# CONTROL OF INTEGRAL PROCESSES WITH DEAD TIME: PRACTICAL ISSUES AND EXPERIMENTAL RESULTS

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Abstract *The problem of controlling an integral process with dead time is addressed in this paper. In particular, various practical issues concerned with the controller implementation are discussed and then verified with experiments carried out on a laboratory-scale setup where a level control problem is concerned. A comparison with a standard Proportional-Integral (PI) controller is also performed.*

Keywords: Integral processes, dead-time compensators, disturbance observer, two degree-of-freedom control

## 1. INTRODUCTION

Integral processes are frequently encountered in the process industries and their dynamics can be often well described by an integrator plus dead time (IPDT) transfer function [Chien and Fruehauf, 1990]. The control of such systems has been investigated widely in the last years. By considering the standard solution in industrial settings, namely the use of Proportional-Integral-Derivative (PID) controllers, many tuning rules and methods for the improvement of the performance have been proposed; see for example [Åström and Hägglund, 2006, Visioli, 2006, O'Dwyer, 2006]. Moreover, the stability region of a PI controller parameters has been characterized in [Wang et al., 2006], where the limitations on the achievable gain and phase margin have been also analyzed.

From a slightly different viewpoint, it is well-known that a two degree-of-freedom control scheme is an effective way to achieve satisfactory performance both in set-point tracking and in load disturbance rejection. In this context, different solutions have been proposed in the literature, developing the basic Smith predictor scheme; see, for example, [Åström et al., 1994, Matausek and Micic, 1999, 1996, Normey-Rico and Camacho, 1999, Zhang and Sun, 1996, Normey-Rico and Camacho, 2002]. In particular, a methodology based on the concept of disturbance observer, *i.e.*, the use of the inverse of the nominal model to observe the disturbances with the aim of canceling their effects directly in the control signal, has been presented in [Zhong and Normey-Rico, 2002, Zhong and Mirkin, 2002]. A two degree-of-freedom control scheme, where the feedback controller is inherently a PID controller which retains the Smith predictor principle, has been proposed in [Zhong and Li, 2002]. The robustness issue can be explicitly analyzed and a tuning parameter that handles the trade-off between aggressiveness and robustness is available.

However, despite the development of these techniques, surprisingly, no experimental results are reported in the literature related to the control of integral processes, to the best knowledge of the authors. Practical issues are often not discussed

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thoroughly and this implies that these techniques are not fully characterized from the industrial point of view. While trying to implement some of the techniques into practice, it turns out that the controller implementation is not straightforward. If the controller is not implemented properly, the system might be internally unstable or there might be a non-zero static error etc. In this paper, these issues are discussed and verified with experimental results obtained with a laboratory-scale level control system, in which the water level is controlled by regulating the difference between the input and output flow rate.

The paper is organized as follows. The control structure under consideration is reviewed in Section 2. The practical issues in controller implementation are discussed in Section 3 and the experimental results are presented in Section 4.

## 2. THE CONTROL STRUCTURE UNDER CONSIDERATION

The integral processes with dead time (IPDT) can be described as,

$$G(s) = G_p(s)e^{-\tau s} = \frac{K}{s}e^{-\tau s}, \qquad (1)$$

where the pure dead time $\tau$ and the static gain $K$ are all positive.

In this paper, the attention is paid to the disturbance observer-based control scheme proposed in [Zhong and Normey-Rico, 2002]. Similar issues exist with other control schemes for integral processes with dead time. The scheme is revisited in Figure 1(a) for the readers' convenience, where $\tau_m$ is the estimated dead time and $G_m(s)$ is the nominal delay-free part. The controller $C(s) = \frac{1}{KT}$ is designed to be proportional and the low-pass filter

$$Q(s) = \frac{(2\lambda + \tau_m)s + 1}{(\lambda s + 1)^2} \qquad (2)$$

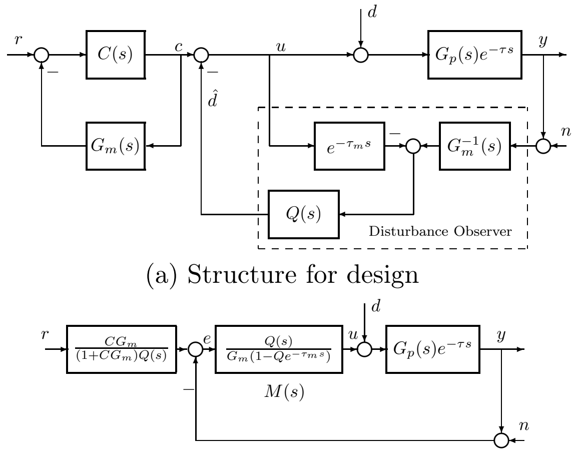
is designed to reject step disturbances with no static error, where $\lambda$ is a design parameter to compromise the disturbance rejection, robustness and, possibly, the control action bound [Zhong and Normey-Rico, 2002].

Under the nominal condition, $G_m(s) = G_p(s) = \frac{K}{s}$ and $\tau_m = \tau$. The setpoint response and disturbance response are respectively:

$$G_{yr}(s) = \frac{1}{Ts+1}e^{-\tau_m s},$$

$$G_{yd}(s) = \frac{K}{s}\left(1 - Q(s)e^{-\tau_m s}\right)e^{-\tau_m s}. \qquad (3)$$

It is quite easy to evaluate the achievable specifications of the set-point response. The disturbance



(a) Structure for design

(b) Equivalent structure for implementation

Figure 1. Disturbance observer-based control scheme

response obtained using (2), which is sub-ideal as having been shown in [Mirkin and Zhong, 2003], can be obtained with several different schemes [Mirkin and Zhong, 2003]. The achievable performance specifications and the robust stability regions of the system can be found in [Zhong and Normey-Rico, 2002].

The scheme shown in Figure 1(a) is not internally stable nor causal. An algebraically equivalent but casual and internally stable structure is shown in Figure 1(b), as proposed in [Zhong and Mirkin, 2002]. The implementation of the block

$$M(s) = \frac{Q(s)}{G_m(s)(1 - Qe^{-\tau_m s})}$$

needs to be very careful. If not properly implemented, then problems may occur.

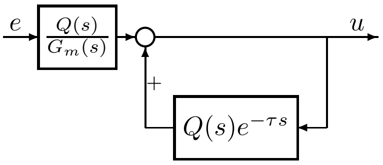
## 3. PRACTICAL ISSUES

### *3.1 Zero static error*

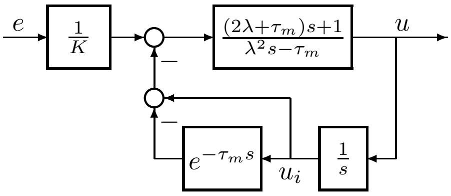
The block diagram shown in Figure 2(a) for $M(s)$ is implementable because the block $\frac{Q(s)}{G_m(s)}$ is causal. However, if the block $M(s)$ is implemented in this way, then the system will not be able to reject step disturbances completely. There will be a non-zero static error because the block

$$\frac{Q(s)}{G_m(s)} = \frac{(2\lambda + \tau_m)s^2 + s}{K(\lambda s + 1)^2}$$

contains a differentiator and any constant error $e$ will be ignored by the controller. This causes a non-zero static error. As a matter of fact, since the static gain of $Q(s)$ is unity, the local positive feedback loop in Figure 2(a) contains an integral action. The differential action in the previous




(a) Non-zero static error




(b) Internally unstable

Figure 2. Wrong implementation schemes for $M(s)$

block cancels this integral action. This should be avoided in practical implementation.

### 3.2 Internal stability

One special property of the control plant is the instability due to the pole at the origin. This makes the structure shown in Figure 1(a) internally unstable because of the pole-zero cancellation at $s = 0$ between the block $G_m^{-1}(s)$, the inverse of the delay-free model, and the plant $G_p(s)e^{-\tau s}$. The equivalent structure for implementation shown in Figure 1(b) guarantees the internal stability if implemented properly. As discussed in the previous subsection, the implementation of $M(s)$ shown in Figure 2(a) causes non-zero static errors when the system is subject to step a disturbance. The cancellation between the differential action in $\frac{Q(s)}{G_m(s)}$ and the integral action in the local loop should be removed in implementation. The block $M(s)$ can be algebraically equivalent to

$$\begin{aligned}
M(s) &= \frac{Q(s)}{G_m(s)(1 - Qe^{-\tau_m s})} \\
&= \frac{1}{K} \frac{(2\lambda + \tau_m)s + 1}{\lambda^2 s - \tau_m + ((2\lambda + \tau_m)s + 1)\,\frac{1-e^{-\tau_m s}}{s}} \\
&= \frac{1}{K} \frac{\frac{(2\lambda+\tau_m)s+1}{\lambda^2 s-\tau_m}}{1 + \frac{(2\lambda+\tau_m)s+1}{\lambda^2 s-\tau_m} \cdot \frac{1-e^{-\tau_m s}}{s}}.
\end{aligned}$$

This can be described as shown in Figure 2(b) as a local feedback loop. The signal $u_i$ in Figure 2(b), i.e., the state of the integrator, is unbounded and hence the system is not internally stable even when the output of the whole system is stable because the block $\frac{1-e^{-\tau_m s}}{s}$ is split into the cascade of an integrator and $1 - e^{-\tau_m s}$, which causes a zero-pole cancellation at $s = 0$. One way to avoid this is to use the approach proposed in [Zhong, 2004, 2006] to approximate $\frac{1-e^{-\tau_m s}}{s}$ as

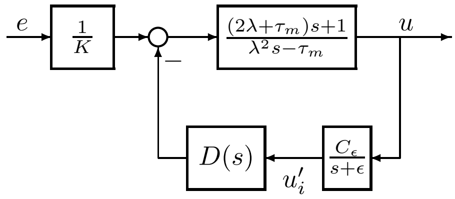



Figure 3. Correct implementation scheme of $M(s)$

$$\frac{1 - e^{-\tau_m s}}{s} \approx \frac{C_\epsilon}{s + \epsilon} \cdot D(s),$$

with

$$C_\epsilon = \frac{\frac{\tau_m}{N}\epsilon}{1 - e^{-\epsilon\tau_m/N}},$$

and

$$D(s) = (1 - e^{-\frac{\tau_m}{N}(s+\epsilon)}) \cdot \sum_{i=0}^{N-1} e^{-i\frac{\tau_m}{N}s},$$

where $\epsilon > 0$ is a small enough number and $N$ is a natural number. The longer the delay, the bigger the $N$ is needed. Then, the block $M(s)$ can be implemented as shown in Figure 3. With this implementation, the closed-loop system is internally stable. Moreover, there is no static error. The only downside is that the performance will be slightly worse than the designed performance due to the approximation. This can be improved by choosing a small $\epsilon$ and a big $N$.

## 4. EXPERIMENTAL RESULTS

### 4.1 The experimental setup

The laboratory experimental setup employed to verify the findings in the previous sections is shown in Figure 4. This is a small perspex tower-type tank with a section area of $A = 40\text{cm}^2$, of which the water level is regulated by a PC-based controller. The tank is filled with water by means of a pump whose speed is controlled by a DC voltage, in the range 0-5 V, through a PWM circuit. Then, an outlet at the base allows the water to return to the reservoir. The measure of the level of the water is given by a capacitive-type probe that provides an output signal between 0 (empty tank) and 5 V (full tank). The process can be modeled by the following differential equation:

$$\frac{dy(t)}{dt} = \frac{1}{A}(q_i(t) - q_o(t))$$

where $y$ is the water level (i.e., the process variable), $q_i$ is the input flow rate and $q_o$ is the output flow rate. The input flow rate $q_i$ is linearly dependent on the voltage applied to the DC pump. The output flow rate can be calculated by measuring


Figure 4. The experimental setup (only one tank has been employed in the experiments).

the level $y(t)$ because $q_o(t) = \alpha\sqrt{2gy(t)}$, where $g$ is the gravitational acceleration constant and $\alpha$ is the cross-sectional area of the output orifice. This is then added to the control variable $u$ via a local feedback loop to obtain the input flow rate $q_i$, i.e., $u(t) + q_0(t) = q_i(t)$, which is then used to determine the speed of the DC motor (pump). Hence, the control variable $u$ is

$$u(t) = q_i(t) - q_o(t).$$

We have now obtained an integral process

$$\frac{dy(t)}{dt} = \frac{1}{A}u(t).$$

Dead time also occurs in the process because of the length of the pipe. The transfer function of the process, from the control variable $u$ to the voltage corresponding to the water level, has been estimated as follows by means of a simple open-loop step response experiment:

$$G(s) = \frac{0.026}{s}e^{-1.5s}.$$

### *4.2 The scheme shown in Figure 2(a)*

The control parameters are chosen as $T = 5$ and $\lambda = 8$. A set-point change from 1.5V to 2.3V has been applied to the control system and, after 100s, a disturbance $d = -2.5$V has been applied to the control variable. The resulting process variable together with the set-point signal are shown in Figure 5. As expected, non-zero static error appears.

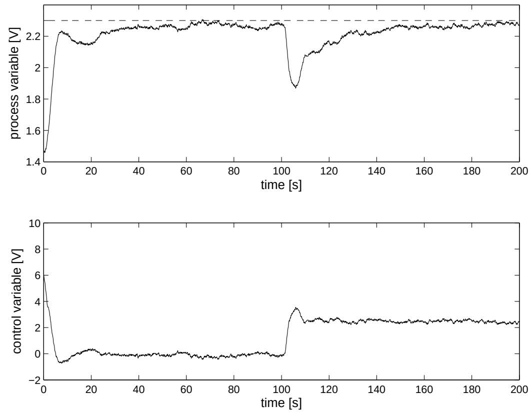
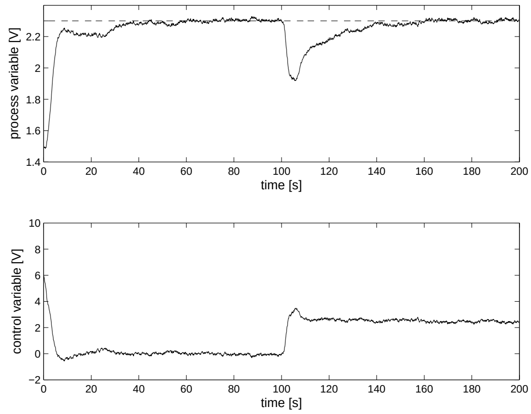
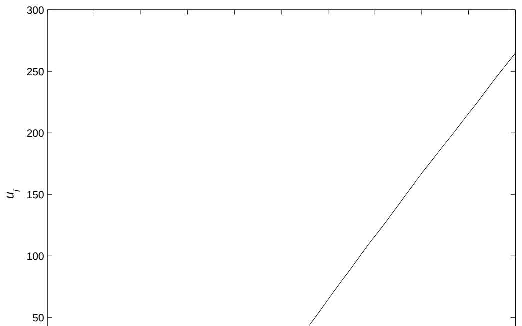



Figure 5. Experimental results with the control scheme of Figure 2(a)




(a) Process variable and control variable




(b) Internal variable $u_i$

Figure 6. Experimental results with the control scheme of Figure 2(b)

### *4.3 The scheme shown in Figure 2(b)*

The same experiment as described in the previous subsection has been repeated for the scheme shown in Figure 2(b). The resulting process and control variables are plotted in Figure 6(a). The static error has disappeared but, as expected, the system is internally unstable as can be seen from the plot of the variable $u_i$ (i.e., the state of the integrator) shown in Figure 6(b). This variable becomes unbounded eventually.

### 4.4 The scheme shown in Figure 3

Then, the scheme of Figure 3 has been employed for the same control task. The control parameters are selected as $T = 5$, $\lambda = 8$, $\epsilon = 0.01$ and $N = 1$. Results are shown in Figure 7(a). It appears that the control scheme is effective in both the set-point response and disturbance rejection. In order to evaluate the physical meaning of the parameter $\lambda$, the same experiment as before has been repeated with different values of $\lambda = 4$ and $\lambda = 12$. The resulting process and control variables are plotted in Figures 7(b) and 7(c) respectively. The control system is more aggressive when the value of $\lambda$ is decreased and it is more sluggish when $\lambda$ is increased, as expected.

### 4.5 Comparison with a PI controller

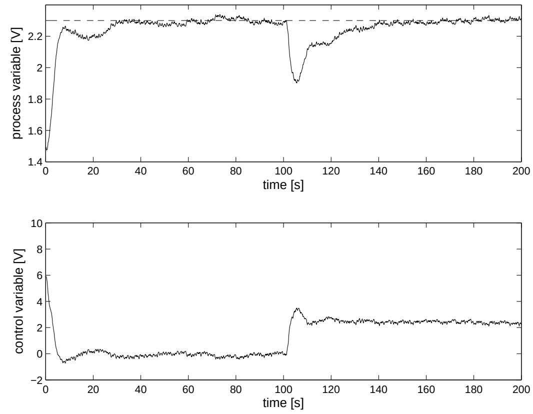
In order to evaluate better the performance of the disturbance observer-based control scheme, comparison with a standard PI controller in the unity-feedback control scheme has been performed. The controller is

$$C_{PI}(s) = K_p \left(1 + \frac{1}{T_i s}\right),$$

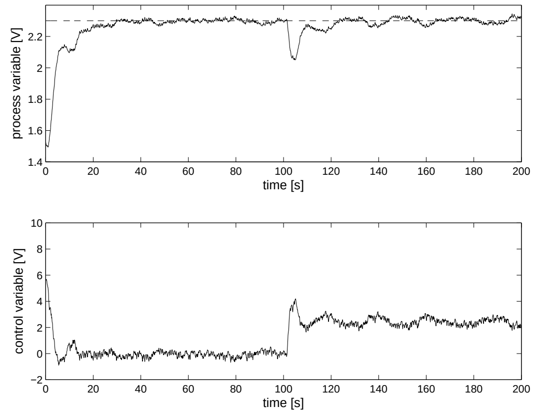
where the proportional gain $K_p$ and the integral time constant $T_i$ have been selected by applying the tuning rule proposed in [Chien and Fruehauf, 1990], i.e., $K_p = \frac{0.3306}{KL} = 8.48$, $T_i = 10L = 15$. The experimental results are shown in Figure 8. The disturbance rejection is very good but the set-point response has a large overshoot. This clearly shows the benefit of the two degree-of-freedom control scheme. It is able to provide good performance for both the set-point response and disturbance rejection. Obviously, being of a single degree-of-freedom, the PI controller is able to obtain satisfactory performance in disturbance rejection at the expense of a significant overshoot in the set-point response. In general, it is not easy to select the most appropriate tuning rule to meet the requirement for both.

### 4.6 Experiments with an additional delay

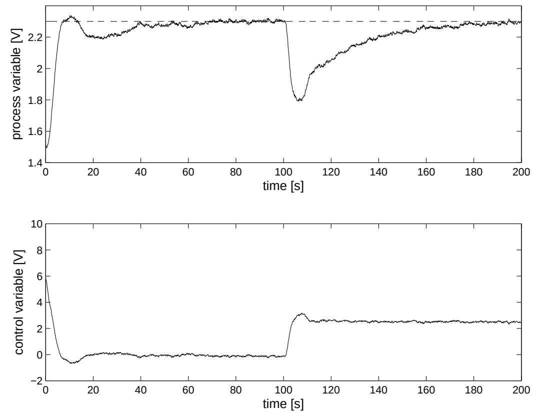
The same experiments have also been performed on the system as before but with an additional input delay of 1s, which is implemented via software. The controller for the scheme shown in Figure 3 has been re-designed with $\lambda$=8 and $\lambda$=12, respectively, and the PI controller is tuned as $K_p$=5.09 and $T_i$=25 using the same tuning rule as before. The results are shown in Figures 9. It can be seen that the observer-based control scheme performs better than the PI controller.




(a) $\lambda$=8




(b) $\lambda$=4




(c) $\lambda$=12

Figure 7. Experimental results with the control scheme of Figure 3.

## 5. CONCLUSIONS

In this paper, some practical issues in controller implementation for integral processes with dead time are reported and verified with experimental results. It is shown that with proper implementation, these problems can be eliminated.

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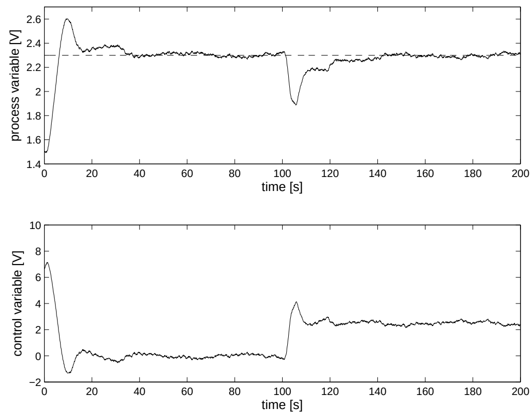
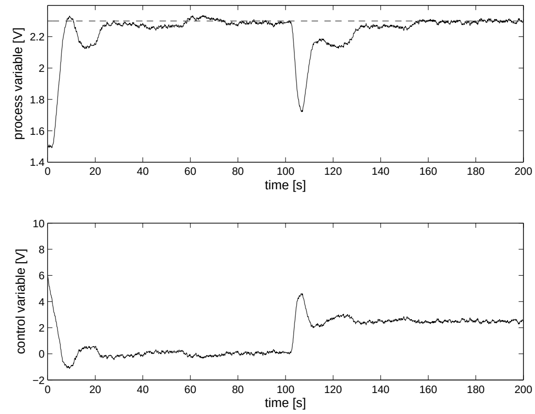
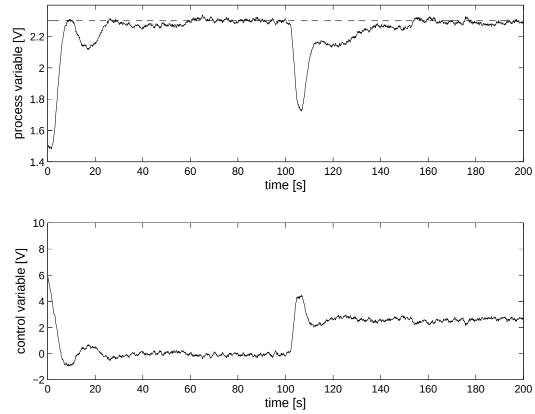


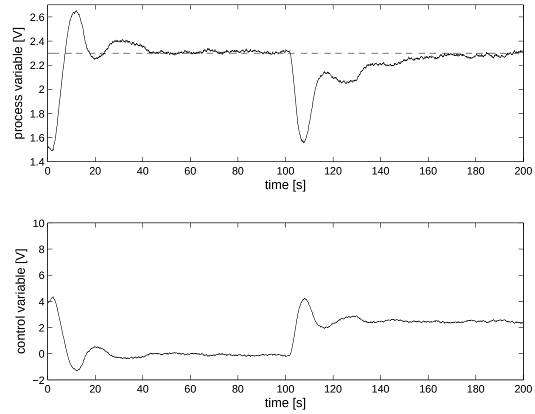

Figure 8. Experimental results with a PI controller




(a) With the control scheme of Figure 3 ($\lambda$=8)




(b) With the control scheme of Figure 3 ($\lambda$=12)




(c) With a PI control scheme

Figure 9. Experimental results for the system with an additional 1s-delay

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