Improved FOPDT model estimation with Delayed-relay feedback for constant time dominant processes

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Abstract: In this paper with reference to analytical results of different well-known relay feedback methods, we illustrate a main deficiency in parameter estimation of processes with a small ratio of time delay to time constant. Then to rectify this problem we introduce a modified relay feedback structure with additional delay to estimate the parameters of the FOPDT transfer function of the system.

The significance of this method lies in the fact that many industrial plants perform fairly such as FOPDT systems, and a wide range of processes have negligible dead time versus their long constant time. Also, the estimated FOPDT transfer function from proposed relay feedback test can be used as a priori knowledge in advanced control strategies which need a FOPDT model of the system. The method is straightforward and simulation results illustrate the effectiveness, and simplicity of the proposed method.

Keywords: Relay feedback; FOPDT; parameter estimation; Autotuning;

1. INTRODUCTION

Process identification and controller autotuning from closed loop relay feedback tests has been widely accepted over the past two decades (Hang *et al.*, 2002). Various methods are proposed in this area since the seminal work of Astrom and Hagglund in 1984. They suggested the relay feedback test to generate sustained oscillations as an alternative to the conventional step response continuous cycling technique, to estimate the ultimate gain and ultimate frequency. The relay feedback method is effective in critical point estimation and has remained attractive because of its simplicity, time efficiency and robustness (Hang *et al.*, 1991).

Relay feedback autotuning automatically extracts the frequency response of the process at an important frequency (ultimate frequency) and the information is usually sufficient to tune the PID controller for many processes. The test is carried out under closed loop so that with an appropriate choice of the relay parameters, the process can be kept close to the setpoint and in the linear region where the frequency response is of interest. Also unlike other autotuning methods, the technique eliminates the need for a careful choice of the sampling rate, which is very useful in initializing a more sophisticated adaptive controller (Hang *et al.*, 2002). These appealing features of the relay feedback autotuning have lead to a number of commercial autotuners and industrial applications (Yu, 2006).

Ultimate gain and ultimate frequency used in ZN types of tuning rules are the most common parameters extracted from relay feedback.

Transfer function process models play an important role in process analysis and control. Several methods to derive such models are available (Majhi et al., 1999, and Luyben et al., 2001). The methods reported in the literature for TF modeling from relay feedback require two tuning tests or some extra knowledge and their results are approximate in nature (Wang et al., 1997, Yu, 2006, and Majhi et al., 2007).

Many industrial processes can be modeled with a stable FOPDT system with small delay time. In this paper, such processes are considered and efficiencies/deficiencies of relay feedback tests on them are analyzed. Then some ideas will be proposed to overcome the problems.

Despite the success of relay feedback tests in most applications, it has a poor performance in some plants. One important factor in accuracy of estimation in relay feedback test is the process dead time (D) rather than its constant time (τ). In processes with small D/τ ratio, nyquist curve cross the negative real axis close to the origin and so the ultimate frequency is high. Also it is very usual to sample the process data with a specific sample time which is selected according to the process time constant and not according to the delay time. If the sample time become more than D or close to it, the output of relay test will switch at each sample time between high and low levels which yields an on/off triangular oscillation. This case, as will be shown in this paper, causes a large error in critical point estimation. However, the proposed Delayed-relay (D-relay) structure, estimate an accurate TF for the processes with small ratio of D/τ.

Also, tuning rules for PID controllers are developed based on D-relay autotuning data and comparative tests are performed.

This paper is organized as follows. In section II, some conventional relay structures and their ability to cope with the problems of D/τ ratio are reviewed and the need for further development is highlighted with detailed experiments. In section III, the proposed Delayed-relay (D-relay) method is presented to estimate the FOPDT TF of systems with small D/τ ratio and some controller tuning methods are evaluated based on D-relay information. Conclusions are drawn in section IV.

2. THE RELAY FEEDBACK METHODS ANDASSOCIATED PROBLEMS

The relay feedback procedure is an effective yet simple technique for critical point estimation that is used in many industrial process controllers. However, while the relay feedback experiment design will yield sufficiently accurate results for many of the processes encountered in the process control industry, there are some potential problems associated with such relay feedback-based estimation techniques, associated with the estimation accuracy. The inaccuracies that may arise in using the existing procedures are a result of the approximations in describing function (DF) method, used in estimation of the critical point.

Many attempts have been done to amplify the effect of fundamental harmonic by using different shapes and structures of relay or by using fast fourier transform (FFT) to entire the effect of other components in calculations. All of these methods are developed to estimate the critical point of systems for ZN or other frequency domain tuning methods, where as it will be shown even these modified structures can not rectify the problem for small or large ratios of D/τ.

Fig. 1 shows the main structure of relay feedback autotuning where y is the controlled output, SP is the setpoint, e is the error and u is the manipulated input. A relay with hysteresis band is placed in the feedback loop. The Astrom-Hagglund relay feedback system is based on the observation, when the output lags behind the input by $-\pi$ radians, the closed loop system may oscillate with a period P_u. Fig. 2 illustrates how the relay feedback system works. The period of the limit cycle is ultimate period and from the fourier series expansion, the amplitude a can be considered to be the result of the primary harmonic of the relay output. Therefore, the ultimate frequency and gain can be approximated as:

$$\omega_u = 2\pi/P_u \quad (1)$$

$$K_u = 4h/\pi a \quad (2)$$

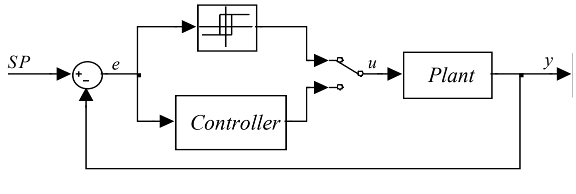

Fig. 1. Conventional relay feedback system

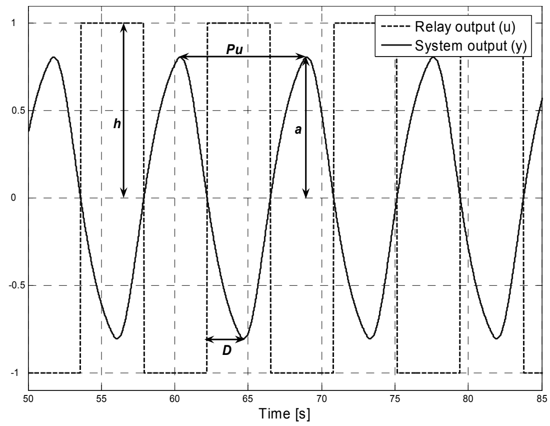

Fig. 2. Limit cycle oscillation for a system with positive steady state gain

The key idea behind the *pre-load relay feedback* is amplification of the fundamental oscillation frequency relative to the other harmonics via an additional periodic signal u_k, added to the relay output signal u_r to form a moderated input signal u to the process. The manipulated signal is defined in (3).

$$u = u_{relay} + u_k \,,\, u_k = k_{\text{constant}} e \quad (3)$$

With this moderation, the amplitude of the fundamental frequency at the output of the preload relay is boosted from $(4\mu/\pi)$ to $(4\mu/\pi + ka)$, while the residual part containing the higher harmonics remains essentially unchanged.

A detailed description of P-relay algorithm is presented by Tan et al. (2006) and from extensive empirical studies, they have recommended that the gain k can be fixed at 20–30% of the relay amplitude h. The ultimate gain and ultimate frequency are calculated from (4) and (1)

$$K_u = \frac{4h}{\pi a} + K \quad (4)$$

Saturation relay is an experiment designed to achieve a better accuracy of critical point estimation from relay feedback test. Since the square-wave u comes from an abrupt slope change in zero point, the saturation relay provides a smooth transition around the zero point. The saturation relay is characterized by two parameters: a relay height h and slope k. therefore the input of the relay is limited by a maximum $\bar{a}$ where:

$$\bar{a} = \frac{h}{k} \quad (5)$$

Saturation relay algorithm has been completely discussed by Yu. According to (YU, 2006), a system dependent critical slope k exists to indicate the success/ failure of the relay test. Also the smaller slope k yields a more accurate estimation of K_u and ω_u, but if it decreases to less than critical slope then the relay fails to generate a sustained oscillation. So a quantitative value of the slope should be given. The summarized procedure is as follows.

First the conventional relay feedback should be performed to obtain the estimated ultimate gain $\hat{K}_u$ from (2) and the slope k is defined as (6). Then the saturation relay is done and the ultimate gain and ultimate frequency are calculated from (7) and (1), based on the fundamental component of fourier transformation.

$$k = 1.4\hat{K}_u \tag{6}$$

$$K_u = \frac{2h}{\pi \bar{a}}\left[\left(\sin^{-1}\frac{\bar{a}}{a}\right)+\left(\frac{\bar{a}}{a}\sqrt{1-\left(\frac{\bar{a}}{a}\right)^2}\right)\right] \tag{7}$$

By using saturation relay, the modified signal of u is not in square shape and it is like a saturated sinusoidal wave and the DF approximation about the fundamental harmonic, has less error than conventional relay.

After the relay feedback experiment, the estimated critical point parameters can be directly used to calculate controller parameters. Alternatively, it is possible to use the relay data to estimate the transfer function of the system. Transfer function modeling is carried out in time or frequency domains. In the time domain procedure, the analytical expressions for relay feedback output response of different types (stable or unstable FOPDT, SOPDT and higher orders) of transfer functions have been developed (Wang et al., 1997, Majhi et al., 1999, and Kaya et al., 2001) and it can be useful if the structure of the model is known. In frequency domain method, the ultimate gain and ultimate frequency are used to back calculate the approximated transfer function, but this method uses the approximated values of the critical point and covers fewer TF types (Yu, 2006). Therefore, in this paper we use the time domain procedure for TF estimation of the FOPDT processes:

$$G_P(s) = \frac{K_P e^{-Ds}}{\tau s + 1} \tag{8}$$

Identification of the FOPDT systems consist of three steps. First, time to the peak amplitude D and a, h, P_u should be recorded. The time constant τ is calculated from (10) in an iterative manner and (9) provides an initial guess for τ. Finally the gain K_P is computed from (11).

$$\tau = \frac{\tan(\pi - D\hat{\omega}_u)}{\hat{\omega}_u} \tag{9}$$

$$\tau = \frac{\pi}{\omega_u \ln(2e^{D/\tau} - 1)} \tag{10}$$

$$K_P = \frac{a}{h(1 - e^{-D/\tau})} \tag{11}$$

Equations (12) and (13) are derived directly from mathematical expression of the first half cycle.

$$y(t) = K_P h(1 - e^{-t/\tau}), 0 < t < D \tag{12}$$

$$y(t) = K_P h(1 - e^{-t/\tau}) - 2K_P h(1 - e^{-(t-D)/\tau}) \\ , D < t < P_u / 2 \tag{13}$$

To compare the presented methods and probe their ability in parameter estimation of FOPDT system *G(s)*, we consider a various range of first order systems with D/τ =0.01-20. At each test τ and K_P are kept constant and the sample time is equal to 0.1τ. Then the critical point and TF parameters of the plant are estimated under different relay tests. Fig. 3 shows that increasing K_P/τ up to 4.5 times, when the D/τ is equal to 1, has no effects on the accuracy of estimation results of conventional relay method, and after that the percentage of error will increase only 10%. If the constant ratio of D/τ increases, then the K_P/τ can increase without any feasible changes in estimation error. But the ratio of D/τ has the most important role in accuracy of the estimations and for the ratio of K_P/τ=1, the results are shown in Table 1 and Fig. 4.

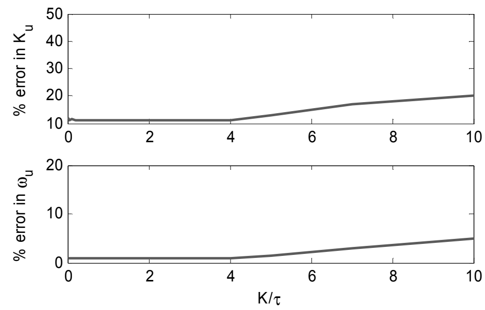

Fig. 3. Accuracy of relay estimation versus K_P/τ ratio

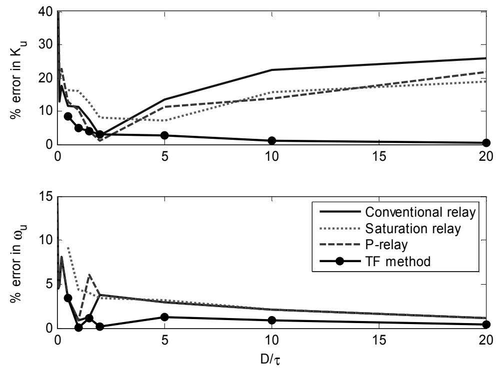

Fig. 4. Accuracy of relay estimation versus D/τ ratio

Results show little improvements in estimation of critical parameters for $G_P(s)$ under pre-load and saturation relay tests and much better estimation in transfer function method, for mediate to large ratios of D/τ. As is shown in Table 1, the saturation relay and TF estimation methods do not result in any sustained oscillation or acceptable parameter estimations for small ratio of D/τ. The salient point is data sampling. The following observations are drawn form the above results:

– The sampling time changes the nyquist curve of systems with small ratios of D/τ.

– The relay feedback test for systems with small dead time generates triangular oscillations with small amplitude rather than the steady state gain. Then the frequency domain methods assume this triangular oscillation as sinusoidal wave and the time domain methods try to fit this triangular oscillation to a first order response.

Table 1. Results of parameter estimation under relay feedback test for FOPDT systems

$G_P : K_P e^{-Ds} / (\tau s+1)$	***D/τ***	**0.01**	**0.05**	**0.1**	**0.2**	**0.5**	**1**	**1.5**	**2**	**5**	**10**	**20**
Continuous plant	K_u	157.08	32.05	16.35	8.50	3.81	2.26	1.76	1.52	1.13	1.04	1.01
	ω_u	157.08	32.04	16.32	8.44	3.67	2.03	1.45	1.14	0.53	0.29	0.15
Discrete plant	K_u	24.75	21.55	10.51	6.78	3.50	2.18	1.73	1.50	1.13	1.04	1.01
	ω_u	31.42	16.44	11.01	6.84	3.37	1.95	1.41	1.12	0.53	0.28	0.15
Conventional relay	K_u	7.62	9.92	9.17	5.58	3.09	1.94	1.6	1.46	1.28	1.27	1.27
	ω_u	15.71	15.71	10.47	6.28	3.49	1.96	1.43	1.16	0.54	0.29	0.15
Saturated relay	K_u	No sustained oscillation				2.92	1.83	1.51	1.38	1.21	1.20	1.20
	ω_u					3.68	2.03	1.47	1.16	0.54	0.29	0.15
P-relay	K_u	6.66	8.35	8.46	5.24	3.06	1.96	1.66	1.49	1.26	1.18	1.23
	ω_u	15.71	15.71	10.47	6.28	3.49	1.96	1.49	1.16	0.54	0.29	0.15
Approximated transfer function	K_u	Negative constant time				3.79	2.29	1.79	1.55	1.16	1.05	1.01
	ω_u					3.49	1.95	1.39	1.12	0.52	0.28	0.15
	K_P					1.85	1.37	1.19	1.06	1	1	1
	τ					2	1.53	1.368	1.16	1.16	1.14	1.14

3. PROPOSED DELAYED-RELAY FEEDBACK FOR ACCURATE FOPDT MODEL ESTIMATION

Having observed the problems associated with different frequency and time domain relay feedback-based estimation methods, we consider next the design of a modified relay feedback structure that addresses the issue of improved estimation accuracy for FOPSDT processes. The modification of the basic relay feedback method is motivated by the accuracy of estimation in systems with different *D/τ* ratios, and the modification is designed to boost the *D/τ* ratio of the system in the period of relay test, under a modified relay feedback configuration. Fig. 5 shows the proposed configuration using the Delayed-relay (abbreviated as D-relay). The D-relay is equivalent to a serial connection of the usual relay with a virtual dead time.

In this section, the operational principles and rationale for the proposed configuration and guidelines for choosing the added delay $\overline{D}$ will be elaborated.

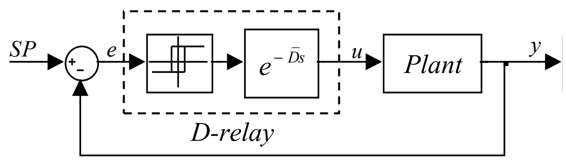

Fig. 5. Proposed configuration of D-relay feedback system

It was shown that none of the methods yields an accurate estimate for FOPDT systems with small *D/τ*. But as it is seen in Table 1, the approximated TF method has distinctly a better estimation rather than the others and according to figure 6 if the dead time rather than constant time increases up to 1, the error of critical point parameters (K_u, ω_u) are less than 8 percent and if it increases to 2 times, the error of FOPDT TF parameters become less than 18 percent for τ and 8 percent for K_P. This implies that while the approximated knowledge about D and τ is available, the added delay $\overline{D}$ can be chosen in order to achieve estimation with desirable accuracy.

3.1 Choice of added delay

From extensive empirical studies and optimization studies, it is recommend that the added delay ($\overline{D}$) be fixed such that the total delay of loop ($D+\overline{D}$) become at least 100-200% of τ. It is obvious that only the approximated values of τ and D are needed to start the modified D-relay feedback test and $\overline{D}=0$ for systems with *D>(1or2)τ*.

To start the identification or tuning procedure with relay test, it is necessary that the output of process be in the *SP* range. This mission is called startup and is usually carried out by open loop step changes that are the best experiments to derive initial required parameters like *D* and *τ*.

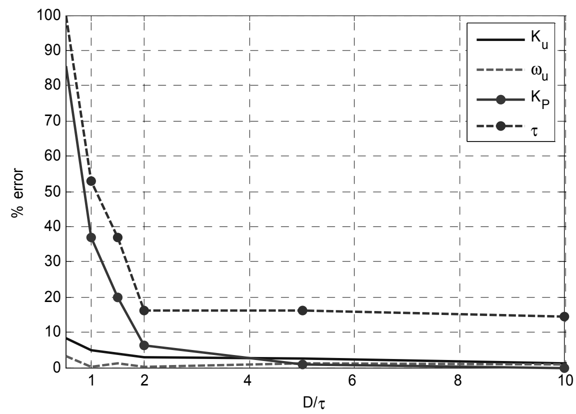

Fig. 6. Reduction of estimation error in TF method

The systematic procedure of D-relay feedback method is as follows:

Table 2. Robustness of D-relay method versus measurement noise

$\frac{e^{-0.1s}}{s+1}, T_{sample}=0.1$		σ^2 of Noise	Conventional relay			Delayed-relay		
K_u	ω_u		K_u % error	ω_u % error	*TF*	K_u % error	ω_u % error	*TF*
10.51	11	0	1.84 (82%)	1.85 (83%)	-	8.26 (20%)	8.5 (23%)	$\frac{0.95e^{-0.1s}}{0.91s+1}$
		0.001	1.49 (86%)	1.65 (85%)	-	9.18 (12.5%)	8.2 (25.5%)	$\frac{1.559e^{-0.1s}}{1.74s+1}$

1. Record the values of D and τ from startup stage or from a simple step test around the *SP*. In this step only the approximated ratio of D/τ is needed and not the exact values.
2. Calculate $\overline{D}$ and add it to the relay structure.
3. Run the D-relay and estimate the approximated first order TF from (9-11).
4. Optimize D, τ and ω_u in (14).

$$\tan \omega_u D_{total} = -\omega_u \tau \tag{14}$$

5. Calculate D from (15) and form the identified $G_P(s)$ according to (16)

$$D = D_{total} - \overline{D} \tag{15}$$

$$G_P(s) = \frac{K_P e^{-Ds}}{\tau s + 1} \tag{16}$$

6. Calculate critical point parameters K_u, ω_u from TF if needed.

3.2 Compensating the noise effect

Fig. 7 shows the output of conventional relay feedback and D-relay feedback (with $D_{total}/\tau=1$) tests for FOPSDT system $G_P(s)$, with same amplitude of relays, but as it is seen the amplitude of output in D-relay is clearly greater. If the measured output contains a noise signal with standard deviation of σ (in this example $\sigma=0.03$), then a hysteresis band of $\pm 3\sigma$ should be considered in relays. In systems with small ratios of D/τ, a is small and in the presence of measurement noise the hysteresis band may become greater than a. In this condition the conventional relay feedback waits till the amplitude of output reaches the Hysteresis band and then switches to the opposite value and therefore it reads the hysteresis band instead of real amplitude and the computations result in incorrect parameters. To avoid this problem, the operator should run the test again with higher relay amplitude that may be damaging for actuators.

But in D-relay with the same amplitude of relay, output has greater oscillations and in equal conditions with conventional relay, stronger measurement noise conducts the mentioned problem in estimation procedure. It turns out that the D-relay feedback method has more robustness to measurement noise rather than conventional structures and in the same conditions, the D-relay experiment has more signal to noise ratio (SNR).

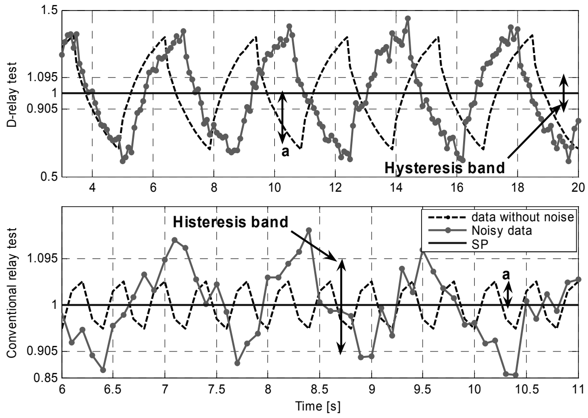

Fig. 7. Comparison between conventional relay and D-relay against measurement noise

Fig. 7 and Table 2 indicate that the D-relay method leads to less deviations in estimating both K_u and ω_u parameters.

3.3 Analysis to optimize the required time of D-relay test

Beyond the fact that the process output in relay feedback test is under control, it is more time efficient than the step test. So in this part we consider the time efficiency of D-relay rather than step test besides its accuracy and ability to handle the FOPDT systems with small time delay.

As regards to Fig. 6 if the total delay of loop increases, the estimated results become more accurate, but the time of relay test will increase and this is undesirable especially for processes with long time constants. So there is a trade off between the period of tuning test and the accuracy of estimated parameters and therefore the final performance of the tuned controller.

To establish a stable oscillation for FOPDT systems in relay feedback test, the required time is $2P_u$. And based on Fig. 8, in D-relay feedback test if the $\overline{D}$ has been selected such that the total delay of loop become τ, then the required time for D-relay test is 6τ, which is same to the step test with 95% accuracy factor. But if $\overline{D}$ increases and $D_{total}/\tau=2$, the time of D-relay test will become 11τ which is longer than step test with 98% accuracy factor. Creating greater ratios of D_{total}/τ, regards to small changes in estimation error, is not reasonable.

So the operator should trade off D_{total}/τ ratio for accurate estimation and although choosing $D_{total}/\tau=1$ leads to 30% error in estimated K_P and 5% error in K_u, but in process control of FOPSDT systems it doesn't result in large differences in the final performance of controlled system.

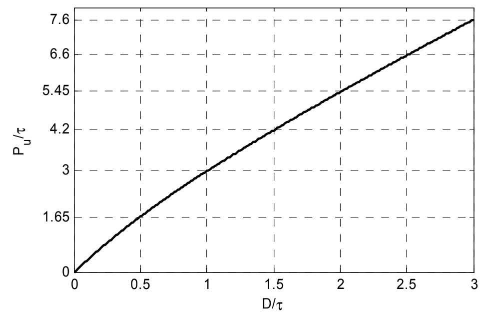

Fig. 8. P_u/τ as function of normalized dead time

As mentioned earlier, FOPSDT systems have large values of K_u and ω_u and therefore the common tuning rule of ZN doesn't result in good performance (D/τ<0.2). But methods like Tyreus-Luyben tuning rule, derived based on the integrator plus dead time system (17), work well for the constant time dominant processes considered here. IMC based PID controllers also yield acceptable output performance (Fig. 9).

$$G(s) = \frac{K_P e^{-Ds}}{s} \tag{17}$$

To investigate the improvement of final performance under D-relay tuning and to validate different tuning methods for FOPSDT systems, some simulations have been done.

In the first simulation example, PI controllers are tuned for the system of Table 2, using the estimated parameters of D-relay and conventional relay tests. Fig. 9 shows the superior performance for tuning with D-relay test.

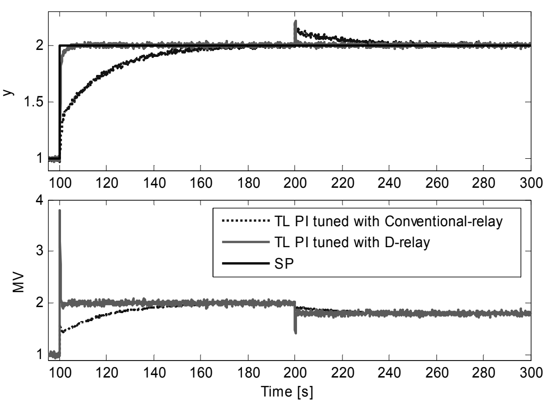

Fig. 9. Controller performance based on conventional and delayed relay tests tuning

In the next simulation the performance of different PI controllers with IMC, TL and ZN tuning methods is validated for the system of Table 2. Tunings have been performed based on accurate parameters of D-relay test and the result is illustrated in Fig. 10. According to the results, the ZN response has overshoot but the outputs and the manipulated variables of TL and IMC perform smoother.

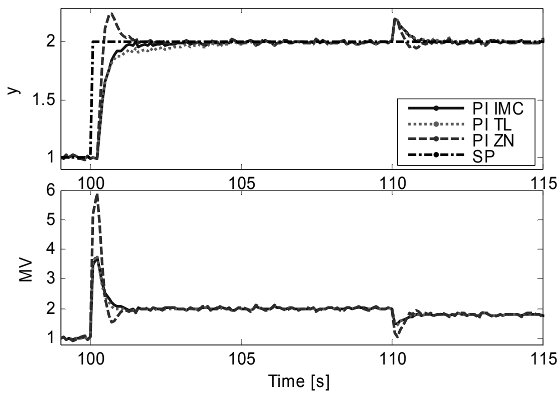

Fig. 10. Controllers' performance for FOPSDT system

4. CONCLUSIONS

This paper proposes a new relay feedback structure named as D-Relay feedback method for accurate estimation of first order transfer functions with small time delay to time constant ratio.

Empirical evidence is also provided to show other benefits of the proposed approach. These include improved identification and performance evaluation of tuned controllers based on an improved estimation, applicability to a class of processes with small D/τ when the conventional relay or other relay modifications fail or result in large inaccuracy, and finally more robustness for measurement noise.

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