A Hands-On Approach Toward Vehicle Velocity Estimation

Ansgar Rehm * , **Hui Fan** **

* *University of Applied Sciences Osnabrck, D-49076 Osnabrck, Germany (Tel: ++49-541-9692156; e-mail: a.rehm@fh-osnabrueck.de).*
** *Tesis Dynaware GmbH, München.*

Abstract: Automotive vehicle velocity estimation based only on steering angle and angular wheel velocity measurements is considered in the paper at hand. The approach is based on stationary Kalman filter design combined with a suitable preprocessing of the wheel velocity signals. A detailed assessment of the results by comparison with measured data is given. Possible applications include hierarchical monitoring of vehicle dynamics sensor networks.

Keywords: vehicle velocity estimation, Kalman filter

1. INTRODUCTION

Vehicle velocity estimation is one of most important ingredients of advanced driver assistant systems. Already the ABS system (anti look braking system) developed more than 25 ago crucially relies on the vehicle velocity. More recent systems with the need of the velocity signal are adaptive cruise control and lateral vehicle dynamic control systems (ESP, VDC, see (van Zanten et al. [1998], van Zanten [2006]) for an overview). These systems are highly safety critical with the need of accurate signals. However, vehicle velocity is also needed in non-critical applications as for example the velocity dependent control of windscreen wipers in luxury vehicles.

Direct measurement of vehicle velocity (over-ground velocity) from within the vehicle is possible, however, expensive. The optical reference measurement system used for assessment of the estimation results in the paper at hand is in the price region of a middle class vehicle. Therefore, vehicle velocity is a signal that is inferred form other measurements within the systems mentioned above.

Details on the estimation algorithm are considered to be secret knowledge of the ABS-, VDC-, ESP-system producers. Recent publications (Isermann [2006]) indicate that estimation is based on a Kalman filter set-up together with a set of Fuzzy rules (as e.g. in (Kiencke and Nielsen [2005], Semmler [2006])) for a weighted velocity update (based on angular wheel velocities). Additionally, all available information on longitudinal forces (braking moment, motor moment, optionally a longitudinal acceleration sensor) is included.

During critical situations (braking) it is possible to actively enhance the quality of the Kalman injection term: one of the rear wheels is under-braked for a short time, i.e. the applied braking moment is less than actually possible. Slipping at this wheel is thus reduced and an accurate velocity information can be inferred from the angular wheel velocity of this wheel. Braking distance is only marginally affected by this procedure due to the small time interval and the fact that the rear wheels only have a minor effect on deceleration.

In summary: velocity estimation is possible. Sophisticated procedures are available and they are applied in real-live applications for at least 10 years. Why consider a new approach toward estimation of vehicle velocity? The main reasons are the following ones:

(1) It is unpopular within the automotive business environment to share information, especially safety critical information. This is even more true if competitors are involved.
(2) It is immediately clear that system-internal “tricks” (especially the under-braking described above) will not be available to other systems requiring velocity information.
(3) The procedure for velocity estimation described above is rather complex in the sense that many sensors are involved. Therefore the estimation procedure is rather susceptible to faults and uncertainties. Uncertainties are especially linked to information from the motor management that may include up to 30% error (depending on car manufacturer specifications).

The paper at hand is especially linked to the last point. With angular velocities and steering angle, only few measurements are considered as a basis for velocity estimation. These sensors are based on a digital measurement principle with a correspondingly small failure rate and uniform time-invariant measurement uncertainty (Bosch [2003]). The corresponding velocity estimate therefore can be considered intrinsically safe (Henry and Clarke [1993]).

This fact is important in a fault detection context for lateral vehicle dynamics sensor networks since all detection algorithms rely on an available velocity estimate (Ding et al. [2004], Halbe [2007]). In turn it is possible to include the proposed algorithm within a supervision architecture that employs a first intrinsically safe velocity estimation as a basis for failure detection of the additional sensors that are mandatory for the complex velocity estimation approach outlined above (Rehm and Hofmann [2004], Rehm [2008]).

However, the proposed velocity estimation is also important in its own right: comparison of the estimated signal (based on wheel velocities and steering angle) with a velocity estimate from a VDC/ESP system shows that the overall difference is small. Therefore the proposed algorithm may serve as a good alternative for many applications as e.g. for adaptive cruise control. On the other hand the algorithm proposed in this paper is no alternative for braking systems as e.g ABS/VDC/ESP. The crucial point is that these systems never ever are allowed to underestimate vehicle velocity. In such a case it may be possible that braking pressure is reduced by the system while the driver still is requiring maximal deceleration. Surely an unpleasant situation with the concrete wall in front of you.

1.1 Contributions and Related Results

Almost all contributions toward longitudinal velocity estimation are based on the evaluation of rigid body dynamics of the car in combination with a model of the force generation at the tires. The main difference in these approaches is the complexity of the assumed model for the tire forces and the assumed sensors available in order to extract information on longitudinal velocity.

In (Kiencke and Nielsen [2005], Semmler [2006]) Fuzzy logic in combination with a Kalman filter is used to generate a velocity estimate based on angular wheel velocities and longitudinal acceleration. In (Canudas de Wit et al. [2003]) wheel torque in combination with angular wheel velocity is used to estimate longitudinal velocity in combination with an adaptation of a friction parameter.

Recent approaches to velocity estimation consider additionally lateral dynamics in order to also estimate lateral velocity. In (von Vietinghoff et al. [2007]) an extended Kalman filter appproach along this line is proposed. Sensor information used in this approach consists of angular wheel velocities, steering angle, yaw rate, and lateral acceleration. With the same sensor information a series of papers (Imsland et al. [2005, 2006, 2007]) advocates nonlinear observer design for vehicle velocity determination. Computational efficiency of this approach is shown by comparison with a extended Kalman filter set-up. Assessment by realistic measurement data shows the applicability of the proposed algorithm in an automotive environment.

Rigid body dynamics is in principle not necessary if GPS measurements are available. However, in practice GPS sensor outage (foliage, tunnels) and multi-path effects (Grewal et al. [2007]) enforce sensor fusion. This approach is examined in (Bevly et al. [2000], Carlson and Gerdes [2005]). With decreasing GPS sensor costs this set-up will become more important in the future. However, actually this approach cannot be considered a competitive cheap sensor configuration for vehicle velocity estimation in an automotive context.

In the paper at hand a minimalistic sensor configuration with only angular wheel velocities and steering angle is considered. Velocity estimation is performed by means of few stationary Kalman filters. Switching between the filters is depending on driving situation. The most important point is a sophisticated preprocessing of the angular wheel velocities in order to provide high quality Kalman filter updates. Compared with the approaches mentioned before this implies a rather efficient velocity estimate with respect to computational effort and sensor configuration. Additionally there are only few parameters to be tuned in order to adapt the proposed algorithm to different vehicles. Application of the approach to real data from series type sensors and comparison with reference measurements shows rather good results. The drawback of the proposed algorithm compared with the work in (Imsland et al. [2005, 2006, 2007]) is that there is no guaranteed convergence of the velocity estimate in a strict mathematical sense. Instead, as usual in automotive practice, the functionality of the proposed algorithm is established by successful testing of driving situations from a representative catalog.

2. ESTIMATION SET-UP

Rigid body dynamics in longitudinal vehicle direction is given by

$$\dot{v}_x = v_y \omega_z + a_x \tag{1}$$

if movement is restricted to a horizontal plane. Here, v_x, v_y denote the components of the velocity vector in body fixed coordinates (v_x longitudinal velocity, v_y lateral velocity), ω_z the angular velocity with respect to the vertical axis of the vehicle (yaw rate), and a_x the measured acceleration.

2.1 Kalman Filter Set-Up

Since only angular wheel velocities and steering angle are available measurements in the proposed approach, none of the quantities on the right hand side of (1) is available. Instead, the simplistic discrete-time model (sampling time T_s)

$$\begin{pmatrix} v_x(k+1) \\ a_x(k+1) \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & T_s \\ 0 & 1 \end{pmatrix}}_{=A} \begin{pmatrix} v_x(k) \\ a_x(k) \end{pmatrix} + \begin{pmatrix} n_1(k) \\ n_2(k) \end{pmatrix}, \quad k = 0, 1, \ldots \tag{2}$$

is used as starting point for Kalman filter design. The equations imply that longitudinal velocity is only a result of acceleration and acceleration is constant. All errors within this model (i.e. non-constant acceleration, influence of the yaw rate, longitudinal or lateral inclination of the road) are assumed to be captured by the process noise $\underline{n}(k) = (n_1(k), n_2(k))^T$.

The model (2) is combined with the measurement equation

$$y(k) = \underbrace{(1,\ 0)}_{=C} \begin{pmatrix} v_x(k) \\ a_x(k) \end{pmatrix} + w(k) \tag{3}$$

with measurement noise $w(k)$. However, the velocity is not directly available. An estimate is computed as the weighted mean of the velocity information inferred from the four angular wheel velocities (see the following sections).

The covariance matrix Q of the process noise is assumed to be a constant matrix with

$$Q = E[\underline{n}\,\underline{n}^T] = \begin{pmatrix} 0.0001 & 0 \\ 0 & 0.1 \end{pmatrix}.$$

The absolute values of the entries in Q are determined heuristically (based on measurements from a broad range

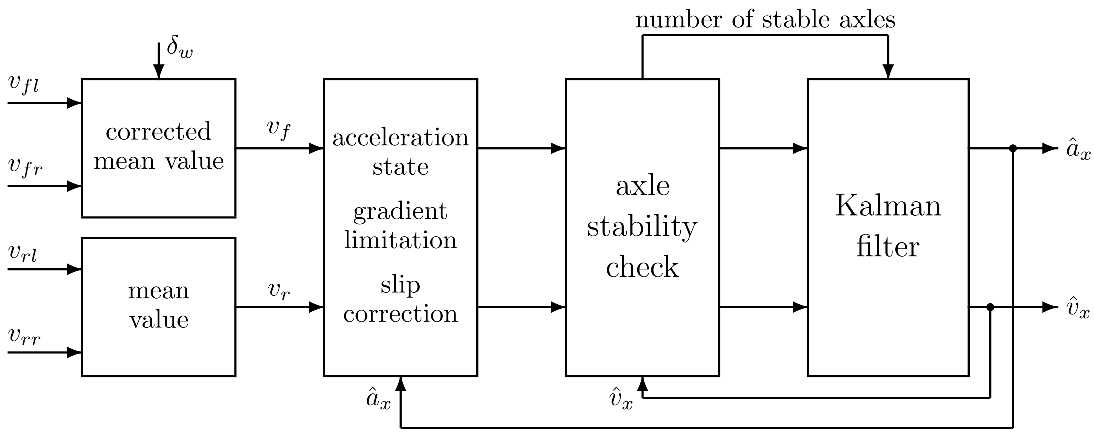

Fig. 1. Velocity estimation based on wheel velocities and steering angle

of different driving situations). The factor 10^3 between the diagonal entries of Q indicates the different validity of the model assumptions of the two equations in (2).

The variance R of the measurement noise is depending on the actual driving situation. Here, $R \in \{r_1, r_2, r_3, r_4\}$ is considered. The constant values r_i reflect whether front/rear angular axle velocities are reliable sources for the determination of the vehicle velocity v_x or not (see Section 2.4.1).

With (2), (3) and covariance matrices Q, R_i, $i = 1(1)4$ it is possible to compute stationary Kalman injection matrices L_i, $L_i \in \mathbb{R}^{2\times 1}$, $i = 1(1)4$ (Kalman [1960]). The corresponding Kalman filter equations are given by

$$\begin{pmatrix} \hat{v}_x(k+1) \\ \hat{a}_x(k+1) \end{pmatrix} = (A - L_i C) \begin{pmatrix} \hat{v}_x(k) \\ \hat{a}_x(k) \end{pmatrix} + L_i y(k). \quad (4)$$

Switching between the four Kalman filters described by (4) depends on driving situation which is classified by means of angular wheel velocity measurement. The overall structure of velocity estimation and the inclusion of the Kalman filter is depicted in Figure 1.

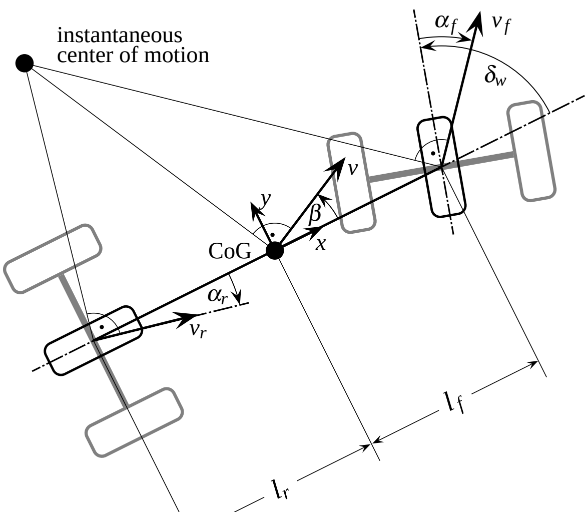

Fig. 2. Scheme of the single track model.

2.2 Axle Velocities

In order to create a velocity "measurement" for the Kalman filter, the available angular wheel velocities are converted into wheel velocities (v_{fl}, v_{fr}, v_{rl}, v_{rr}, subscript "fl" for "front right" etc.) by means of the wheel radius.

This is already the main source of uncertainty in a realistic automotive environment, since the radius will change due to tire wear. Even different tire diameters are in general approved for a given wheel rim. Therefore the wheel radius is typically not directly given. This problem can be overcome by on-line estimation of the wheel radius (see Gustavsson et al. [2007] for a least squares regression approach).

Here, availability of wheel velocities is assumed. Evaluation of the proposed estimation scheme to experimental data in the next section is based on measured tire diameters for new tires. In order to exclude the influence of varying wheel radius as far as possible, the approximately 350 examined driving maneuvers were carried out with 10 sets of tires from one production line.

The first step to fuse the four wheel velocities is to consider the single track model (Figure 2), i.e to consider axle velocities (average axle velocity of a fictitious single track vehicle). With the front steering configuration in Figure 2 the mean value

$$v_r = \frac{1}{2}(v_{rl} + v_{rr}) \quad (5)$$

at the rear axle directly renders a longitudinal wheel velocity information. However, due to steering ($\delta_w \neq 0$) and rotation of the vehicle ($\omega_z \neq 0$), the mean value at the front axle in general will contain lateral velocity components. Therefore rigid body kinematics (Wong [2001]) is used to compute the fictitious velocity at the middle of the rear axle which corresponds to the available mean velocity at the front axle. This value

$$v_f = \frac{1}{2\cos\delta_w}(v_{fl} + v_{fr}) - l\omega_z \tan\delta_w \quad (6)$$

(with wheel base $l = l_f + l_r$, see Fig. 2) contains compatible information to v_r from (5) provided that the lateral vehicle velocity at the rear wheels is zero. This may happen during unstable driving situations. However, possible errors only have a small influence on the Kalman estimate since in these situations the Kalman up-date is mainly based on the integration of the estimated acceleration (see Section 2.4.1).

A mayor problem with Eq. (6) is that only wheel velocities and steering angle are within the assumed measurement configuration, but not the yaw rate ω_z. However, the steering angle δ_w at the wheels (computed from the driver steering angle δ by means of the characteristic curve of the transmission linkage) is in general small (exception: low velocity situations like parking). Thus small angle arguments apply and with the approximation $\omega_z \approx v_f \cdot \delta_w / l$ (stationary driving, see Rajamani [2006]) we get

$$v_f \approx \frac{1}{2}(v_{fl} + v_{fr}) \cos \delta, \tag{7}$$

i.e. a possibility to express v_f without the yaw rate information.

2.3 Acceleration State, Gradient Limitation, and Slip Correction

Wheel velocities or more precisely axle velocities were corrected from kinematic influences in the previous subsection. In this subsection correction from kinetic effects is considered. Simply speaking the aim is to extract the longitudinal velocity component within the axle velocities as accurate as possible.

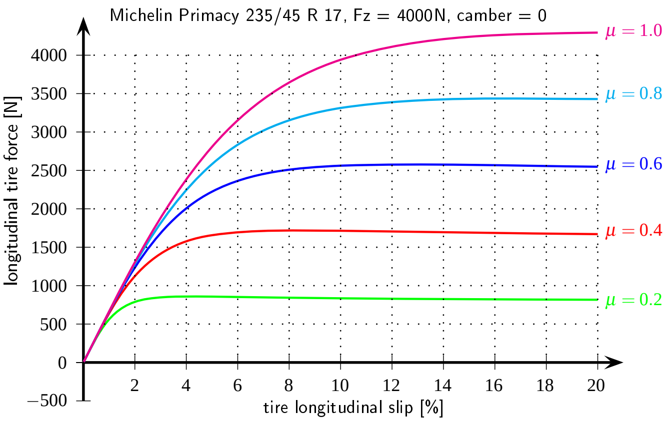

Fig. 3. Wheel forces due to slip and friction coefficient.

Acceleration State A state automaton is introduced to distinct the situations acceleration, constant driving, and deceleration. The structure and switching conditions are given in Figure fig:stateautomata. Switching is based on the estimated acceleration. The non-symmetric conditions establish a hysteresis characteristic in order to reduce scattering. Furthermore, switching back to the "constant driving state" is only possible if the two axle velocities are close together, i.e. $\sigma = |v_f - v_r|$ is limited. The three driving states indicate the connection of axle velocity to vehicle velocity: for acceleration the axle velocity of the driven axle will be greater than vehicle velocity, for constant driving both axle velocities will be close to vehicle velocity, and for braking both axle velocities will be less than vehicle velocity.

Gradient Limitation A second glance at axle velocities reveals that the rate of change of axle velocities may

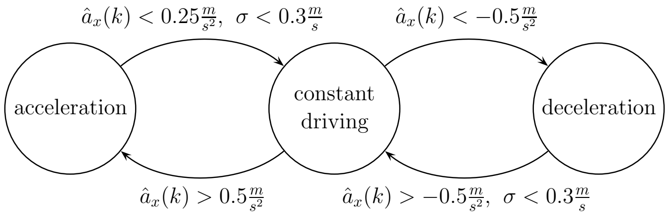

Fig. 4. Acceleration state automaton.

be much larger (acceleration) or much smaller (braking) than the corresponding acceleration of the vehicle. Typical values for the interval $[a_{min}, a_{max}]$ of vehicle accelerations are $a_{min} = -14\frac{\text{m}}{\text{s}^2}$ (uphill braking on a dry road) and $a_{max} = 10\frac{\text{m}}{\text{s}^2}$ (downhill acceleration on dry road).

Therefore gradient limited axle velocities $v_{f,g}$ and $v_{r,g}$ are closer to vehicle velocity than the axle velocities v_f and v_r. Gradient limitation for v_f is given in the following fashion (gradient limitation for v_r is analogous):

$$v_{f,g}(k) = \begin{cases} v_{f,g}(k-1) + T_a \cdot a_{min}, & \text{for deceleration and} \\ & v_f(k) - v_{f,g}(k-1) < T_a \cdot a_{min} \\ v_{f,g}(k-1) + T_a \cdot a_{max}, & \text{for acceleration and} \\ & v_f(k) - v_{f,g}(k-1) > T_a \cdot a_{max} \\ v_f(k), & \text{otherwise.} \end{cases} \tag{8}$$

Differences between wheel velocity v_x and axle velocities v_f and v_r indicate that there is (longitudinal) slip

$$\lambda_f = 1 - \frac{v_f}{v_x}, \quad \lambda_r = 1 - \frac{v_r}{v_x} \tag{9}$$

at the wheels. Slip is responsible for force transmission at the tire-road contact. A typical force characteristic is almost linear for small slip values and saturates for larger slip values (Canudas de Wit et al. [2003]). Absolute values of saturating forces depend on friction coefficient, see also Fig. 3. Gradient limitation as described above basically can be interpreted as a method to prevent a hypothetical slip, computed with $v_{f,g}$, $v_{r,g}$ instead of v_f, v_r in (9), to enter the saturation region.

Slip Correction If saturation can be avoided by gradient limitation and the slope of the linear force generation would be known, an inversion would be possible, i.e v_x could be computed from slip and $v_{f,g}$, $v_{r,g}$. However, the slope is typically unknown due to wear or unknown tires (winter and summer tires). The influence of friction coefficient is less important. As a worst case estimate the following assumptions are made:

- deceleration: $\hat{a}_x = -10\frac{\text{m}}{\text{s}^2}$ imply 5% slip at the front axle and 2% slip at the rear axle (due to dynamical displacement of normal forces toward front axle).
- acceleration: $\hat{a}_x = 10\frac{\text{m}}{\text{s}^2}$ imply 5% slip at the driven axle (and no slip at the non-driven axle).

These assumptions (linear interpolation) result into the following slip corrected axle velocities $v_{f,s}$, $v_{r,s}$ (for a rear driven vehicle):

$$v_{f,s} = \begin{cases} v_{f,g} & \text{for } \hat{a}_x \geq 0 \\ \dfrac{v_{f,g}}{1 + \frac{0.05 \hat{a}_x}{10 \frac{\text{m}}{\text{s}^2}}} & \text{for } \hat{a}_x < 0 \end{cases} \tag{10}$$

$$v_{r,s} = \begin{cases} \dfrac{v_{r,g}}{1 + \frac{0.05\hat{a}_x}{10\frac{\mathrm{m}}{\mathrm{s}^2}}} & \text{for } \hat{a}_x \geq 0 \\ \dfrac{v_{r,g}}{1 + \frac{0.02\hat{a}_x}{10\frac{\mathrm{m}}{\mathrm{s}^2}}} & \text{for } \hat{a}_x < 0 \end{cases} \tag{11}$$

2.4 Axle Stability

If an axle velocity is close to vehicle velocity the corresponding axle is termed *stable*. Here, only the flags f_f, f_r

$$f_f = \begin{cases} 1 \text{ front axle stable} \\ 0 \text{ front axle unstable} \end{cases}$$

$$f_r = \begin{cases} 1 \text{ rear axle stable} \\ 0 \text{ rear axle unstable} \end{cases}$$

are assigned. Assignment is due to numerical evaluation of axle slip and axle acceleration. An unstable axle implies a large variance for the corresponding axle velocity and vice versa.

Variance Computation With only two considered variance values

$$\sigma_f^2 \in \{0.1, 10\}, \quad \sigma_r^2 \in \{0.1, 10\}$$

for every axle and the weighted velocity (measurement for the Kalman filter)

$$y = \alpha_1 v_{f,s} + (1 - \alpha_1) v_{r,s}$$

the variance of y is given as

$$\sigma_y^2 = \alpha_1^2 \sigma_f^2 + (1 - \alpha_1)^2 \sigma_r^2.$$

Normalization finally implies

$$\alpha_1 = \frac{\sigma_r^2}{\sigma_f^2 + \sigma_r^2}.$$

Therefore weighting of the axle velocity and variance of the resulting signal are determined and thus every information needed for Kalman filter computation is available. The injection matrices are computed off-line. The on-line computational burden is thus only the integration of the filter equation plus filtering of wheel velocity signals.

3. EXPERIMENTAL RESULTS

The proposed algorithm for velocity estimation is applied to sensor signals from a series production car (upper class vehicle). The estimated velocity is compared to an internal VDC estimate and a reference velocity signal from an optical sensor (Correvit from Corrsys-Datron). Due to problems of the reference sensor on wet surfaces only high friction results are shown here.

In Figure 5 estimation results for driving on a handling course of a test range are shown. There is almost no difference between the developed simple algorithm and the internal VDC estimate and the reference sensor.

Cut-off of the VDC signal and the proposed estimation algorithm for small velocities is due to the resolution of the angular wheel velocity sensors.

The second driving situation (Figure 6) is a ABS braking. With beginning of braking both velocity estimates (proposed algorithm and VDC signal) differ from the reference sensor. Here, saturation cannot fully be compensated by gradient limitation. However, the absolute value is small and the estimate recovers in course of time. Experiments on low friction (ice) show a certain degradation of the velocity estimate. This is mainly due to a missing friction estimate (and conservative approximation of wheel force generation) which is not possible within the assumed sensor configuration.

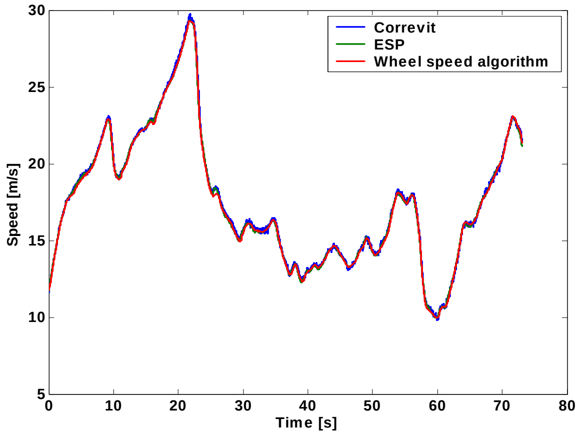

Fig. 5. Handling course on a test range. Driving with large lateral acceleration.

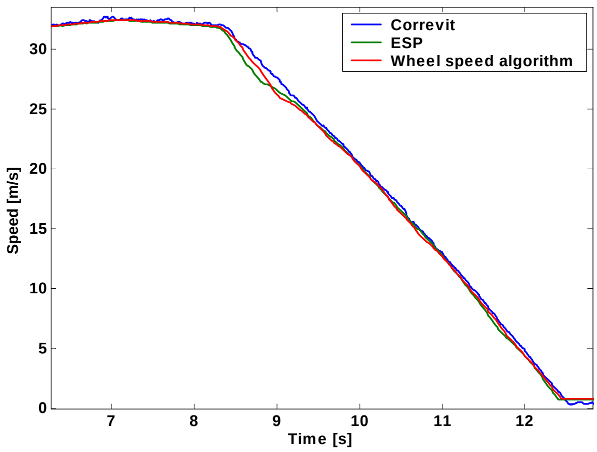

Fig. 6. Velocity estimate for ABS braking to stand-still.

Overall the presented algorithm is applied to almost 350 different driving situations. The final paper will contain a detailed statistical assessment on the estimation error.

4. CONCLUSIONS

In the paper at hand a simple Kalman filter approach toward vehicle velocity estimation is presented. Compared to other approaches only a minimalistic sensor configuration (steering angle and angular wheel velocities) is assumed.

The velocity estimate shows very good results on high friction surfaces. Degradation of the estimate on low friction surfaces (ice) can be traced back to the restrictions in the sensor configuration. However, even driving on snow shows rather good results.

Future work will extend the approach toward lateral velocity estimation in combination with an extended sensor configuration.

REFERENCES

B. M. Bevly, R. Sheridan, and J. C. Gerdes. The use of GPS based velocity measurements for improved vehicle state estimation. In *Proceedings of the American Control Conference, June 28-30*, pages 2538–2542, Chicago, IL, 2000.

Bosch. *Bosch Automotive Handbook*. Bentley Publishers, 5th edition, 2003.

C. Canudas de Wit, M. L. Petersen, and A. Shiriaev. A new nonlinear observer for tire/road distributed contact friction. In *Proc. 42nd IEEE Conf. Decision Contr.*, Maui, HI, 2003.

C. R. Carlson and J. C. Gerdes. Consistent nonlinear estimation of longitudinal tire stiffness and effective radius. *IEEE Transactions on Control Systems Technology*, 13 (6):1010–1020, 2005.

E. L. Ding, H. Fennel, and S. X. Ding. Model based diagnosis of sensor faults for ESP systems. *Control Engineering Practice*, 12(7):847–856, 2004.

M. S. Grewal, L. R. Weill, and A. P. Andrews. *Global Positioning Systems, Inertial Navigation, and Integration*. John Wiley & Sons, Inc., Hoboken, NJ, 2nd edition, 2007.

F. Gustavsson, M. Drevö, and N. Persson. Tire preasure estimation. Patent, July 10 2007. United States Patent, Patent No.: 7240542.

I. Halbe. Model-based fault-detection of vehicle dynamics sensors. *Automatisierungstechnik*, 55(6):322–329, 2007. in German.

M. P. Henry and D. W. Clarke. The self-validating sensor: rationale, definitions, and examples. *Control Engineering Practice*, 1(4):585–610, 1993.

L. Imsland, T. A. Johansen, T. I. Fossen, J. C. Kalkkuhl, and A. Suissa. Vehicle velocity estimation using modular nonlinear observers. In *Proceedings of the 44th IEEE Conference on Decision and Control and European Control Conference ECC 2005, December 12-15*, pages 6728–6733, Seville, Spain, 2005.

L. Imsland, T. A. Johansen, T. I. Fossen, H. F. Grip, J. C. Kalkkuhl, and A. Suissa. Vehicle velocity estimation using nonlinear observers. *Automatica*, 42(12):2091–2103, 2006.

L. Imsland, H. F. Grip, T. A. Johansen, T. I. Fossen, J. C. Kalkkuhl, and A. Suissa. Nonlinear observer for vehicle velocity with friction and road bank angle adaptation - validation and comparison with an extended Kalman filter. In *Proc. SAE World Congress*, Detroit, MI, 2007.

R. Isermann, editor. *Fahrdynamik-Regelung*. Vieweg, ATZ/MTZ-Fachbuch, 2006.

R. E. Kalman. A new approach to linear filtering and prediction problems. *Transactions of the ASME-Journal of Basic Engineering*, 82(Series D):35–45, 1960.

U. Kiencke and L. Nielsen. *Automotive Control Systems*. Springer, Berlin, 2nd edition, 2005.

A. Pinnel, T. Butz, M. Ehmann, and H. Fan. Vehicle dynamics simulation with active yaw control using variable drive torque distribution. *JSAE Paper 200445411*, 2004.

R. Rajamani. *Vehicle Dynamics and Control*. Springer, New York, NY, 2006.

A. Rehm. Sensor networks for vehicle dynamics control systems: A monitoring perspective. In *Third International Symposium on Communications, Control and Signal Processing (ISCCSP), St. Julians, Malta, March 12-14*, pages 263–268, 2008.

A. Rehm and D. Hofmann. Design of a central sensor plausibility check for the vehicle movement. In *Proceedings of the World Automotive Conference FISITA 2004, May 23 - 27*, Barcelona, Spain, 2004. CD-ROM, Paper No. F2004IO52.

S. Semmler. Bremsregelungen fü mechatronische Bremsen. In R. Isermann, editor, *Fahrdynamik-Regelung*, pages 137–168. Vieweg, ATZ/MTZ-Fachbuch, 2006.

A. T. van Zanten. Elektronisches Stabilitätsprogramm (ESP). In R. Isermann, editor, *Fahrdynamik-Regelung*, pages 169–211. Vieweg, ATZ/MTZ-Fachbuch, 2006.

A. T. van Zanten, R. Erhardt, K. Landesfeind, and G. Pfaff. VDC systems development and perspective. *SAE Paper No. 980235*, 1998.

A. von Vietinghoff, S. Olbrich, and U. Kiencke. Extended Kalman filter for vehicle dynamics determination based on a nonlinear model combining longitudinal and lateral dynamics. In *SAE World Congress*, Detroit, 2007. SAE Paper 2007-01-0834.

J. Y. Wong. *Theory of Ground Vehicles*. John Wiley & Sons Inc., 3rd edition, 2001.