Low Complexity Control of Hybrid Systems with Application to Control of Step-down DC-DC Converters

Jalal Habibi * **Behzad Moshiri** ** **Ali Khaki Sedigh** ***

* *School of Electrical and Computer Engineering, University of Tehran, Tehran, Iran,(e-mail: habibi@control.ee.ethz.ch).*
** *Control and Intelligent Processing Center of Excellence, University of Tehran, Tehran, Iran (e-mail: moshiri@ut.ac.ir)*
*** *Electrical Engineering Department, K. N. Toosi University of Technology, Tehran, Iran, (e-mail: sedigh@kntu.ac.ir)*

Abstract: Control of hybrid systems as those systems with mixed time-driven and event-driven dynamics faces the computational complexity as a main challenging problem. Explicit solution to the optimal control problems has been proposed as a tool to reduce the on-line computational burden. The complexity of the explicit solution is again prohibitive for large problems. This paper shows that how a recently-proposed approach by the authors can be utilized to reduce the computational complexity in explicit predictive control of hybrid systems. The proposed approach generates a family of suboptimal controllers for which the complexity and error can be controlled by a tuning parameter. The closed loop stability is guaranteed by a contractive constraint and is preserved in all suboptimal controllers. Application of the proposed scheme to hybrid control of synchronous step-down DC-DC converters clarifies the steps for modeling and controller design as well as the achieved computational benefits.

Keywords: Hybrid Systems; Predictive Control; Mixed Logical Dynamical Systems; Multiparamtric Programming; Mixed Integer Linear Programming; Computational Complexity; Contractive Predictive Control.

1. INTRODUCTION

Advanced control of real world complex processes face the problem of computation time as a main obstacle for real-time control task. Usually these advanced methods need to solve some numerical optimization problems to find the control action in each time instant and these computations should be performed in a short sampling time interval. Model predictive control as an outstanding member of advanced control strategies also faces the problem of computational complexity specially for large-scale systems. Development of efficient predictive controllers to relief the computational burden has been a major field of research for the last decade.

Besides the predictive control, introduction of hybrid systems theory as a deep theory to analyze those systems with mixed event-driven and time-driven dynamics, requires more computational effort for control synthesis tasks.

The nature of numerical optimization problems is affected when hybrid models are used in the formulation of predictive controllers. A major reason is that the hybrid models usually contain some logical/discrete variables and this in many cases leads to the presence of integer variables in the optimization problems. So the typical quadratic or linear programming problems in MPC formulation are changed to mixed-integer quadratic programming (MIQP) or mixed-integer linear programming (MILP) problems. Computational complexity of MIQP or MILP problems is also prohibitive for control of hybrid systems as will be addressed in the current paper.

The main goal of this paper is to show how an approximate multi-parametric MILP (mp-MILP) approach (Habibi [2008a]) can be used to develop a family of suboptimal predictive controllers for hybrid systems.

The selected framework for hybrid systems here is mixed logical dynamical modeling (Bemporad [1999]). In the following section some theoretical background about predictive control of MLD systems has been reviewed. Design steps is mentioned to formulate the optimization problem for predictive control of hybrid systems as a multi-parametric MILP problem. As an important part of this paper, the exact and approximate algorithms for mp-MILP are briefly reviewed. An illustrative example is mentioned to highlight the complexity reduction achieved by the proposed approach. section 5 concludes the paper.

2. PREDICTIVE CONTROL OF MLD SYSTEMS

2.1 Theoretical Background

In the mixed logical dynamical modeling of hybrid systems, dynamics, logic and constraints of a complex system are transformed into an integrated model. Logical and dynamical constraints are translated into mixed integer inequalities. Mixed integer inequalities, are those inequalities which contain continuous and integer variables. Using binary and auxiliary variables, hybrid systems are modeled as linear-like dynamical systems subject to mixed integer inequalities as constraints.

By using these techniques a broad range of hybrid systems could be modeled as following MLD structure:

$$\begin{array}{l} x[k+1] = Ax[k] + B_1u[k] + B_2\delta[k] + B_3z[k] \\ y[k] = Cx[k] + D_1u[k] + D_2\delta[k] + D_3z[k] \\ E_2\delta[k] + E_3z[k] \leq E_1u[k] + E_4x[k] + E_5 \end{array} \tag{1}$$

where the state, input and output vectors of the system have been denoted by x, u and y and binary and auxiliary variables by δ and z respectively.

In a basic general formulation of predictive control for MLD systems the objective function at time t is defined as follows:

$$\begin{array}{l} J(\{U\}_0^{N-1}, x(0\,|t)) = \sum\limits_{k=0}^{N-1} \|u(k\,|t) - u_e\|_{Q_1}^1 + \\ \|\delta(k|t) - \delta_e\|_{Q_3}^1 + \ \|z(k|t) - z_e\|_{Q_3}^1 + \|x(k|t) - x_e\|_{Q_4}^1 \end{array} \tag{2}$$

where N is the control horizon, $\{U\}_0^{N-1}$ denotes the optimization variables over control horizon, $x(0\,|t)$ is the measured state vector at time t and x_e, u_e, z_e and δ_e are steady state values for state, input and auxiliary variables.

The auxiliary variables are treated as optimization variables as well as input variables. Also the constraints have to be considered in the optimization task. So, based on predictive control scheme the following optimization problem is solved in each time step t.

$$\min_{\{U\}_0^{N-1}} \ J(\{U\}_0^{N-1}, x(0\,|t)) \tag{3}$$

$$\begin{array}{l} \text{s.t.} \ \|x(1\,|t) - x_e\|_1 \leq \alpha\,\|x(0\,|t) - x_e\|_1 \\ x(k+1\,|t) = Ax(k\,|t) + B_1u(k\,|t) + \\ \qquad\qquad B_2\delta(k\,|t) + B_3z(k\,|t) \\ E_2\delta(k\,|t) + E_3z(k\,|t) \leq E_1u(k\,|t) + E_4x(k\,|t) + E_5 \\ \text{for } k = 0, 1, \ldots, N-1 \end{array}$$

Note that the $x(0\,|t)$ is the measured state at time t. The first input sequence obtained during optimization is applied to the system and the whole procedure will be repeated in the next step based on the new measurement from the system. In the above formulation, the contraction constraint is used to guarantee the stability of the closed loop system (Habibi [2008b]). The proposed form of contraction constraint states that the state vector in the next step should have smaller L_1 norm with respect to $x(0\,|t)$.

2.2 The Problem of Computational Complexity

Different approaches have been proposed to reduce the computational burden for hybrid system optimization. Some methods exploit multi-parametric mixed integer programming approaches (Bemporad [2002], Dua [2000], Dua [2002]). In multi-parametric mixed integer programming, some parameters are present in the objective function or in the constraints and the goal is to find the optimal solution of a general MILP problem *as a function of parameters.*

The computational tasks of optimization problem in predictive control of MLD systems could be decomposed into on-line and off-line tasks. The *parametric* solution for a family of optimization problems is obtained using multi-parametric programming in an off-line manner. This parametric solution (which is called *explicit* solution) is computed off-line for different regions of the state space (which are called critical regions) and the solution is stored in a look-up table. Then the on-line computations is reduced down to distinguish the correct value of the optimizer in the table and then computing the control action based on this value. The main difficulty with explicit approaches is that the number of critical regions grows rapidly when the problem size increases and this results in a tremendous off-line computations.

An approximate algorithm has been proposed in (Habibi [2008a]) to reduce the complexity of explicit solution to mp-MILP problems. The main goal of this paper is *not* to address the theoretical properties of this approximate algorithm but to show that how the task of low-complexity predictive control of MLD systems could be performed using the proposed approximate mp-MILP algorithm.

2.3 MPC and parametric optimization problems

Transformation of general optimization problem (3) into the right-hand side parametric optimization problem requires several tricky steps. In the first step the norm operators are removed from objective function using a well known fact from the optimization theory. This fact states that the optimization problem with L_1 norm in the objective function

$$\min_x \|x\|_Q^1 \tag{4}$$

is equivalent to the following problem.

$$\begin{array}{l} \min\limits_{x,\varepsilon} \varepsilon(1) + \varepsilon(2) + \cdots + \varepsilon(n) \\ \text{s.t.} \ -\varepsilon \leq Qx \leq \varepsilon \end{array} \tag{5}$$

Based on this fact some auxiliary variables have to be defined for the optimization problem (15) and the following constraints should be included in the set of constraints.

$$-\varepsilon_u(k\,|t) \leq Q_1\left(u(k\,|t) - u_e\right) \leq \varepsilon_u(k\,|t) \tag{6}$$

$$-\varepsilon_\delta(k\,|t) \leq Q_2\left(\delta(k\,|t) - \delta_e\right) \leq \varepsilon_\delta(k\,|t) \tag{7}$$

$$-\varepsilon_z(k\,|t) \leq Q_3\left(z(k\,|t) - z_e\right) \leq \varepsilon_z(k\,|t) \tag{8}$$

$$-\varepsilon_x(k\,|t) \leq Q_4\left(x(k\,|t) - x_e\right) \leq \varepsilon_x(k\,|t) \tag{9}$$

where $\varepsilon_u(k\,|t)$, $\varepsilon_\delta(k\,|t)$, $\varepsilon_z(k\,|t)$ and $\varepsilon_x(k\,|t)$ are vectors of auxiliary variables of proper dimensions for $k = 0, 1, \ldots, N-1$. Let's introduce the vector of auxiliary variables for step k as follows.

$$\varepsilon_p(k) = \left[\varepsilon_u(k\,|t)^T, \varepsilon_\delta(k\,|t)^T, \varepsilon_z(k\,|t)^T, \varepsilon_x(k\,|t)^T\right]^T \tag{10}$$

Also the contraction constraint could be reformulated using some auxiliary variables. It can be shown that the introduced form of contraction constraint is equivalent to the following inequalities.

$$\begin{array}{l} -\varepsilon_c \leq x(1\,|t) - x_e \leq \varepsilon_c \\ \varepsilon_c(1) + \cdots + \varepsilon_c(n) \leq \alpha\,\|x(0\,|t) - x_e\|_1 \end{array} \tag{11}$$

where ε_c is the vector of auxiliary variables for contraction constraint. The auxiliary variables have to be included in the vector of optimization variables. So a tractable equivalent of the optimization problem is found as follows.

$$\min_{\{U\}_{k=0}^{N-1},\{\varepsilon_p(k)\}_{k=0}^{N-1},\varepsilon_c} \sum_{k=0}^{N-1} sum(\varepsilon_p(k)) \qquad (12)$$

s.t. (6), (7), (8), (9), (11)
$x(k+1\,|t) = Ax(k\,|t) + B_1u(k\,|t) + B_2\delta(k\,|t) + B_3z(k\,|t)$
$E_2\delta(k\,|t) + E_3z(k\,|t) \le E_1u(k\,|t) + E_4x(k\,|t) + E_5$
for $k = 0, 1, \ldots, N-1$

Here the operator $sum(.)$ stands for the sum of the elements of the operand vector.

In the next step the solution to the state equation for MLD systems is used to replace predicted states in the constraints by equivalent relation of optimization variables. The solution to state equation can be easily derived from MLD state equation as follows.

$$\begin{aligned} x(k\,|t) = A^k x(0\,|t) + \sum_{i=0}^{k-1} A^i [B_1 u(k-1-i\,|t) + \\ B_2\delta(k-1-i\,|t) + B_3 z(k-1-i\,|t)] \end{aligned} \qquad (13)$$

Substitution of $x(k\,|t)$ from above equation into (12) and assuming the vector of parameters as $\theta = \left[x(0\,|t)^T, \alpha \left\|x(0\,|t) - x_e\right\|_1\right]^T$ results in the following form of optimization problem.

$$\min_{U_r,U_b} C_rU_r + C_bU_b \qquad (14)$$

s.t. $G_rU_r + G_bU_b \le W + E\theta$

where U_r and U_b are the vectors of real and binary optimization variables respectively. So the optimization problem is now in special form of the general multi-parametric MILP problem for which the vector of parameters appears linearly in the set of constraints.

3. MULTI-PARAMETRIC MILP PROBLEMS

3.1 Exact Solution to mp-MILP Problem

Here a special class of mp-MILP is considered as follows.

$$z(\theta) = \min_{U_r,U_b} C_rU_r + C_bU_b \qquad (15a)$$

$$\text{s.t. } G_rU_r + G_bU_b \le W + E\theta \qquad (15b)$$

$$\theta_{\min}(s) \le \theta(s) \le \theta_{\max}(s),\ s = 1, \ldots, n_\theta \qquad (15c)$$

$U_r \in D_{U_r}$, $U_b \in \{0,1\}^{n_b}$

where θ is the vector of parameters with n_θ elements and D_{U_r} is the region of interest for real variables (usually a polytope) and n_b is the number of binary variables. Basically the problem is solved using a three-part procedure (Dua [2000]). Three parts in the exact mp-MILP procedure are initialization, mp-LP subproblem and MILP subproblem. The following MILP problem is solved in the initialization phase to find an initial feasible integer vector $\bar{U}_b$:

$$\min_{U_r,U_b,\theta} C_rU_r + C_bU_b \qquad (16a)$$

$$\text{s.t. } G_rU_r + G_bU_b \le W + E\theta \qquad (16b)$$

$$\theta_{\min}(s) \le \theta(s) \le \theta_{\max}(s),\ s = 1, \ldots, n_\theta \qquad (16c)$$

$U_r \in D_{U_r}$, $U_b \in \{0,1\}^{n_b}$

In the mp-LP part of the procedure, the binary vector U_b is fixed to $\bar{U}_b$ and the space of real variables is partitioned by solving the following mp-LP problem:

$$\hat{z}(\theta) = \min_{U_r} C_rU_r + C_b\bar{U}_b \qquad (17a)$$

$$\text{s.t. } G_rU_r + G_b\bar{U}_b \le W + E\theta \qquad (17b)$$

$$\theta_{\min}(s) \le \theta(s) \le \theta_{\max}(s),\ s = 1, \ldots, n_\theta \qquad (17c)$$

$U_r \in D_{U_r}$.

In the third part of the algorithm an MILP problem is solved in each critical region CR^i to determine if there exists a feasible binary vector U_b yielding a lower objective function.

$$\min_{U_r,U_b,\theta} C_rU_r + C_bU_b \qquad (18a)$$

$$\text{s.t. } G_rU_r + G_bU_b \le W + E\theta \qquad (18b)$$

$$C_rU_r + C_bU_b \le \hat{z}(\theta)^i \qquad (18c)$$

$$\sum_{j\in J^{ik}} (U_b)_j^{ik} - \sum_{j\in L^{ik}} (U_b)_j^{ik} \le \left|J^{ik}\right| - 1, \qquad (18d)$$

$k = 1, \ldots, K^i$, $\theta \in CR^i$, $U_r \in D_{U_r}$, $U_b \in \{0,1\}^{n_b}$.

Note that the constraints on the elements of vector U_b (18d) are used to exclude already analyzed binary solutions from all binary candidates for the critical region CR^i. Here, K^i is the number of binary solutions that have already been analyzed in CR^i and for each analyzed binary solution $(U_b)^{ik}$, the sets J^{ik} and L^{ik} are defined as follows:

$$J^{ik} = \{j\,|(U_b)_j^{ik} = 1\} \qquad (19)$$

$$L^{ik} = \{j\,|(U_b)_j^{ik} = 0\} \qquad (20)$$

We use $\left|J^{ik}\right|$ to denote the cardinality of the set J^{ik}.

The algorithm terminates whenever infeasibility occurs for master MILP sub-problems in all critical regions.

3.2 Approximate Solution to mp-MILP

A recently-proposed approximate approach for mp-MILP neglects all feasible solutions in the MILP subproblem which do not provide *important* improvement (Habibi [2008a]). The approximate algorithm is described as follows.

Approximate mp-MILP Algorithm

Step 0: Initialize the approximate solution $z(\theta,\mu)$ with ∞, the critical region CR^i with the initial description of the parameter space like $\theta_{\min}(s) \le \theta(s) \le \theta_{\max}(s)$, $s = 1, \ldots, n_\theta$.

Solve the MILP optimization problem (18). If the problem is not feasible, stop. The original mp-MILP problem is also infeasible.

If (18) is feasible, choose the binary part of the solution as the initial $\bar{U}_b$.

Step 1: (mp-LP subproblem) for each critical region with an associated binary vector $\bar{U}_b$:

1-1 - solve the mp-LP problem (17) to find a set of critical regions CR^i and their corresponding parametric solution $\hat{z}(\theta,\mu)^i$.

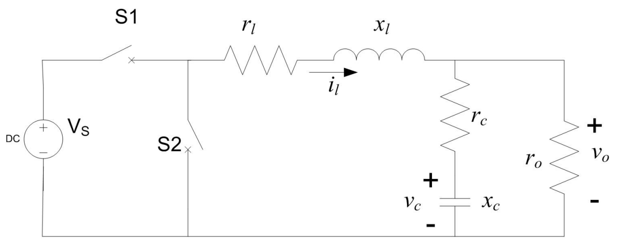

Fig. 1. The synchronous step-down DC-DC converter

1-2 - for each critical region CR^i compare the parametric solution $\hat{z}(\theta,\mu)^i$ and $z(\theta,\mu)$. If $\hat{z}(\theta,\mu)^i \le z(\theta,\mu)$ update the approximate solution for the region CR^i.

Step 2: (MILP subproblem) for each critical region solve the following problem.

$$\min_{U_r,U_b,\theta} C_rU_r + C_bU_b \tag{21a}$$

$$\text{s.t. } G_rU_r + G_bU_b \le W + E\theta \tag{21b}$$

$$C_rU_r + C_bU_b \le z(\theta,\mu) - \mu \tag{21c}$$

$$\sum_{j\in J^{ik}} (U_b)_j^{ik} - \sum_{j\in L^{ik}} (U_b)_j^{ik} \le \left|J^{ik}\right| - 1, \tag{21d}$$

$k = 1,\ldots,K^i$, $\theta \in CR^i$, $U_r \in D_{U_r}$, $U_b \in \{0,1\}^{n_b}$.

If the problem is infeasible the algorithm terminates in the current CR^i. if not, return to step 1 with new binary solution $\bar{U}_b$ and critical region CR^i. □

The result of the approximate mp-MILP algorithm is the approximate objective function $z(\theta,\mu)$ and the approximate parametric optimizers $U_r^*(\theta,\mu)$ and $U_b^*(\theta,\mu)$.

4. SUBOPTIMAL PREDICTIVE CONTROL

4.1 The Tradeoff Between Suboptimality and Complexity

Based on the formulation of the optimization problem mentioned in section 2.3 and the approximate mp-MILP algorithm, a family of suboptimal predictive controllers can be developed. The tuning parameter μ is used to control the level of suboptimality and complexity of the explicit solution of mp-MILP problems. Small values of μ will result in more accurate parametric solutions with partial improvement in the computational complexity compared to the exact solution. On the other hand, low complexity controllers can be achieved with large values of the tuning parameter.

4.2 Application to DC-DC Converters

High performance control of power electronics converters is a well-suited area for studying the complexity reduction schemes for hybrid systems. The reason is that from one side the system contains different switching and event-driven phenomena which makes usage of the hybrid modeling strategies desirable and at the same time due to high frequency switching, the control signal has to be provided in a short time interval.

Here we consider the control of step down buck DC-DC converter shown in Fig. 1. The switches S_1 and S_2 work alternatively with the same switching frequency. The switching frequency is assumed constant and the duty cycle as manipulated variable determines the interval for which each switch is conducting.

Hybrid modeling and control of DC-DC converters have been deeply studied in (Geyer [2005a]) as well as other applications of hybrid systems theory in power systems. A special approach - called ν-resolution modeling - has been introduced in (Geyer [2005b]) which aims at obtaining a precise model of the converter (compared to the traditional average models) in order to achieve high performance controllers. Here we use the same ideas to obtain an MLD model for the DC-DC converter but our approach contains considerable reduction in the complexity of the model itself as will be shown later.

Assuming the inductor current i_l and the output voltage v_o as state variables and the value of the duty cycle $d(k)$ as the manipulated variable, the state vector evolves as follows based on the situation of the S_1 switch.

$$\frac{dx(t)}{dt} = \begin{cases} Ax(t) + BV_s, & (kT_s \le t < (k+d(k))T_s) \\ Ax(t), & ((k+d(k))T_s \le t < (k+1)T_s) \end{cases} \tag{22}$$

where: $A = \begin{bmatrix} -\dfrac{r_l}{x_l} & -\dfrac{1}{x_l} \\ \dfrac{r_o(1 - x_c r_c \frac{r_l}{x_l})}{x_c(r_o+r_c)} & -\dfrac{(1 + x_c r_c \frac{r_o}{x_l})}{x_c(r_o+r_c)} \end{bmatrix}$ and

$$B = \begin{bmatrix} \dfrac{1}{x_l} \\ \dfrac{r_o}{r_o+r_c}.\dfrac{r_c}{x_l} \end{bmatrix}$$

Since the circuit might work with different voltage sources, it would be suitable to normalize the state vector with respect to the magnitude of the input source V_s as $x'(t) = x(t)/V_s$. So the state equation would be as follows:

$$\frac{dx'(t)}{dt} = \begin{cases} Ax'(t) + B & ,kT_s \le t < (k+d(k))T_s \\ Ax'(t) & ,(k+d(k))T_s \le t < (k+1)T_s \end{cases} \tag{23}$$

There are basically reference values for the state vector and an upper limit for the inductor current which are also normalized for consistency. We denote these normalized values by $i'_{l,ref}$, $v'_{o,ref}$ and $i'_{l,\max}$ respectively.

The main idea in the ν-resolution modeling approach is to consider ν sub-interval in the kth switching period. The state vector evolves in the intermediate sub-intervals based on the fact that whether the switching occurs before, after or during each specific sub-interval. To track the information of the state vector during evolution, some auxiliary vectors denoted by $\gamma(n)$ ($n = 0,1,\ldots,\nu$) are used. Also, for the kth step ν auxiliary binary variables have to be defined as follows:

$$\delta_n(k) = 1 \Leftrightarrow d(k) \ge \frac{n}{\nu} \quad \text{, for } n = 1,\ldots,\nu \tag{24}$$

The $\gamma(n)$ vectors are initialized in the kth switching period by $\gamma(0) = x(k)$ and the value of $x(k+1)$ is obtained after evolution during the intermediate sub-intervals by $x(k+1) = \gamma(\nu)$. The evolution of $\gamma(n)$ variables is stated as follows:

$$\gamma(1) = \begin{cases} \Phi\gamma(0) + \Psi & \delta_1 = 1 \\ \Phi\gamma(0) + \Psi\nu d(k) & \delta_1 = 0 \end{cases} \tag{25}$$

and:

$$\gamma(n+1) = \begin{cases} \Phi\gamma(n) + \Psi & ,[\delta_n = 1] \wedge [\delta_{n+1} = 1] \\ \Phi\gamma(n) & ,\delta_n = 0 \\ \Phi\gamma(n) + \Psi[\nu d(k) - n] & ,[\delta_n = 1] \wedge [\delta_{n+1} = 0] \end{cases} \quad (26)$$

for $n = 1, \ldots, \nu - 1$ and where $\Phi = e^{A\frac{T_s}{\nu}}$ and $\Psi = -A^{-1}(I - e^{A\frac{T_s}{\nu}})B$.

Based on MLD formalism we define $\nu - 1$ binary variables as follows:

$$\begin{aligned} &[\delta_{\nu+1}(k) = 1] \Leftrightarrow [\delta_1(k) = 1,\, \delta_2(k) = 1] \\ &[\delta_{\nu+2}(k) = 1] \Leftrightarrow [\delta_2(k) = 1, \delta_3(k) = 1] \\ &\quad \vdots \qquad\qquad \vdots \qquad\qquad \vdots \\ &[\delta_{2\nu-1}(k) = 1] \Leftrightarrow [\delta_{\nu-1}(k) = 1,\, \delta_\nu(k) = 1] \end{aligned} \quad (27)$$

With a definition like $[\delta'_n(k) = 1] \Leftrightarrow [\delta_n(k) = 1,\, \delta_{n+1}(k) = 1]$ (where $\delta'_n(k)$ is one of the $\nu - 1$ newly-defined variables) the equation 26 can be rewritten as follows:

$$\begin{aligned} \gamma(n+1) = \Phi\gamma(n) + (n+1)\Psi\delta'_n(k) - (n)\Psi\delta_n(k) - \\ \Psi\nu d(k)\delta'_n(k) + \Psi\nu d(k)\delta_n(k) \end{aligned} \quad (28)$$

Again based on MLD modeling approach the products of real and binary variables are defined as auxiliary continuous variables. Here we define $2\nu - 2$ auxiliary variables as follows:

$$\begin{aligned} z_1(k) = d(k)\delta_1(k), \ldots, z_{\nu-1}(k) = d(k)\delta_{\nu-1}(k) \\ z_\nu(k) = d(k)\delta_{\nu+1}(k), \ldots, z_{2\nu-2}(k) = d(k)\delta_{2\nu-1}(k) \end{aligned} \quad (29)$$

The equation 28 is rewritten using the auxiliary variables as follows:

$$\begin{aligned} \gamma(n+1) = \Phi\gamma(n) + (n+1)\Psi\delta'_n(k) - (n)\Psi\delta_n(k) - \\ \Psi\nu z'_n(k) + \Psi\nu z_n(k) \end{aligned} \quad (30)$$

So for each switching period there are $2\nu - 1$ binary variables and $2\nu - 2$ auxiliary variables which are assumed to be concatenated in vectors $\delta(k)$ and $z(k)$ respectively.

The MLD state equation is obtained from $\gamma(n)$ variables. The matrices are as: $A = \Phi^\nu$, $B_1 = \Phi^{\nu-1}\Psi\nu$,

$B_2 = [\Phi^{\nu-2}(\Phi\Psi - \Psi), -2\Phi^{\nu-3}\Psi, -3\Phi^{\nu-4}\Psi, \ldots, -(\nu-1)\Psi, 0, 2\Phi^{\nu-2}\Psi, 3\Phi^{\nu-3}\Psi, 4\Phi^{\nu-4}\Psi, \ldots, \nu\Psi]$

and $B_3 = [\Phi^{\nu-2}(\nu I - \Phi\nu)\Psi, \Phi^{\nu-3}\Psi\nu, \Phi^{\nu-4}\Psi\nu, \ldots, \Psi\nu, -\Phi^{\nu-2}\Psi\nu, -\Phi^{\nu-3}\Psi\nu, \ldots, -\Psi\nu]$.

The constraints in the MLD model are formed by those inequalities which are required to model the definition of auxiliary and binary variables and also by operational constraints. The main operational constraints are limitation of the duty cycle ($0 \le d(k) \le 1$) and limitation of the inductor current. The latter couls be considered more precisely in the intermediate sub-interval by including the current limit as follows:

$$-i'_{l,\max} \le [1, 0]\gamma(n) \le i'_{l,\max} \quad ,\text{for } n = 0, \ldots, \nu - 1 \quad (31)$$

In the sequel the problem of predictive control for the step-down converter is considered. Basically the control goal is to track the output reference voltage while respecting the operational constraints. Reducing the ripple on the duty cycle is also generally desirable but for the sake of simplicity, the tracking problem is considered here as the main objective. So the objective function over the following N-step horizon is assumed as follows:

$$J(k) = \sum_{l=0}^{N-1} \left\| v'_o(k + l\,|k) - v'_{o,ref} \right\|_1 \quad (32)$$

The optimization in each switching period consists of minimization of this objective function subject to the constraints of the MLD model. The first element from obtained sequence of duty cycles ($d(k + l\,|k)$ for $l = 0, 1, \ldots, N - 1$ is applied to the system and the procedure will be repeated in the next step.

Note that based on the contractive predictive control approach, a contraction constraint is also considered in the problem to ensure the closed loop stability. Here the steady state value for the state vector is obtained a priori from static transfer function analysis.

Now we consider the problem with the typical numerical values $x_c = 70\,p.u.$, $r_c = 0.001\,p.u.$, $x_l = 3\,p.u.$, $r_l = 0.05\,p.u.$, $i_{l,max} = 3\,p.u.$, $r_o = 1\,p.u.$, $N = 2$ and $\nu = 3$.

The inductor current, the output voltage and the duty cycle for the closed loop system are shown in Figs. 2 to 4. The green dashed graphs on the first two figures shows the response of the ν-resolution model to the optimized duty cycle. The exact explicit solution to the optimization problem (obtained with $\mu = 0$)consists of 325 regions as depicted in Fig. 5. The exact solution requires solving 2296 MILP and 915 mp-LP subproblems. The number of critical regions is reduced down to 157 region by selecting the tuning parameter as $\mu = 10$ as is shown in Fig. 6.

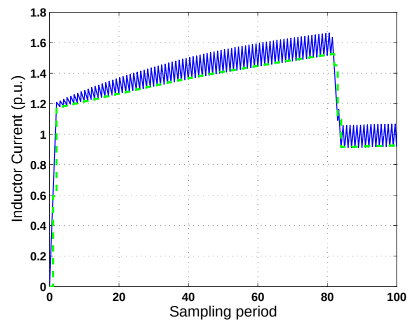

Fig. 2. The inductor current

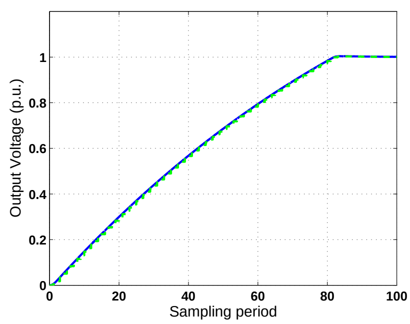

Fig. 3. The output voltage

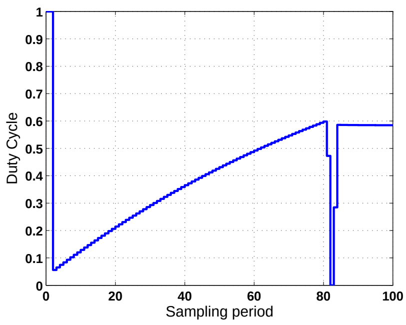

Fig. 4. The optimized duty cycle

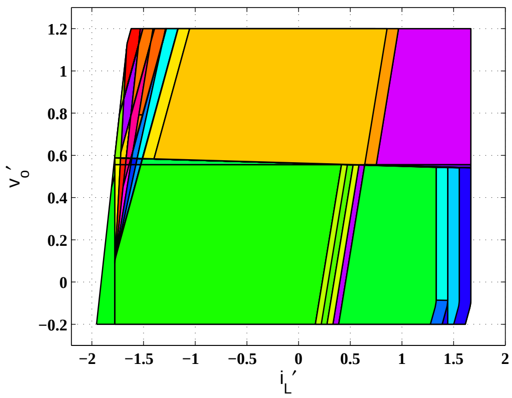

Fig. 5. Critical regions for exact explicit predictive controller

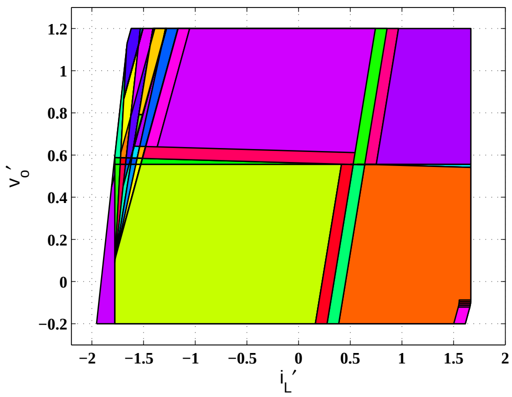

Fig. 6. Critical regions for suboptimal explicit predictive controller

To find this low complexity controller 858 MILP and 47 mp-LP subproblems have to be solved. So clearly the measure for the complexity of the "solution" (i.e. the number of the critical regions) has been reduced by a factor of 2. At the same time the measures for the complexity of the procedure to obtain the solution (i.e. the number of MILP and mp-LP subproblems) have been reduced by factors of 2.67 and 19.46 respectively.

5. CONCLUSION

A novel approximation algorithm for multi-parametric mixed integer linear programming problems, utilized to develop suboptimal controllers with adjustable complexity and optimality. The step-by-step procedure to formulate the optimization problem with the contractive controller as an mp-MILP problem discussed. Since all suboptimal controllers respect the constraints, the closed loop stability is preserved using the proposed scheme. Hybrid modeling of the step-down DC-DC converter in the MLD framework and the procedure for the design of the explicit controller highlights the achieved computational benefits using proposed low complexity approach.

ACKNOWLEDGEMENTS

The authors would like to thank Prof. Manfred Morari and Automatic Control Laboratory, ETH Zurich University for perfect environment to perform this research work, Michal Kvasnica for excellent support of MPT Toolbox and Miroslav Baric for his helpful comments and suggestions.

REFERENCES

A. Bemporad and M. Morari. Control of systems integrating logic, dynamics and constraints. *Automatica*, volume 35, number 3, pages 407-427, 1999.

A. Bemporad, M. Morari, V. Dua and N. Pistikopoulos. The Explicit Linear Quadratic Regulator for Constrained Systems. *Automatica*, volume 38, number 1, pages 3-20, 2002.

V. Dua and E. N. Pistikopoulos. An algorithm for the solution of multi-parametric mixed integer linear programming problems. *Annals of Operations Research*, volume 99, pages 123-139, 2000.

V. Dua, A. Bozinis and E. N. Pistikopoulos. An algorithm for the solution of multi-parametric mixed integer linear programming problems. *A Multi-parametric Programming Approach for Mixed-Integer Quadratic Engineering Problems*, volume 26, pages 715-733, 2002.

T. Geyer. Low complexity model preictive control in power electronics and power systems. Ph.D. Dissertation, Swiss Federal Institute of Technology (ETH) Zurich, Switzerland, 2005.

T. Geyer, G. Papafotiou and M. Morari. Model predictive control in power electronics: A hybrid systems approach. Proc. 44th IEEE Conference on Decision & Control and European Control Conference, volume 1, pages 5606-5611, 2005.

J. Habibi, B. Moshiri and A. Khaki Sedigh. Suboptimal control of hybrid systems using approximate Multi-Parametric MILP. To appear in Proc. 47th IEEE Conference on Decision & Control and European Control Conference, 2008.

J. Habibi, B. Moshiri and A. Khaki Sedigh. Contractive Predictive Control of Mixed Logical Dynamical Hybrid Systems. *International Journal of Innovative Computing, Information and Control*, volume 4, number 6, pages 1283-1297, 2008.

M. Kvasnica, P. Grieder and and M. Baotic. Muti-Parametric Toolbox (MPT). 2004, available from http://control.ee.ethz.ch/ mpt/.