epsilon(i)^2 = \frac{1}{2}\sum_{i=1}^{t}(y(i) - \phi^T(i)\theta)^2 \quad (19)$$
$$= \frac{1}{2}E^T E \quad = \frac{1}{2}||E||^2 \quad (20)$$

The least-squares error is minimal for parameters $\hat{\theta}$ such that

$$\Phi^T\Phi\hat{\theta} = \Phi^T Y \quad (21)$$

If the matrix $\Phi^T\Phi$ is non-singular, the minimum is unique and given by:

$$\hat{\theta} = (\Phi^T\Phi)^{-1}\Phi^T Y \quad (22)$$

5.1 Known Terms in the Regression Model

The regression model in 12 is shown to be made up of a linear combination of unknown parameters and known regressors. In many cases some of these parameters will be known, and it would be more convenient if only the unknown parameters are estimated. In the railway vehicle application presented in this paper this is the case. Better results are obtained if the least squares method estimates the nonlinear function alone, rather than all the parameters in the regression model.

A new regression model is defined as:

$$\hat{y}(i) = \phi(i)\theta + \omega(i) \quad (23)$$

where ω are the combined known parameter and regressor terms. The complete set of these known terms can be now be defined as

$$\Omega = [\omega(1)\omega(2)... \quad \omega(t)]^T \quad (24)$$

Now the same least squares error, $V(\theta, t)$, is minimal with the parameters $\hat{\theta}$ such that

$$\Phi^T(\Phi\hat{\theta} + \Omega) = \Phi^T Y \quad (25)$$

If the matrix $\Phi^T\Phi$ is non-singular, the minimum is unique and given by:

$$\hat{\theta} = (\Phi^T\Phi)^{-1}(\Phi^T Y - \Phi^T\Omega) \quad (26)$$

5.2 Estimating Complex Nonlinear Functions

Typically any nonlinear function that is to be estimated by this method needs to be made up of an analytical function, for example, a cubic function of one of the regressors. This is shown to give large errors in the application presented in this paper. The nonlinear function is too complex to be estimated with a simple function. One approach is to use higher order equations to match the nonlinear function, but this adds complexity and can give false results. An alternative approach is to use a piecewise cubic function which fits the data in a least squares sense over each piece of the function whilst preserving the continuity of function and gradient between each piece of the function. The process is briefly described here, however full details can be found in Ichida et al. [1976].

The piecewise cubic function is given by:

$$S_i(x) = m_i a_i(x) + m_{i+1} b_i(x) + y_i c_i(x) + y_{i+1} d_i(x) \quad (27)$$
$$a_i(x) = (x_{i+1} - x)^2 (x - x_i)/h_i^2 \quad (28)$$
$$b_i(x) = -(x_{i+1} - x)(x - x_i)^2/h_i^2 \quad (29)$$
$$c_i(x) = (x_{i+1} - x)^2 \left\{2(x - x_i) + h_i\right\}/h_i^3 \quad (30)$$
$$d_i(x) = (x - x_i)^2 \left\{2(x_{i+1} - x) + h_i\right\}/h_i^3 \quad (31)$$
$$h_i = x_{i+1} - xi \quad (32)$$

where $S_i(x)$ is the cubic function for piece i, (x_i, y_i) is the coordinates of knot point i, m_i is the gradient at knot point i.

Let the least and largest x data in the interval $[x_i, x_{i+1})$ be x_{pi} and x_{qi}, the the problem becomes one of minimising the sum square of the errors for the n sections in the piecewise function:

$$E = \sum_{i=1}^{n} \left[\sum_{k=pi}^{qi} \left\{S_i(x_k) - f_k\right\}^2 \right] \quad (33)$$

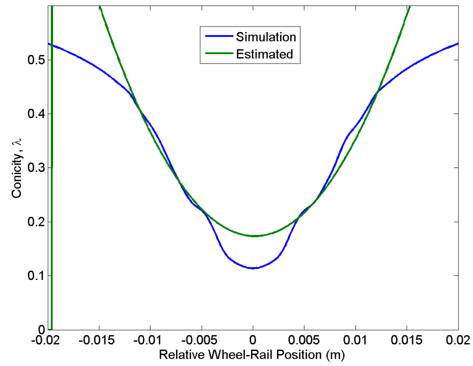
where f_k is the data for the function to be estimated. Differentiating E w.r.t. the cubic function parameters and setting to zero reduces the minimisation problem to the solution of the following relationship:

$$Az = g \quad (34)$$

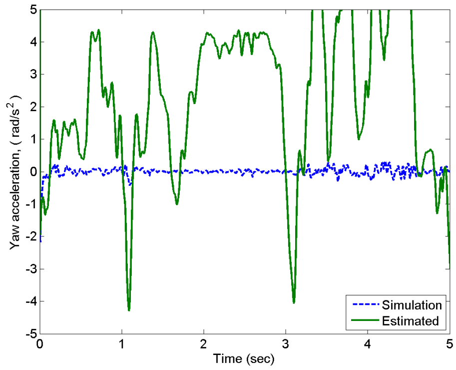
where z is a vector of the cubic function parameters, A is a matrix containing sum terms of x_i, x_k and g is a vector containing sum terms of x_i, x_k, f_k. Details on the specific structure of these matrices can be found in Ichida et al. [1976].

5.3 Nonlinear Conicity Estimation

The first step in applying the least squares estimation method is to determine the regression model and regressors. The observed variable taken here is $\ddot{\psi}$, and the assumption that all the state and system parameters other than that being estimated are available to construct the regressors. Track irregularities are also assumed to be known at this point in the work. As in all model based condition monitoring approaches, it is necessary to select an estimation model that is sufficiently complex to account for all the dynamics of interest. Here, a reduced model is sufficient to capture these dynamics, given by:



(a) Estimated conicity function



(b) Regression model response

Fig. 8. Cubic least squares estimation.

$$\ddot{\psi} = \frac{1}{I} \left\{ -\frac{2 f_{11} l \lambda}{r_0}(y - d) - \frac{2 l^2 f_{11}}{v}\dot{\psi} - k_\psi \psi - f_\psi \dot{\psi} \right\} \quad (35)$$

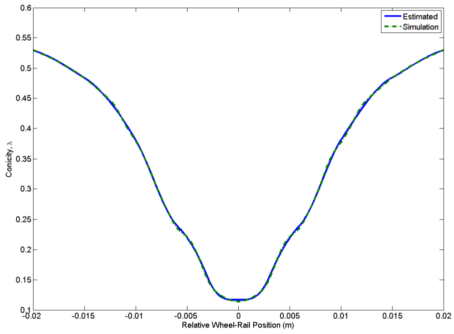
This can be converted into a regression sense by considering the conicity function to be estimated using a cubic function of the relative wheel-rail position:

$$\begin{aligned} \ddot{\psi} &= A_1 \lambda (y - d) + A_2 \psi + A_3 \dot{\psi} && (36)\\ &= A_1 (a_1 (y - d)^2 + a_2 (y - d) + a_3)(y - d) \\ &\quad + A_2 \psi + A_3 \dot{\psi} && (37)\\ &= B_1 (y - d)^3 + B_2 (y - d)^2 + B_3 (y - d) \\ &\quad + A2\psi + A_3 \dot{\psi} && (38) \end{aligned}$$

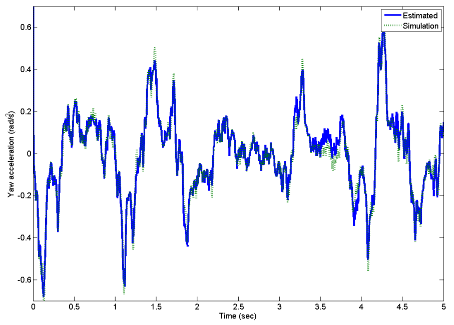
where A_1,A_2 and A_3 are known constants, a_1,a_2 and a_3 are the cubic function coefficients, B_1,B_2 are B_3 are the regression parameters. Hence the vector of regressors, vector of unknown parameters and known terms are given by:

$$\phi = [(y - d)^3 \quad (y - d)^2 \quad (y - d)]^T \quad (39)$$
$$\theta = [B_1 \quad B_2 \quad B_3]^T \quad (40)$$
$$\omega = A_2 \psi + A_3 \dot{\psi} \quad (41)$$

Figure 8(a) shows the results from applying this cubic best fit to the railway vehicle simulation. The correlation is reasonable, and gives a rough approximation for monitoring



(a) Estimated conicity function



(b) Regression model response

Fig. 9. Piecewise cubic least squares estimation.

the condition of the wheel-rail geometry. However, this is not accurate enough to gain a complete insight into the wheel-rail contact conditions. Notice that, although the simulation conicity is symmetric about zero relative wheel-rail position, the estimated curve is biased slightly to one side. This is a result of the sampled data being not evenly distributed. The simulation model is driven by track irregularities, which result in the contact patch varying in position across the wheel and rail. Figure 8(b) shows that the regression model output given the same track irregularities is very different from the rail vehicle simulation. This is a result of the large effect that the wheel-rail contact shape and condition have upon the vehicle dynamics. Even slight differences in the estimated conicity function from the 'real' simulation conicity causes large errors in the model response.

The complexity in the conicity function shape means that a single cubic fit is not good enough. The application of the piecewise cubic function aims to allow more flexibility to making the least squares fit of the data. Effectively a cubic function is fitted to each section of the conicity data, between designated knot points. Figure 9(a) shows the result of this approach and it can be seen that the correlation is very good to the simulation conicity function. Furthermore, figure 9(b) shows that the dynamic response of the regression model matches well with the simulation response, in spite of the simplified model used in the estimation process.

6. FURTHER APPROACHES

Currently work is progressing on approaches to combine the benefits of least square approaches and the Kalman Filter approaches to form a hybrid estimation approach that uses a Kalman Filter to estimate the track irregularities and the Least Mean Squares to estimate the conicity function. The authors also hope to assess further approaches to estimating the wheel-rail profile, including the application of multiple model Kalman Filters, Particle Filters. Further results are expected to be produced in time for the final submission date.

7. CONCLUSIONS

This paper presents some initial work in the feasibility of model-based condition monitoring applied to estimating a nonlinear conicity function as an indicator of the wheel-rail profile shape. Initial results obtained using Kalman Filter and Least Mean Square approaches are encouraging. The best results were obtained using the Kalman Filter by included self updated information about the conicity function. The best results were obtained for the Least Mean Squares approach by using a piecewise cubic function.

Further development and investigation work is being carried out into both these approaches and alternatives such as Particle Filters or multiple model Kalman Filters. Testing will continue in simulation on these and more complex models, and it is hoped that the assessment will develop to use data from track test measurements.

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