A reduced structure controller for a Grinder Circuit system

Amin Nobakhti, Hong Wang

Control Systems Center, The University of Manchester, Manchester, M60 1QD, UK (e-mail: a.nobakhti@manchester.ac.uk)

Abstract: In this paper a reduced structure multivariable controller is developed for the model of a closed-system grinding circuit. The said controller is developed using a novel technique which employs basis pursuit regularization in order to generate a family of solutions which together span the entire range from decentralized to centralized controllers. Using this information, and the performance-cost trade off which is also computed, the designer is then able to choose the required amount of controller complexity which can achieve the desired closed-loop performance levels.

1. INTRODUCTION

In mineral processing industry, a grinding circuit (GC) is used to liberate the valuable minerals from the discardable gangue so as to help the subsequent beneficiation process Zhou et al. [2006].The most important quality index of GC is the grinding product particle size (GPPS), which is usually characterized by the particle filtration under 200 mesh. The quality of the GPPS will thus directly affect the efficiency of the subsequent beneficiation process in terms of product concentrate grade and metal recovery rate. Therefore, it is important to effectively control the GC in order to provide an optimal GPPS for downstream operation. This is also the precondition for improvement of production rate and optimal utilization of energy Zhou et al. [2006].

Such systems, as is common to the industry, are often controlled with decentralized SISO low-level controllers which are responsible for control of individual control variables, ignoring the inherent interactions of the system which arise from recycle streams, or closed circuit system as in the case of the GC. Whilst it is known that centralized higher level controllers can lead to marked improvements in the closed-loop performance, very few applications of this have appeared in the industry. This is generally the result of several factors, but experts point out to the wholly unstructured approach of modern multivariable control as one of the main hurdles Skogestad [2004]. Specifically, while much of the research has been focused towards pushing the performance optimality envelope, only a very insignificant amount of research has been dedicated to the study of control structure for multivariable systems with the consequence being that multivariable controllers are often either performing much worse than easily attainable, or are much more complex than required. This sharp divide becomes more important when one considers that often, the majority of the performance gains of a centralized controller over its decentralized counterpart, can be attributed to only a few loops Nobakhti et al. [2007b]. In other words such 'sparse' controllers are able to offer performances close to the fully centralised controllers, but with a reduced structure.

The Reduced Structure was initially proposed in Nobakhti and Munro [2003]. In Nobakhti and Wang [2006a] and Nobakhti et al. [2007b] MINLP solutions for the problem were developed and in Nobakhti et al. [2007a] the problem was formulated as a Basis Pursuit Regularisation Brown and Costen [2005], Efron et al. [2004] which is a quadratic and convex optimization problem and thus substantially easier to solve. In this paper the methodology is used to developed a Reduced Structure controller for the GC model.

2. BRIEF OUTLINE OF REDUCED STRUCTURE CONTROL

Let $G(s) \in \mathcal{R}_p^{m\times m}$ be the transfer function matrix of a given multivariable plant, where $\mathcal{R}_p^{m\times m}$ is the set of $m \times m$ rational proper linear time invariant transfer function matrices. Let $K(s) \in \mathcal{R}_p^{m\times m}$ be a controller to be designed for this plant so as to meet a set of performance criteria. Two properties are of interest about the pair (G, K), namely, the performance of the controller acting on the system, and the structure of $K(s)$. These two properties are linked together through a natural comprise, whereby a larger (more complex) structure is expected to yield a smaller (better) performance. Let the performance of the (G, K) pair be denoted by $\Gamma(G, K)$. For any chosen performance index, Γ can be chosen to be a mapping onto the positive real space $\mathbf{R}^+$. i.e., $\Gamma(G, K) : K(s) \rightarrowtail \mathbf{R}^+$ and it is a measure of how well the controller performs in reference to the chosen performance index. This can be any measure specified by the designer, such as the H_∞ norm of some weighted measurement, the Integral Squared Error (ISE) of some error, or indeed any other measure. The structure of $K(s)$ will be denoted by $\Upsilon(K)$ which is a mapping onto the positive integer space $\mathbf{I}^+$. i.e. $\Upsilon(K) : K(s) \rightarrowtail I^+$ and it is the number of the input-output channels of the controller $K(s)$. In the context of the Reduced Structure control, $\Upsilon(K)$ will be referred to as the 'rank' of K. $\Gamma(G, K)$ is a non-surjective and non-injective morphism. However, for $z_i \in \mathbf{I}^+$, $n \leq z_i \leq n^2$, $\Upsilon(K) : K(s) \rightarrowtail z_i$ is a non-injective surjection. Most importantly this implies that topological information (i.e. 'which' input-output channel) is not captured by Υ, i.e.,

$$\Upsilon\left(\begin{bmatrix} PI & 0 \\ 0 & PI \end{bmatrix}\right) = \Upsilon\left(\begin{bmatrix} PI & PI \\ 0 & 0 \end{bmatrix}\right). \quad (1)$$

Most traditional controller design methods (e.g., H_∞, Diagonal Dominance and LQR, etc.) may be explained in terms of the generalised form of $\Gamma(G, K)$ and $\Upsilon(K)$. In each technique a given structure is assigned to the controller, and the controller $K(s)$ is designed to meet the chosen performance criteria. i.e.,

$$K(s) = \underset{\tilde{K}(s)\in\mathcal{S}, \Theta(\tilde{K})=\mathcal{T}}{\arg\min} \Gamma(G, \tilde{K}), \quad (2)$$

$$s.t. \quad R(G, \tilde{K}) \leq 1. \quad (3)$$

where $R(G, K)$ is a function which incorporates any linear (or non-linear) constraints that must be satisfied, $\mathcal{S}$ is the set of all stabilising controllers for plant $G(s)$, and Θ is a connectivity matrix such that $\theta_{ij} = 1$ if $k_{ij}(s) \neq 0$. Therefore, the search is performed over all the controllers which have the same rank *and* the same topology of cross-coupling channels. In the Reduced Structure control approach, the design problem is released from this constraint and the problem is defined as that of finding the lowest rank controller which delivers some minimum level of performance,

$$K_r(s) = \underset{\tilde{K}(s)\in\mathcal{S},\ \theta_{ij}}{\arg\min} f\left(\Upsilon(\tilde{K}), \Gamma(G, \tilde{K})\right) \quad (4)$$

$$s.t. \quad R(G, \tilde{K}) \leq 1$$

$$\Gamma(G, \tilde{K}) \leq \beta$$

where $K_r(s) \in \mathcal{R}^{n\times n}(s)$ and $k_{r_{ij}}(s)$ is the dynamic connection from the j^{th} input to the i^{th} output of the controller. f is an objective function to be determined which is related to the controller performance and structure and β (with $\beta > \min \Gamma(G, K)$, $\forall K \in \mathcal{S}$) resembles the amount of desired performance. This resembles a more realistic approach to controller design because both the controller and its structure are subject to consideration.

It has been shown elsewhere Nobakhti and Wang [2006b] that to find the Reduced Structure controller as described by (4), the form of $f(.)$ is a weighed constrained sum. However, it is fairly trivial to see that regardless of the form of $f(.)$, the problem will be a MINLP. In Nobakhti et al. [2007a] it has been shown that it is possible to relax the above problem into a convex basis pursuit regularization problem. The Basis pursuit regularisation problem is then not only much easier to solve, but instead of providing one single solution, will generate a set of solution for different controller ranks. In this paper this novel approach to controller structure selection is utilized to develop a controller for a particle grinding circuit.

3. OVERVIEW OF THE SPARSE PARAMETER LOCUS AND BASIS PURSUIT REGULARISATION

3.1 Basis pursuit regularisation (BPR)

Basis pursuit regularisation (BPR) Brown and Costen [2005],Efron et al. [2004] involves minimising a function which involves both a performance and a complexity component. The performance component is a sum squared residual term, which is quadratic, and the complexity term is a 1-norm of the parameters. Therefore, the BPR optimisation function can be expressed as,

$$f(\theta, \rho) = \frac{1}{2}\theta^T H\theta + \theta^T g + \rho||\theta||_1 \quad (5)$$

where θ is the n-dimensional parameter vector, ρ is the non-negative weighting coefficient which reflects the relative importance of the performance and complexity components, H is a positive definite $n \times n$ matrix and g is the n-dimensional vector of the first order linear terms. The BPR optimisation function is a piecewise quadratic function of θ. As the weighing coefficient is varied, the optimal parameter values trace out a continuous path which is known as the sparse parameter locus $\theta(\rho)$. By examining the form of the sparse parameter locus, it is possible to obtain a useful insight into the stability of the sparse model design process. It is also possible to see which parameters are conditionally independent from other parameters (their value remains approximately constant when new parameters enter the model), and it is possible to determine which parameters are conditionally dependent on others (their value changes substantially, possibly going to zero or even reversing their sign, when new parameters enter the model). Otherwise known as the Simpson's paradox, such inter-parameter interactions can lead to incorrect conclusions if one only considers local information. For example in Figure 1 it can be seen that whilst θ_2 is the first parameter to be activated, the introduction of θ_1 causes θ_2 to initially go back to zero and then reverse sign. The parameter locus can provide the designer with information about how stable/sensitive the selected model is to small changes in the model complexity.

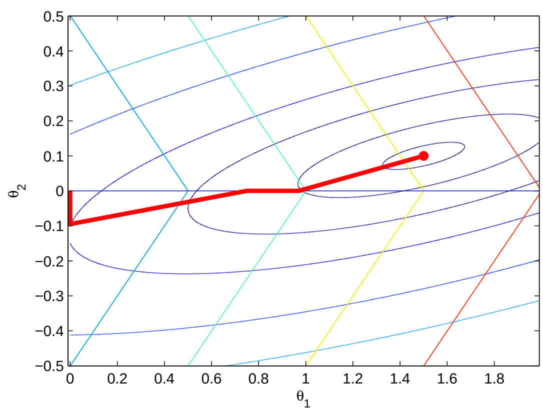

Fig. 1. An illustration of the piecewise linear optimal parameter locus,$\theta(\rho)$, as the regularization parameter ρ is altered from ∞ to 0. The contours of the quadratic loss function are also shown.

4. FORMULATING THE REDUCED STRUCTURE CONTROL AS A BASIS PURSUIT REGULARISATION

The main developmental work to develop the Reduced Structure control as a basis pursuit regularisation has been reported in Nobakhti et al. [2007a]. Here for the sake of completeness, only the final results are reported. As reported in the said reference, the BPR formulation of the Reduced Structure Control is as follows;

$$K = \arg\min \frac{1}{2}\tilde{\mathcal{K}}_p \tilde{H}_{22} \tilde{\mathcal{K}}_p^T + \tilde{K}_p \tilde{H}_{12}^T Y^T + \rho \left|\left|\tilde{\mathcal{K}}_p\right|\right|_1 \quad (6)$$

where,

$$\tilde{G} = G(s) \otimes I^{m \times m} \quad (7)$$

$$\tilde{K}_p = \{K_{p_1}^T, \ldots, K_{p_m}^T\} \quad (8)$$

$$= \{1, \tilde{k}_{p_1}, \ldots \tilde{k}_{p_{\mathcal{N}_1 - 1}}, 1, \tilde{k}_{p_{\mathcal{N}_1 + 1}}, \ldots,$$
$$1, \tilde{k}_{p_{\mathcal{N}_2 - 1}}, 1, \ldots, \tilde{k}_{p_{\mathcal{N}_m - 1}}, 1\} \quad (9)$$

$$\tilde{I}_{ij} = \begin{cases} 0, & \text{for } \; i \neq j, i \in \mathcal{N}, \\ 1, & \text{else.} \end{cases}$$

where K_{p_i} is the i^{th} column of the precompensator K_p, and,

$$\mathcal{N} = \{z_i \mid z_i \leq m^2, ... \\ z_i = (i-1)(m+2) - (i-2), i = 1, \ldots, m\}. \quad (10)$$

$$H = \left[\tilde{G}(j\omega_1), \ldots, \tilde{G}(j\omega_N)\right] \tilde{I} \begin{bmatrix} \tilde{G}(j\omega_1)^H \\ \vdots \\ \tilde{G}(j\omega_N)^H \end{bmatrix}. \quad (11)$$

The matrix H is partitioned according to the dimensions of a unitary permutation matrix $\mathcal{U} \in \mathbf{R}^{m \times m}$ such that,

$$\tilde{K}_p \mathcal{U} = \{Y, \tilde{\mathcal{K}}_p\} \quad (12)$$

where $Y \in \mathbf{R}^m$ is a vector of 1s. Since $\mathcal{U}$ is unitary, then,

$$\tilde{K}_p H \tilde{K}_p^T = \tilde{K}_p \mathcal{U}\mathcal{U}^T H \mathcal{U}\mathcal{U}^T \tilde{K}_p^T = \left(Y, \tilde{\mathcal{K}}_p\right) \tilde{H} \begin{pmatrix} Y^T \\ \tilde{\mathcal{K}}_p^T \end{pmatrix} \quad (13)$$

where $\tilde{H} = \mathcal{U}^T H \mathcal{U}$. The matrix $\tilde{H}$ can then be partitioned according to the dimensions of Y and $\tilde{\mathcal{K}}_p$ to give,

$$\tilde{H} = \begin{pmatrix} \tilde{H}_{11} & \tilde{H}_{12} \\ \tilde{H}_{12}^T & \tilde{H}_{22} \end{pmatrix} \quad (14)$$

5. APPLICATION TO AN ORE GRINDING SYSTEM

The Closed-circuit (GC) consisting of a ball mill and a spiral classifier is widely used in most concentration plants in China. A typical configuration of such a system is shown in Figure (2). Fresh ore from an ore bin is fed onto the conveyer belt by a vibratory feeder at a certain speed, and then is conveyed into the ball mill inlet, together with certain amount of water flow (called mill water). The knocking and tumbling action of iron balls within the revolving mill crush the ore inside to fine particles. Flow of the mixed ore slurry is discharged from the mill, diluted by the dilution water, and then flowed into the spiral classifier, which is used to separate the fine and coarse particles. The fine particles in the slurry, which is the final product of this grinding procedure, is overflowed across the overflow weir and then transported to the next procedure, while the coarse particles (called recycle) are enriched in the bottom and then fed back into the mill for regrinding.

Table (1) lists the inputs and outputs of the Grinder Circuit with their units.

	Output	Units
1	Flux (accept rate) from classifier	(m^3/h)
2	Consistency of flux from the classifier	(%)
3	Maximum particle size in accepts	
	Inputs	Units
1	Ore feed rate to grinder	(T/h)
2	Water feed to grinder	(m^3/h)
3	Water feed to classifier	(m^3/h)

Table 1. Grinder-circuit inputs and outputs

5.1 *Model prepration*

A multivariable linearized model of the GC has been derived from real runtime plant data. Before a reduces structure controller may be developed for this system the inputs and the outputs of the plant need to be paired and scaled. Scaling is an important feature of multivariable controller design because without proper consideration to the relative units of measurements, false interaction information could be assumed.

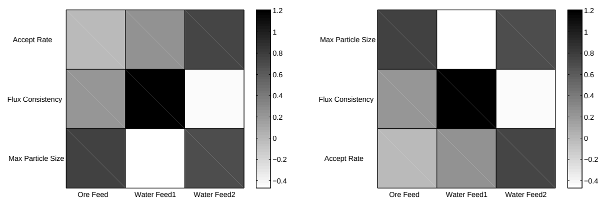

Fig. 3. Showing the RGA of the original (left) and re-ordered system (right)

Figure (3) shows the RGA matrix for the original system with the inputs and outputs indicated. Note that the RGA matrix agrees well with our physical understanding of the system. Water Feed 1 which is the grinder dilution water has the highest gain with slurry consistency from the grinder. Ore fee on the other hand has little influence on the final accept from the classifier. Water Feed 2 which is the dilution water for the classifier has - as expected- a large gain with the accept rate. The RGA matrix suggest that the inputs and outputs are not correctly paired. An optimal pairing can be archived by simply reordering the outputs in the reverse order. The RGA for the newly ordered system is also shown in Figure (3).

With the correct pairing, the system's inputs are then scaled to normalize the outputs of the system. Note that input scaling is carried out for numerical robustness and is an operation which effect the signals entering the system, unlike output scaling (which is a point of contention) cannot be used to hide system interactions. The correctly paired and scaled system in then used for the basis pursuit analysis presented in the next section.

5.2 *Results Basis Pursuit Regularization analysis*

The Parameter root loci The primary tool used for analysis purposes is the "parameter locus", which shows how the gain of each controller connection evolves as the

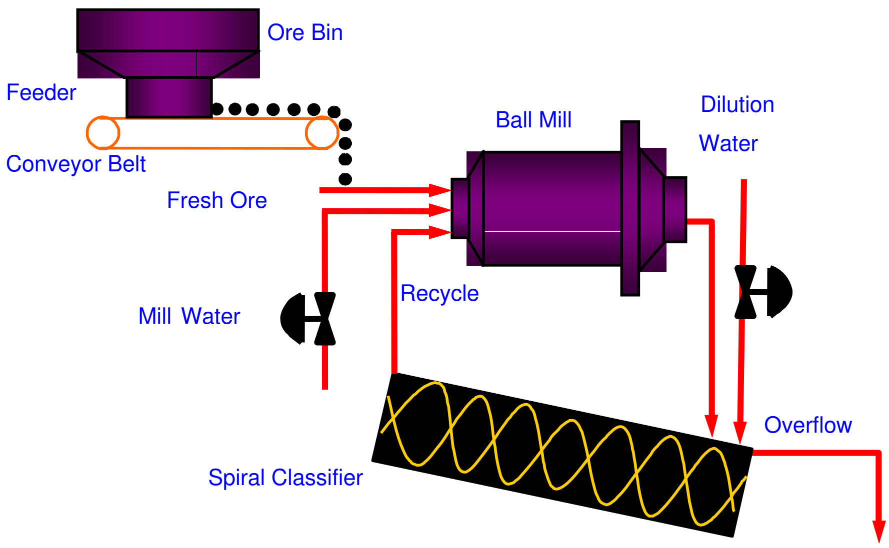

Fig. 2. Schematic of the Grinder Circuit

regularisation parameter is reduced from infinity to zero. When the regularisation parameter is set to zero, this indicates that the model is required to be as accurate as possible, regardless of the resulting complexity. However, when the regularisation parameter is set to infinity, this indicates that the model is required to be as simple as possible, regardless of the deterioration in performance. Therefore, the parameter locus of each parameter starts at zero and ends at some non-zero value for $\rho = 0$. The entire parameter root loci is shown in Figure 4. Figure (5) shows how according to the parameter locus, the connections evolve from the decentralized to the centralized structure as the regularisation parameter is reduced. At each iteration the optimal next best loop is added to the connectivity structure.

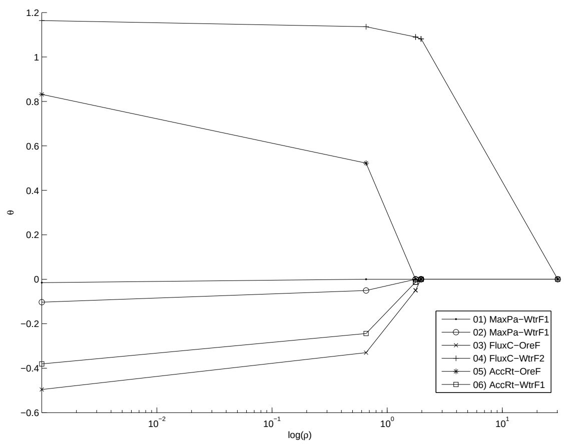

Fig. 4. The entire parameter locus for the GC

Cost performance trade off The parameter loci gives a detailed picture of the evolution of the controller structure as it evolves from a basic decentralized controller to a fully centralized architecture. It is however sometimes also useful to have a more macroscopic view of the regularisation which is especially useful to identify general areas of

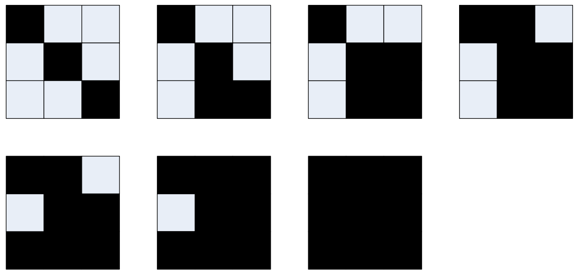

Fig. 5. Showing hoe the connectivity structure evolves from decentralized to centralized

desirability of controller complexity. An important piece of information in this regard is the tradeoff in performance with the overall activation of controller connections. For the GC, this plot is shown in Figure 6. This figure indicates that the the first two connection activations alone are responsible for over 75% of the performance increase of a centralized controller as opposed to a decentralized controller. This is significant because it means that one may have a reduced structure controller with a performance which is comparable to a centralized version.

Connection correlation analysis Two loci with strongly similar or opposite traces might indicate two control loops which are working together, or are fighting each other (i.e. working to cancel each others effect). Similarly, a group of loci with similar characteristics might indicate a strongly coupled subsystem. Note that in the physical sense, a subsystem will not necessarily be local and it may bring together elements from across the plant. The ability to extract a linked sub-systems which may not be a physical sub-system is especially important in plantwide studies of large processes where a particular initial process is interacting with a process much further down the line. Figure 7 displays the correlation coefficient matrix of the de trended parameter locus. Since correlation to a single number is not possible, the figure does not include the last loop which is activated. To eliminate the effect of the regularisation parameter from the loci, the active portion

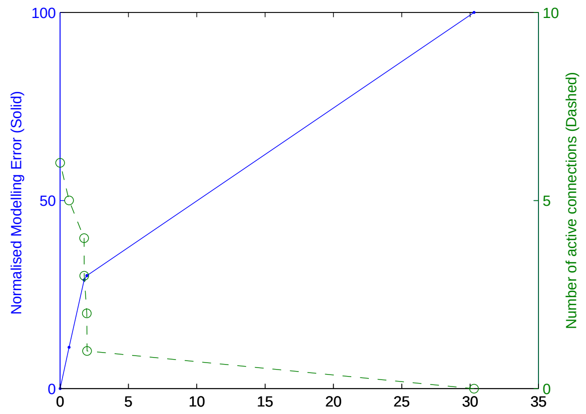

Fig. 6. Rate of connection activation versus the resulting reduction in the residual error.

of each loci has been de-trended. The correlation data have been clustered using the K-Nearest Neighbor method. The results clearly show blocks of connections with strong correlation and also those with strong anti-correlation. It is interesting to note that two loops, namely slurry consistency to classifier water feed and Accepted rate to Ore feed are not correlated with any other loop. This gives these two loops high integrity.

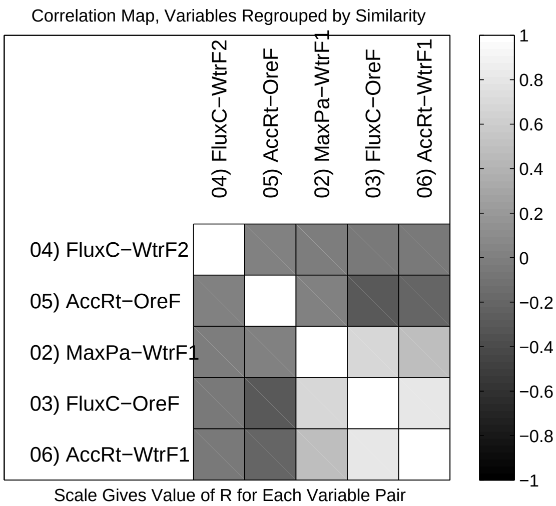

Fig. 7. Correlation analysis of the parameter loci

Sensitivity matrix A high integrity control structure would comprise of controller connections which are activated early and which maintain a high gain value throughout. Connections which for example activate very late and 'spike' to a high value very quickly are not desirable due to the high sensitivity of their gain value to the number of active connections (and thus the controller structure). To identify such connection, the sensitivity matrix (SM) may be used which is computed as follows,

$$SM(i,j) = \frac{\overline{|\theta_{ij}(\rho)|}}{\max(|\theta_{ij}(\rho)|)}, \tag{15}$$

The SM for the GC is shown in Figure 8.

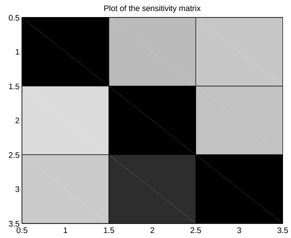

Fig. 8. Colour plot of the sensitivity matrix

Differential matrix Observe that the sensitively matrix gives an indication of the general variability of a connection. For example a connection with large gain value and high mean will have the same sensitivity as a connection with low gain value and low mean. However an important property of a connection gains is the rate at which it is increased once it has been activated. This may be found out by computing the differential matrix (DM) which is given by,

$$DM(i,j) = \text{mean}\left(\left. \frac{\Delta\theta(\rho)_{i,j}}{\Delta\rho} \right|_{pa} \right) \tag{16}$$

where $_{pa}$ denotes that the differential is computed only for the active portions of the connection value. The higher the DM value for a connection, the faster its rate of gain increase or decrease when it is activated. The DM for the GC is shown in Figure (9).

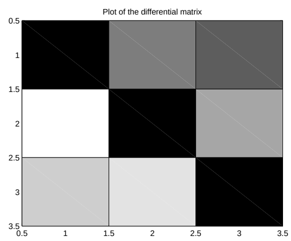

Fig. 9. Colour plot of the differential matrix

5.3 Controller Structure Selection and design

The analysis in the previous section shows that it is possible to design a Reduced Structure controller for this system. Such a controller would be comprised of a baseline decentralized controller (according to the paired system) and any additional loops selected form the Basis Pursuit regularisation analysis. In this case the analysis suggests that only adding two loops (loops 4 and 5) will result in significant improvements in performance over the decentralized controller. Therefore, the reduced structure has 5 loops in total, three of which are the standard decentralized paired loops and two additional loops. To confirm this, three PI based controllers, decentralized, reduced structure, and centralized, are designed for this system. All controllers are designed to minimize the H_2 norm of the model matching error to the same closed loop reference functions and have been designed using the optimization algorithm developed in Nobakhti and Wang [2007].

Figure (10) shows the step responses of the three respective closed loop systems. Figures (11-13) show the closed loop Nyquist arrays of the systems together with the open loops Nyquist array of the original system.

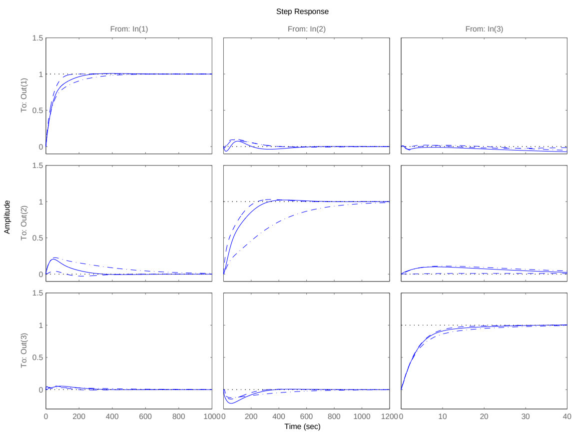

Fig. 10. Step response of the closed loop systems. Solid: Reduced Structure controller, Dashed: Centralised Controller, Dashed-Dot: Decentralized controller

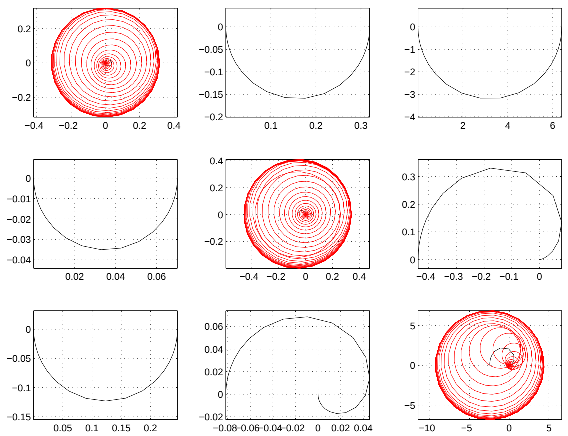

Fig. 11. Nyquist array of the open loop GC

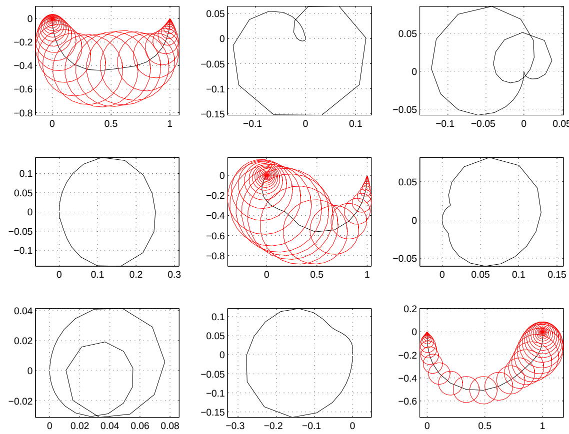

Fig. 12. Nyquist array of the GC with Reduced Strcutre control

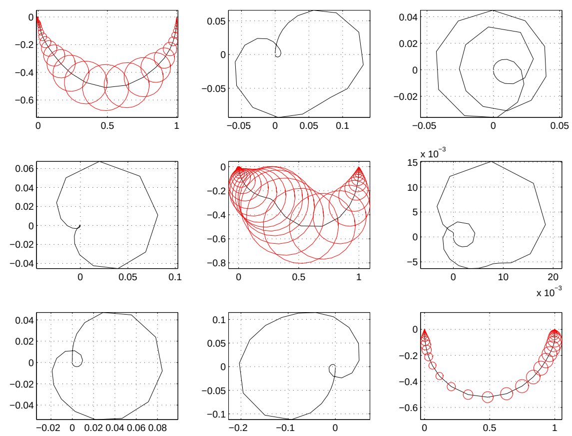

Fig. 13. Nyquist array of the GC with centralized control

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