Data-driven Direct Adaptive Model Based Predictive Control

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Abstract: This paper is concerned with the design of Direct Adaptive Model Based Predictive Control (DAMBPC) using subspace identification technique to identify and implement the controller parameters. The direct identification of controller parameters reduces the design effort and computation load which is usually involved with classical adaptive control techniques. The proposed method requires a single QR-decomposition for obtaining controller parameters directly from input-output data when the model dynamic changes. The method uses receding horizon approaches to collect data and identify the controller. The paper presents a comparison of performance given by proposed control scheme when applied to a 4-tank nonlinear system with that of a linear model predictive control scheme and multi-loop PID controllers.

1. INTRODUCTION

Often, adaptive control has been applied to control systems with large nonlinearity and /or time varying systems where modelling uncertainty is too large for successful linear design. Basically, adaptive control is used to estimate plant model parameters based on the measured outputs/inputs to design a controller for calculating the control inputs (Isermann R et al., 1992). In the presence of smooth model dynamic variation, adaptive control can maintain consistent performance of the system and only requires limited a priory knowledge of the plant to be controlled.

Adaptive predictive control has been shown to be efficient and successful in many applications (Clark, 1996). Adaptive predictive control from subspace-based technique combines the advantages of on-line subspace identification and the model predictive controller design. Subspace identification techniques have emerged as one of the more popular identification methods for the estimation of state space models from measurement data. Using these techniques, subspace matrices can be constructed and used to obtain prediction of the process outputs. These predictions can subsequently serve as a basis for model predictive controller design. By continuously updating these predictions models an adaptive predictive control method can be obtained. As an alternative to the two-step adaptive predictive control method that results when a model is explicitly estimated as shown in Fig.1, it is also possible to estimate the control parameters directly from the measurements. This direct adaptive control method was introduced by the adaptive control community in the early 70s (Åström and Wittenmark) and has been widely deployed. Such algorithm combines system identification and control design simultaneously (see Fig.2) and it has the advantage of requiring less design and computation effort.

Some previous work has been reported on the design of Model Based Predictive Control (MBPC) using subspace matrices such as model-free LQG and subspace predictive controller (Favoreel, De Moor & Overschee 1998; Favoreel, De moor & Gevers, 1999; Favoreel et al., 2000; Kadali et al., 2003; H.Yang et al., 2005), or using the state space model identified through subspace approach (Ruscio, 1997a,b; Ruscio & Foss, 1998;X.Wang et al., 2007). In Favoreel et al., (1998, 1999 and 2000), the main result is that the system identification and the calculation of controller parameters are replaced by a single QR decomposition. Previous developments in this area also include subspace based constrained model predictive controllers (Kadali et al., 2002).

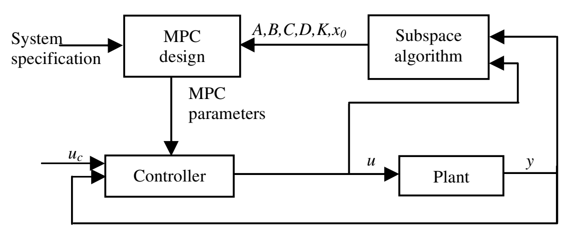

Fig.1. Indirect Adaptive Control

In this paper, a new combination of subspace identification methods and MPC is studied in an adaptive control context. The objective is to develop subspace-based adaptive Model Predictive Control that can cope with nonlinear systems. Other approaches for dealing with these types of processes include linear MPC, nonlinear MPC and neural network based MPC approaches. In practice, however, linear MPC approaches tend to favoured. Linear MPC approaches include linearisation approaches, where a nonlinear model is linearised at each sampling instance (e.g. Krishnan & Kosanovich, 1998), and multiple model based approaches (for example see Narenda & Xiang, 2000). Previous efforts in the area of adaptive predictive control have also seen the application of neural networks (Wang & J.Huang, 2002), however, due to the complexity and computational load typically associated with these methods they have made few inroads in practice.

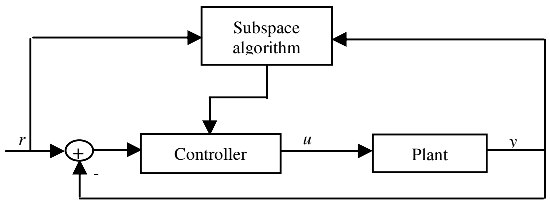

Fig. 2. Direct Adaptive Control

The proposed adaptive MBPC method can offer an attractive alternative to existing adaptive control methods for the nonlinear systems. The proposed method combines the simplicity of linear model predictive control with the power of a self-tuning. The main advantages of the proposed approach are that the usually tedious and time-consuming modelling task can be eliminated and that the controller can adapt to changing process conditions.

The paper is organised as follows: In Section 2 we briefly recapitulate the main concepts of subspace identification and QR-decomposition. The proposed subspace-based MPC approach is developed in Section 3. Section 4 introduces the implementation of adaptive MPC. Section 5 describes the application to a quadruple-tank process, which demonstrates both Minimum Phase (MP) and Non-Minimum Phase (NMP) dynamic behaviour. In Section 6 the proposed controller is compared to a linear MPC and to a multi-loop PID controllers through simulations of a nonlinear model for the quadruple-tank process. The report ends with some conclusion.

2. THE SUBSPACE IDENTIFICATION METHOD

A linear discrete time-invariant state space system can be represented as,

$$x(k+1) = Ax(k) + B\Delta u(k) + Ke(k) \tag{1}$$

$$y(k) = Cx(k) + D\Delta u(k) + e(k) \tag{2}$$

where $y(k)$ and $x(k)$ are the outputs and states vectors and $\Delta u(k)$ is the control input increment vector. $e(k)$ is a white noise sequence with zero mean and variance $E[e_p e_q^T] = S\delta_{pq}$. The following matrix input-output equations (De Moor, 1988) play an important role in the problem treated in linear subspace identification:

$$Y_f = \Gamma_i X_f + H_i U_f \tag{3}$$

where data block Hankel matrices for $u(k)$ represented as U_p and U_f are defined as:

$$U_p = \begin{pmatrix} u_0 & u_1 & \cdots & u_{j-1} \\ u_1 & u_2 & \cdots & u_j \\ \vdots & \vdots & \ddots & \vdots \\ u_{i-1} & u_i & \cdots & u_{i+j-2} \end{pmatrix} \tag{4}$$

$$U_f = \begin{pmatrix} u_i & u_{i+1} & \cdots & u_{i+j-1} \\ u_{i+1} & u_{i+2} & \cdots & u_{i+j} \\ \vdots & \vdots & \ddots & \vdots \\ u_{2i-1} & u_{2i} & \cdots & u_{2i+j-2} \end{pmatrix} \tag{5}$$

where the subscripts p and f represent 'past' and 'future' time. The same way can be defined for outputs block Hankel matrices Y_p and Y_f. The i and j are assumed to be prediction horizon, H_p and receding window size, n respectively. The extended observability matrix, Γ_i and the lower block triangular Toeplitz matrix, H_i are defined as:

$$\Gamma_i = \begin{pmatrix} C \\ CA \\ \vdots \\ CA^i \end{pmatrix} H = \begin{pmatrix} CB & 0 & \cdots & 0 \\ CAB & CB & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA^{i-1}B & CA^{i-2}B & \cdots & CB \end{pmatrix} \tag{6}$$

The linear predictor equation is defined as:

$$\hat{Y}_f = L_w W_p + L_u U_f \tag{7}$$

where for given past inputs and outputs W_p and future inputs U_f , the problem of subspace identification can be expressed as the solution to the following minimisatiojn problem:

$$\min_{Lw,Lu} \left\| Y_f - (L_w, L_u) \begin{pmatrix} W_p \\ U_f \end{pmatrix} \right\|_F^2 \tag{8}$$

The solution to the above optimisation problem can be found by applying an orthogonal projection of the row space of Y_f into the row space spanned by W_p and U_f as:

$$\hat{Y}_f = Y_f \Big/ \begin{pmatrix} W_p \\ U_f \end{pmatrix} = \underbrace{Y_f /_{U_f} W_p}_{L_w W_p} + \underbrace{Y_f /_{W_p} U_f}_{L_u U_f} \tag{9}$$

The most efficient way to calculate this projection is by applying a QR-decomposition to (9):

$$\begin{pmatrix} W_p \\ U_f \\ Y_f \end{pmatrix} = \begin{pmatrix} R_{11} & 0 & 0 \\ R_{21} & R_{22} & 0 \\ R_{31} & R_{32} & R_{33} \end{pmatrix} \begin{pmatrix} Q_1^T \\ Q_2^T \\ Q_3^T \end{pmatrix} \tag{10}$$

By posing:

$$L = \begin{pmatrix} R_{31} & R_{32} \end{pmatrix} \begin{pmatrix} R_{11} & 0 \\ R_{21} & R_{22} \end{pmatrix}^+ \tag{11}$$

where + denotes the Penrose-Moore pseudo-inverse, equation (9) can be written as:

$$Y_f \Big/ \begin{pmatrix} W_p \\ \hline U_f \end{pmatrix} = L \begin{pmatrix} W_p \\ U_f \end{pmatrix} \tag{12}$$

Then, L_w and L_u can be found from L using (written in Matlab notation):

$$L_w = L(:,1:i(N_u + N_y)) \tag{13}$$

$$L_u = L(:,i(N_u + N_y)+1:i(2*N_u + N_y)) \tag{14}$$

where N_u and N_y represent the number of input and output, respectively.

3. PREDICTIVE CONTROL METHOD

This section describes the development of the data driven adaptive control method by combining the subspace identification and model predictive control. Note that the steps of identification and control design can be carried out simultaneously by applying a single QR-decomposition to the input-output data. This stands in contrast to the design of conventional MPC controllers, where modelling and control design is usually distinctly separated tasks.

The model predictive control problem can in mathematical terms be expressed as the minimization of

$$J = \sum_{i=1}^{H_p} \left(\hat{y}(k+i) - r(k+i)\right)^T Q \left(\hat{y}(k+i) - r(k+i)\right) + \sum_{i=0}^{H_c - 1} \Delta u(k+i)^T R \Delta u(k+i) \tag{15}$$

where H_p and H_c denote the prediction and control horizons, respectively. The output and input weighting matrices Q and R are assumed positive definite.

By using the linear predictor in equation (7), rewrite the output sequence to include integral action in the predictor:

$$\Delta \hat{y}_f = \tilde{L}_w \Delta w_p + \tilde{L}_u \Delta u_f \tag{16}$$

where $\tilde{L}_w$ and $\tilde{L}_u$ are obtained directly from the previous identification of L_w and L_u while $\Delta w_p = \begin{bmatrix} \Delta y_p{}^T & \Delta u_p{}^T \end{bmatrix}^T$ and $\Delta \hat{y}_f = [\Delta \hat{y}_1, \ \cdots, \ \Delta \hat{y}_i]^T$. Thus, for a k-step ahead predictor as:

$$\hat{y}_f = y_t + \tilde{L}_w \Delta w_p + \tilde{L}_u \Delta u_f \tag{17}$$

and the current output:

$$y_t = [y_t \quad y_t \quad \cdots \quad y_t]^T \tag{18}$$

By substitution of the new integrated linear predictor in equation (17) into the cost function J, differentiate it with respect to Δu_f and equating it to zero gives the control law:

$$\Delta u_f = \left(\tilde{L}_u{}^T Q \tilde{L}_u + R\right)^{-1} \tilde{L}_u{}^T Q \left(r_f - y_t - \tilde{L}_w \Delta w_p\right) \tag{19}$$

At each sample time, only the first element of Δu_f is used as the control input and the calculation is repeated for the next sample time. The input *u(t)* as:

$$u(t) = u(t-1) + \Delta u(t) \tag{20}$$

4. DIRECT ADAPTIVE MPC

This section considers the on-line implementation of a subspace-based model predictive in which the subspace identification data is collected over a sliding (receding) window.

4.1 Sliding (receding) Window

The procedure of using a sliding window for identification is illustrated in Fig.3. The main advantage of this approach is that the controller parameters are updated at each sample time, which usually means a quicker response to process changes. The controller update can also be monitored such that when there is a significant change in the process, a new controller is identified and implemented. Also, using the control increment as the model input provides integral action ensuring zero static error. The QR-decomposition needs to be computed at each sample instance but with efficient real time QR decompositions methods available, this should not be a major issue.

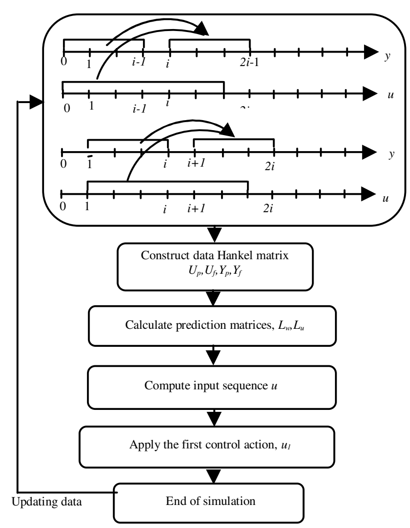

Fig. 3. Sliding window for *i=4*

As shown in Fig.3 only the current control input is applied to the system at each iteration. Therefore, from a measurement input-output data of nonlinear system at a particular operating point, the proposed controller satisfy the minimization of the objective function (15) and present offset free tracking system.

4.2 The prediction horizons

The prediction horizon for MBPC and the length of the identification window are the two parameters which be tuned to meet the control design objectives. Longer horizon windows will of course increase the computational load. A trade-off between the length of the prediction horizon and the computational load of the controller must therefore be employed.

5. APPLICATION TO QUADRUPPLE TANK

The quadruple-tank process consists of four interconnected water tanks and two pumps as shown in Fig. 4. This laboratory process illustrates many issues in multivariable control (K.H. Johansson, 2000). The mass balances and Bernoulli's law that yields the following set of nonlinear differential equations was applied.

$$\dot{h}_1(t) = -\frac{a_1}{A_1}\sqrt{2gh_1} + \frac{a_3}{A_1}\sqrt{2gh_3} + \frac{\gamma_1 k_1}{A_1}v_1 \qquad (21)$$

$$\dot{h}_2(t) = -\frac{a_2}{A_2}\sqrt{2gh_2} + \frac{a_4}{A_2}\sqrt{2gh_4} + \frac{\gamma_2 k_2}{A_2}v_2 \qquad (22)$$

$$\dot{h}_3(t) = -\frac{a_3}{A_3}\sqrt{2gh_3} + \frac{(1-\gamma_2)k_2}{A_3}v_2 \qquad (23)$$

$$\dot{h}_4(t) = -\frac{a_4}{A_4}\sqrt{2gh_4} + \frac{(1-\gamma_1)k_1}{A_4}v_1 \qquad (24)$$

where A_i is cross-section of tank i, a_i is cross-section of the outlet hole and h_i is the water level. The control signals are the input voltage to the two pumps, v_1 and v_2 and the outputs are the voltage from level measurement device y_1 and y_2 or the water levels in the lower two tanks. The flow to tank i is $k_i v_i$ and the parameters γ_1, $\gamma_2 \in (0,1)$ determine the corresponding flows to each tank. The linearized model of the quadruple-tank process has a multivariable zero. By simply changing a valve will affect the location and direction of this multivariable zero, hence determine either the system is MP or NMP.

6. SIMULATION RESULTS

Simulations were carried out from a steady-state operating point at outputs y_1 =6.2 V, y_2 = 6.35 V and inputs u_1= u_2 = 3 V for MP whilst y_1 =6.3 V, y_2 = 6.5 V and inputs u_1= u_2 = 3.15 V for NMP system. The comparison is between the adaptive MPC, the linear MPC and the multi-loop PID controller for set-point tracking performance. The set-point given for the outputs were allowed to vary approximately 10% around the system's steady state condition.

The prediction horizon and control horizon that been employed in the simulations is H_p=35 and H_c=5. In this example the 'best' prediction horizon length is found by fixing the length of the identification window to n=400. Then several different prediction horizons where benchmarked and it was eventually found that those given provided the best performance for both MP and NMP system.

6.1 Choice of weighting matrices Q and R

The weighting matrices were tuned using a trial and error approach, and were chosen as $Q = diag\{0.1,0.1\}$ and $R = diag\{0.008,0.008\}$ for MP case. Since R is often chosen to correspond to the open-loop settling time of a process response to a step change in the control input, hence give $Q = diag\{0.001,0.001\}$ and $R = diag\{0.04,0.04\}$ that demonstrated good performance to NMP case.

6.2 The multi-loop PID controllers

The controller parameters were adjusted to give a good response for the set-point change. The optimal selection of controller tuning parameters were found to be, ρ = 5, ε =0.5 and α = 0.998 for MP case, whilst ρ = 0.3, ε =0.003 and α = 0.995 for NMP case.

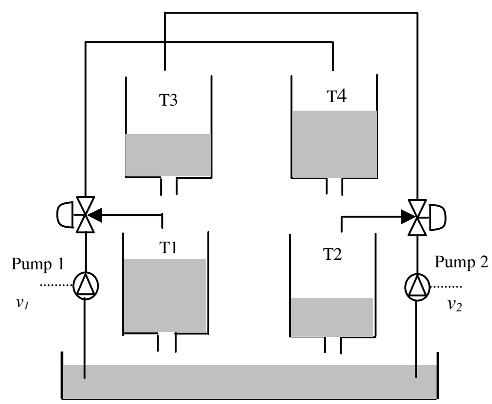

Fig.4. Schematic diagram of the quadruple-tank process

Fig. 5 shows a comparison of the three controllers for MP case. The control performance delivered by the adaptive sliding window control algorithm is better than that delivered from linear MPC control algorithms and multi-loop PID controllers. The proposed sliding window algorithm also demonstrates less interaction and good tracking properties for the MP system.

Fig. 6 shows a comparison of the same three controllers for NMP case. The adaptive sliding window, although not yielding smooth response, critically damped responses, still settles fairly quickly. The linear MPC perform well and the multi-loop PID controller depicts slow tracking performance. The controllers given by NMP system demonstrate more interaction when compared to MP system, though still addressed a good tracking properties.

7. CONCLUSION

In this paper, the design of adaptive model based predictive controller from subspace matrices in a framework of adaptive controller is addressed and successfully applied to a quadruple-tank process. The persistently excitation of the NMP system is harder than MP counterpart for good parameter estimates and hence a good control performance. The data driven adaptive MBPC is shown to be more efficient and robust than a linear MPC controller and a multi-loop PID controllers when applied to the nonlinear systems.

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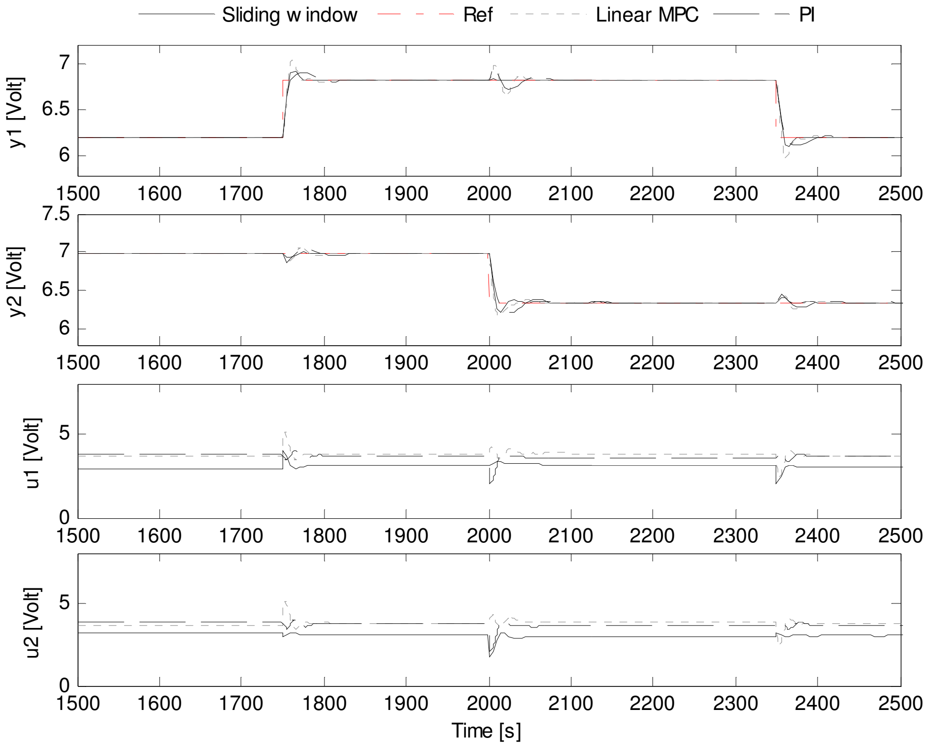

Fig.5 The comparison of control performance MP system

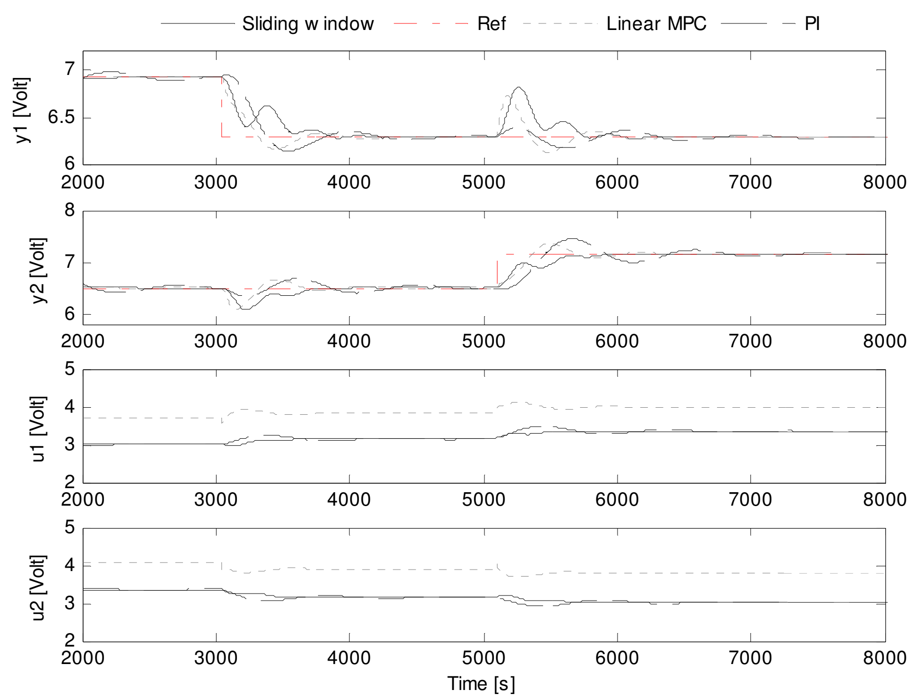

Fig.6 The comparison of control performance for NMP system.