Time-delay in high-gain observer based disturbance estimation

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Abstract: In this paper, the properties of a high gain observer-based disturbance estimation are analysed, and a time delay calculation approach is proposed for improving the identification of model parameter variation. The focus of this paper is the time delay between the actual disturbance and its estimate in a high gain disturbance estimation observer. It is proved, in this paper, the delay depends on the observer gain, but is independent from the model uncertainties. Thus, a novel algorithm is proposed to calculate the delay according to the phase response of disturbance estimation transfer function. The correctness of this algorithm has been verified by the simulation based on a servo motor model.

1. INTRODUCTION

Robust control techniques require a dynamic model of the plant and bounds on model uncertainties coming from parameter variations, unmodelled dynamics. Such model uncertainties can be represented in the form of disturbances. By specifying a nominal model and the associated 'bounds', the robust control laws can be formulated with guaranteed stabilities (Zhou and Doyle [1998]). Although some procedures have been proposed for identifying the model and it quality (see for instance Ninness and Goodwin [1995], Reinelt et al. [2002], Giarre et al. [1997], stochastic embedding (Goodwin [1999],Goodwin and Salgado [1989]), model error modeling (Ljung [2000], Ljung [1999]) and set membership identification (Garulli and Reinelt [1999], Walter and Piet-Lahanier [1990])), there have been no formal procedures in textbooks for estimating the model uncertainty. Recently, the high-gain observer technique has been employed for estimating the model uncertainty parameters.

One of these, an interesting high gain observer (Gao et al. [2007]) was proposed to identify the model parameter perturbation, where both the unmeasured state and disturbance variables are estimated. Then the parameter perturbation can be identified within an ARX framework. With slight modifications, this technique is applied for disturbance estimation and parameter identification (Gao et al. [2008]). Using the estimated state $\hat{x}(t)$, disturbance $\hat{d}(t)$ and the measured input $u(t)$, the variation of the plant parameters are identified within the framework of an multi-input ARX model $\hat{d}(t) = [\hat{x}^T(t)\ u^T(t)]\theta$, where θ is the parameter perturbation.

In such an application, it is assumed that there is no time delay between the actual disturbance signal $d(t)$ and its estimate $\hat{d}(t)$ given by the observer (Gao et al. [2008]). It is true in the case of step input, where no time delay exists at the steady state. However, one interesting phenomenon occurs when the input signal is a sinusoidal signal (e.g., a multi-sine wave which is widely adopted in practice). The phase of the estimated signal $\hat{d}(t)$ differs from that of the actual signal $d(t)$, although the estimate is able to follow the changes of actual disturbance. The phase differences result in $\hat{d}(t)$ lags from $d(t)$. Furthermore, the delay does not disappear at the steady state. In fact, it keeps constant. Due to the time delay, the parameter identification method in (Gao et al. [2008]) may reach an incorrect result.

In order to improve the accuracy of parameter uncertainty identification, this paper will study the properties of the disturbance estimation delay and the possibility of delay compensation. For the problem of disturbance estimation delay, one possible solution is to optimise the high-gain observer to remove the time delay. It seems intuitively simple, but the difficulty is to form the objective function for optimisation. Another way is to employ some time delay estimation (TDE) techniques in the system identification procedure. System identification provides the ability to estimation delay between inputs and outputs. However, a good delay estimation depends heavily on the signal-noise ratio (SNR). Due to the inherent nature of observer and the exoterical disturbance, the SNR of the estimated signal may become more worse which results in an unreliable delay estimate.

Alternatively, in this paper, we admit the existence of delay, and try to calculate the delay by analysing the transfer function matrix (TFM) of the observer. It will be proved that the delay depends on the high-gain observer, and is immune from the model uncertainty. The main benefit of the proposed delay calculation method is the applicability to deal with any exoterical disturbances. It works well even in the case of severe model uncertainty

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and exoterical disturbances. This method relaxes the requirement on high-gain observer and TDE, and makes the high-gain observer technique more applicable.

This paper is organised as follows. The problem is formulated first in section 2. And the disturbance estimation properties of the high gain observer is analysed in section 3, where it is proved that the time delay depends on the observer parameters, but has nothing to do with the model uncertainties. Thus, in section 4, a novel algorithm is proposed to calculate the delay. Based on the delay calculation, and a phase shifter can be used to compensate the delay. As illustrated in section 5, the application to a servo motor verifies the existence of time delay and the correctness of our calculation.

2. PROBLEM FORMULATION

2.1 High gain observer for state and disturbance estimation

A high gain observer is used for estimating the disturbances and states of a dynamic system

$$\begin{cases} \dot{x}(t) = (A_0 + \Delta A)x(t) + (B_0 + \Delta B)u(t) + \omega_i(t) \\ y(t) = Cx(t) + \omega_o(t) \end{cases} \tag{1}$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^p$ the input, $y \in \mathbb{R}^n$ the output, $\omega_i(t) \in \mathbb{R}^n$ and $\omega_o(t) \in \mathbb{R}^n$ are input and output noise vectors, respectively. Note that, for the sake of simplicity, it is assumed that C is an identity matrix. $A_0 \in \mathbb{R}^{n\times n}$, $B_0 \in \mathbb{R}^{n\times p}$ are known matrices of the nominal model,ΔA and ΔB are the parameter variations. Such a system can be rewritten as

$$\begin{cases} \dot{x}(t) = A_0x(t) + B_0u(t) + B_d d(t) + \omega_i(t) \\ y(t) = Cx(t) + \omega_o(t) \end{cases} \tag{2}$$

where

$$d(t) = \Delta Ax(t) + \Delta Bu(t) \tag{3}$$

and

$$B_d = I \tag{4}$$

Equation (3) means that the model uncertainty is integrated in to the extended plant system (2) in form of some disturbance.

It can be seen that the disturbance model (3) is a (static) 2-input-1-output ARX model without time delay, where $x(t)$ and $u(t)$ are regarded as the inputs and $d(t)$ the output. In order to estimate the parameter variation $\{\Delta A, \Delta B\}$, a disturbance observer is used for estimating the unmeasurable $x(t)$ and $d(t)$. Denote

$$\bar{x}(t) = \begin{bmatrix} x(t) \\ d(t) \\ w_o(t) \end{bmatrix} \in \mathbb{R}^{3n} \tag{5}$$

and

$$\bar{A} = \begin{bmatrix} A_0 & I_n & 0 \\ 0 & 0_{n\times n} & 0 \\ 0 & 0 & -I_n \end{bmatrix}, \bar{B} = \begin{bmatrix} B_0 \\ 0_{n\times p} \\ 0_{n\times p} \end{bmatrix}, \tag{6}$$

$$\bar{E} = \begin{bmatrix} I_n & 0 & 0 \\ 0 & I_n & 0 \\ 0 & 0 & 0_{n\times n} \end{bmatrix}, \bar{C} = [\, I_n \;\; 0 \;\; I_n \,]$$

$$\bar{G} = \begin{bmatrix} I_n \\ 0_{n\times n} \\ 0_{n\times n} \end{bmatrix}, \bar{H} = \begin{bmatrix} 0_{n\times n} \\ I_{n\times n} \\ 0_{n\times p} \end{bmatrix}, \bar{N} = \begin{bmatrix} 0 \\ 0_{n\times n} \\ I_n \end{bmatrix}. \tag{7}$$

As a result, an augmented descriptor system can be obtained from (2) and (6) to give

$$\begin{cases} \overline{E}\dot{\bar{x}}(t) = \bar{A}\bar{x} + \bar{B}u(t) + \bar{G}\omega_i(t) + \bar{H}\dot{d}(t) - \bar{N}\omega_o(t) \\ y(t) = \bar{C}\bar{x}(t). \end{cases} \tag{8}$$

In this study, $\dot{d}(t)$, $\omega_i(t)$ and $\omega_o(t)$ are all assumed to bounded. In this context, the following disturbance estimation observer can be constructed

$$\begin{cases} \overline{S}\dot{\xi}(t) = (\bar{A} - \bar{K}\bar{C})\xi(t) + \bar{B}u(t) - \bar{N}y(t) \\ \hat{\bar{x}}(t) = \xi(t) + \bar{S}^{-1}\bar{L}y(t) \end{cases} \tag{9}$$

where $\xi(t) \in \mathbb{R}^{3n}$ is the state vector of the dynamic system above, $\hat{\bar{x}} \in \mathbb{R}^{3n}$ is the estimate of $\bar{x}$, $\bar{S} = \bar{E} + \bar{L}\bar{C}$, and $\overline{K}$, $\overline{L} \in \mathbb{R}^{(3n)\times n}$ are the gain matrices to be designed.

According to the design algorithm proposed by Gao et al. [2007], the high-gain matrix $\bar{K}$ can be computed as

$$\bar{K} = \bar{S}\bar{P}^{-1}\bar{C}^T,$$

where $\bar{P}$ is solved from the following Lyapunov equation

$$-(\mu I + \bar{S}^{-1}\bar{A})^T\bar{P} - \bar{P}(\mu I + \bar{S}^{-1}\bar{A}) = -\bar{C}^T\bar{C} \tag{10}$$

with $\mu > 0$ satisfying $\Re[\lambda_i(\bar{S}^{-1}\bar{A}] > -\mu, \forall i \in \{1, 2, ..., 3n\}$.

Now, the state x, the input uncertainty d and the output noise ω_o can be estimated simultaneously:

$$\hat{x}(t) = [I_n \; 0_n \; 0_p]\hat{\bar{x}}(t) \tag{11}$$

$$\hat{d}(t) = [0_n \; I_n \; 0_p]\hat{\bar{x}}(t) \tag{12}$$

$$\hat{\omega}(t) = [0_n \; 0_n \; I_n]\hat{\bar{x}}(t) \tag{13}$$

It is worthy noting that the estimates of unmeasurable $x(t)$ and $d(t)$ pave the way for the identification of ΔA and ΔB in the succeeding subsection.

2.2 Time-delay in disturbance estimation

In order identify $[\Delta A \; \Delta B]$, the unmeasurable plant state $x(t)$ and disturbance $d(t)$ are replace with their esitmates given by the observer. By considering the disturbance estimation delay, the disturbance model (3) is rewritten in a general form:

$$\hat{d}(t + \tau) = \Delta A\hat{x}(t) + \Delta Bu(t) \tag{14}$$

where τ is the time delay between $\hat{d}(t)$ and $d(t)$. If there is no delay ($\tau = 0$) or the delay τ is so small that it can be ignored, a good parameter estimation can be obtained, as shown in Gao et al. [2008]. Although τ can be made very small by properly selecting μ in (10), which may require more computation on refining the observer design and puts more constraints on the observer design.

One can also apply some time delay estimation (TDE) technique to determine τ, followed by using mature ARX identification algorithms to obtain $[\Delta\hat{A} \; \Delta\hat{B}]$. However, the estimation of τ is not an easy task, in particular when the system is corrupted by external/measurement noises. In our high-gain observer approach, $\hat{d}(t)$ and $\hat{x}(t)$ are used for parameter identification, rather than $d(t)$ and $x(t)$. Estimation uncertainties make the estimation of τ more difficult. Hence, these TDE algorithm may fail to give the correct estimation of τ and $[\Delta A_i \; \Delta B_i]$. In the following sections, a novel approach is proposed to calculate τ.

3. TFMS RELATING THE DISTURBANCE ESTIMATION

In order to derive the mathematical expression of τ, the TFMs relating the plant and the observer are first formulated, then the relationship between $d(t)$ and $\hat{d}(t)$ is presented. The main advantage of this approach is that τ can be calculated accurately at the observer design stage and be removed by phase compensation techniques. Furthermore, the calculation of τ is immune from the model uncertainty $[\Delta A, \Delta B]$ and measurement/external noises.

3.1 TFM $G_{\hat{d}u}(s)$ relating $u(t)$ to $\hat{d}(t)$

From (9) and (12), one can write the estimation of $d(t)$ as

$$\begin{cases} \bar{S}\dot{\hat{\bar{x}}}(t) = (\bar{A} - \bar{K}\bar{C})\hat{\bar{x}} + \bar{B}u(t) + \bar{K}y(t) + \bar{L}\dot{y}(t) \\ \hat{d}(t) = [0_{n\times n} \; I_{n\times n} \; 0_{n\times n}]\hat{\bar{x}} \end{cases} \quad (15)$$

Using the Laplace transform gives the TFMs relating $u(t), y(t), \dot{y}(t)$ to $\hat{d}(t)$

$$\hat{d}(s) = G_1(s)u(s) + G_2(s)y(s) + G_3(s)\dot{y}(s) \quad (16)$$

where

$$G_1(s) = [0 \; I_n \; 0] \cdot (s\bar{S} - (\bar{A} - \bar{K}\bar{C}))^{-1} \cdot \bar{B} \quad (17)$$

$$G_2(s) = [0 \; I_n \; 0] \cdot (s\bar{S} - (\bar{A} - \bar{K}\bar{C}))^{-1} \cdot \bar{K} \quad (18)$$

$$G_3(s) = [0 \; I_n \; 0] \cdot (s\bar{S} - (\bar{A} - \bar{K}\bar{C}))^{-1} \cdot \bar{L} \quad (19)$$

Consider the plant system (1) with the uncertainty $[\Delta A, \Delta B]$, the TFM relating $u(t)$ to $y(t)$ is

$$G_{yu}(s) = C[sI - (A_0 + \Delta A)^{-1}](B_0 + \Delta B) \quad (20)$$

and

$$y(s) = G_{yu}(s) \cdot u(s). \quad (21)$$

By differentiating the output equation in (1), we obtain the derivatives of $y(t)$

$$\dot{y}(t) = C\dot{x}(t) + \dot{\omega}_o \quad (22)$$

Substituting the state equation of (1) into (22) gives

$$\begin{cases} \dot{x}(t) = (A_0 + \Delta A)x(t) + (B_0 + \Delta B)u(t) + \omega_i(t) \\ \dot{y}(t) = C[(A_0 + \Delta A)x + (B_0 + \Delta B)u] + \dot{\omega}_o \end{cases} \quad (23)$$

It follows that the TFM relating $u(t)$ to $\dot{y}(t)$ is

$$\dot{y}(s) = G_{\dot{y}u}(s) \cdot u(s) \quad (24)$$

where

$$G_{\dot{y}u}(s) = C(A_0 + \Delta A)[sI - (A_0 + \Delta A)]^{-1}(B_0 + \Delta B) + C(B_0 + \Delta B) \quad (25)$$

Substituting equations (21), (24) into (16) gives the TFM $G_{\hat{d}u}(s)$ relating $u(t)$ to $\hat{d}(t)$:

$$G_{\hat{d}u}(s) = G_1(s) + G_2(s)G_{yu}(s) + G_3(s)G_{\dot{y}u}(s). \quad (26)$$

3.2 TFM $H_{du}(s)$ relating $u(t)$ to $d(t)$

Recalling the plant (1) with model uncertainty $d(t) = \Delta Ax(t) + \Delta Bu(t)$, the dynamics of $d(t)$ are governed by

$$\begin{cases} \dot{x}(t) = (A_0 + \Delta A)x(t) + (B_0 + \Delta B)u(t) + \omega_i(t) \\ d(t) = \Delta Ax(t) + \Delta Bu(t) \end{cases} \quad (27)$$

Thus, the TFM relating $u(t)$ to $d(t)$ is

$$d(s) = H_{du}(s) \cdot u(s) \quad (28)$$

where

$$H_{du}(s) = \Delta A[sI - (A_0 + \Delta A)]^{-1}(B_0 + \Delta B) + \Delta B \quad (29)$$

Now both the TFMs $H_{du}(s)$ and $G_{\hat{d}u}(s)$ relating $u(t)$ to $d(t)$ and $\hat{d}(t)$ have been obtained, respectively. Next, the time delay between $\hat{d}$ and d will be given by analysing the relationship between $H_{du}(s)$ and $G_{\hat{d}u}(s)$.

4. TIME DELAY ANALYSIS

For the sake of notation, some abbreviations are first defined as follows:

$$\Lambda = (sI - A_0) \quad (30)$$

$$\Psi = [sI - (A_0 + \Delta A)] \quad (31)$$

$$F(s) = G_2(s)C\Lambda^{-1} + G_3(s)[CA\Lambda^{-1} + C] \quad (32)$$

where $F(s)$ is a $n \times n$ TFM.

In this section, a constructive proof of $G_{\hat{d}d}(s) = F(s)$ is presented, where $G_{\hat{d}d}(s)$ is the TFM relating $d(t)$ to its estimate $\hat{d}(t)$. Before giving the main theorem, three lemmas are proved as follows.

Lemma 1. Given the plant (A_0, B_0, C), if its corresponding high gain observer (9) is stable, then the following TFM

$$G_1(s)+G_2(s)[C\Lambda^{-1}B]+G_3(s)[CA_0\Lambda^{-1}B+CB] = 0 \quad (33)$$

Proof: This can be proved by studying the dynamics of the high-gain observer. The the plant system (with model uncertainties) and its corresponding observer can be decomposed as Fig. 1. Note that, $y(t)$ is measurable,

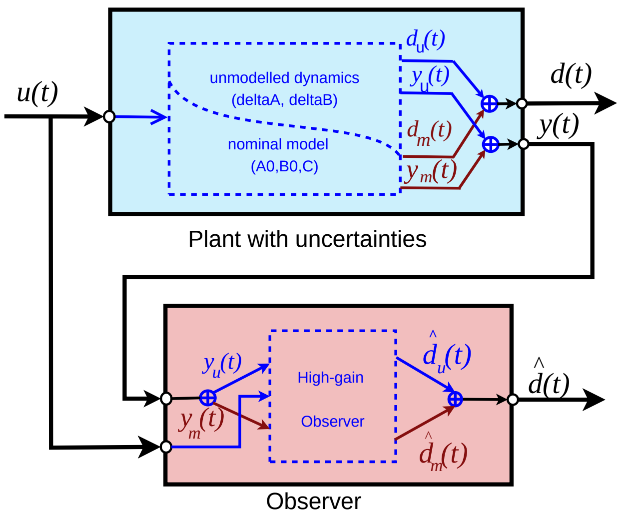

Fig. 1. Decomposition of the plant with uncertainties and its high-gain observer, where the plant consists of two parts: nominal model (A_0, B_0) and unmodelled dynamics.

but $d(t)$ is unmeasurable. However, $d_m(t)$ is always zero, because there is no model uncertainty within the nominal model (A_0, B_0, C).

According to the additionality of linear system, the output of the high-gain observer can be divided in a similar way: $\hat{d}(t) = \hat{d}_m(t) + \hat{d}_u(t)$, where $\hat{d}_m(t)$ is the estimate of $d_m(t)$, $\hat{d}_u(t)$ the estimates of $d_u(t)$.

Observe that, the terms appearing on the left-hand side of (33) have a special meaning. $C\Lambda^{-1}B_0$ is the TFM relating $u(t)$ to $y_m(t)$ and $CA_0\Lambda^{-1}B_0 + CB_0$ is the TFM relating $u(t)$ to $\dot{y}_m(t)$. Compared to the TFM (16), these terms on the left-hand side of (33) are exactly the TFM relating $u(t)$ to $\hat{d}_m(t)$.

Furthermore, if the observer is stable, the disturbance estimation $\hat{d}(t)$ is asymptotically stable. With the decomposition as Fig. 1, it can be stated that $\hat{d}_m(t)$ approaches $d_m(t)$ asymptotically. Since $d_m(t) = 0$, for a stable observer, $\hat{d}_m(t)$ is also zero asymptotically. This means that, no matter what the values of $u(t)$ and $[\Delta A \; \Delta B]$ are, the $\hat{d}_m(t)$ is zero. This implies that $\hat{d}_m(t)$ has nothing to do with $u(t)$. Thus, the corresponding transfer function matrix relating $u(t)$ to $\hat{d}_m(t)$ is a zero matrix. And the result of Lemma 1 follows. Q.E.D.

Lemma 2. If the high gain observer (9) is stable, then

$$\begin{aligned} G_1(s) + G_2(s)[C\Psi^{-1}B_0] + G_3(s)[CA_0\Psi^{-1}B_0 + CB_0] \\ +G_3(s)[C\Delta A\Psi^{-1}B_0] \\ = F(s)\cdot \Delta A^{-1}\Psi^{-1}B_0 \end{aligned} \tag{34}$$

Lemma 3. If the high gain observer (9) is stable, then

$$\begin{aligned} G_2(s)C\Psi^{-1}\Delta B + G_3(s)C[A\Psi^{-1} + I]\Delta B \\ +G_3(s)C\Delta A\Psi^{-1}\Delta B \\ = F(s)\cdot(\Delta A\Psi^{-1}\Delta B + \Delta B) \end{aligned} \tag{35}$$

The proofs of Lemma 2 and Lemma 3 can be done simply by algebraical manipulations. Some equations which may be useful for these proofs are listed as follows:

$$\begin{aligned} \Delta A + \Psi = \Lambda \\ \Delta A\Psi^{-1} + I = \Lambda\Psi^{-1} \\ \Lambda^{-1}(\Delta A\Psi^{-1} + I) = \Psi^{-1} \end{aligned}$$

Theorem 4. Given plant (1) and its corresponding high gain observer with $\bar{K}$, $\bar{L}$ (9), the relationship between the TFMs $G_{\hat{d}u}(s)$ and $H_{du}(s)$ is

$$G_{\hat{d}u}(s) = F(s)\cdot H_{du}(s) \tag{36}$$

where $F(s)$, $G_{\hat{d}u}(s)$ and $H_{du}(s)$ are defined in Eq. (32), (26) and (29), respectively.

Proof: Substituting $\Psi = sI - (A + \Delta A)$, $\Lambda = sI - A$ into $G_{\hat{d}u}(s)$ (26) gives

$$\begin{aligned} & G_{\hat{d}u}(s) \\ =& \underline{G_1(s)} + \underline{G_2(s)C\Psi^{-1}B} + G_2(s)C\Psi^{-1}\Delta B \\ &+ \underline{G_3(s)\cdot C[A\Psi^{-1} + I]B} + \underline{G_3(s)\cdot C\triangle A\Psi^{-1}B} \\ &+ G_3(s)\cdot C[A\Psi^{-1} + I]\Delta B + G_3(s)\cdot C\triangle A\Psi^{-1}\Delta B \end{aligned} \tag{37}$$

Notice that, the terms underlined are identical to the terms on the left-hand side of (34) in Lemma 2. Furthermore, observe that the rest terms in (37) are the same as the left-hand side of (35) in Lemma 3. Substituting (34), (35) into (37) gives

$$\begin{aligned} & G_{\hat{d}u}(s) \\ &= F(s)\cdot\triangle A^{-1}\Psi^{-1}B + F(s)\cdot(\triangle A\Psi^{-1}\Delta B + \Delta B) \\ &= F(s)\cdot(\triangle A^{-1}\Psi^{-1}B + \triangle A\Psi^{-1}\Delta B + \Delta B) \end{aligned} \tag{38}$$

Thus, by Eq.(29), we have

$$G_{\hat{d}u}(s) = F(s)\cdot H_{du}(s) \tag{39}$$

and the result of Theorem 4 follows. Q.E.D.

Based on the result of Theorem 4, one can have the following theorem:

Theorem 5. Given a plant (A_0, B_0, C) with uncertainty $[\Delta A, \; \Delta B]$ and the corresponding high-gain observer (9), for any value of $[\Delta A, \; \Delta B]$, the TFM $G_{\hat{d}d}(s)$ relating $d(t)$ to $\hat{d}(t)$ is equivalent to $F(s)$:

$$G_{\hat{d}d}(s) = F(s). \tag{40}$$

Proof: With the aid of Theorem 4, it can be proved that

$$\begin{aligned} \hat{d}(s) &= G_{\hat{d}u}(s)u(s) \\ &= F(s)H_{du}(s)u(s) \\ &= F(s)d(s) \end{aligned} \tag{41}$$

According to the definition of TFM, equation (41) states that $F(s)$ is the TFM relating $d(t)$ to $\hat{d}(t)$, and the result of Theorem 5 follows. Q.E.D.

Two useful propositions come straight from the result of Theorem 5.

Corollary 6. The TFM $G_{\hat{d}d}(s)$ is independent from model uncertainty $[\Delta A, \; \Delta B]$.

Corollary 7. The time delay τ between $d(t)$ and its estimate $\hat{d}(t)$ is independent from model uncertainty $[\Delta A, \Delta B]$.

The TFM $F(s)$ is a $n \times n$ matrix and can be expressed in terms of magnitude and phase response matrices

$$F(s) = M(s)\underline{/\Phi(s)} \tag{42}$$

where $M(s)$ is the magnitude response matrix and $\Phi(s)$ the phase response matrix, respectively, with the size of $n \times n$. The amplitude of $\hat{d}(t)$ is given by the product of $M(s)$ and $d(s)$, while the phase angle of $\hat{d}(t)$ differs from that of $d(t)$ by the amount of $\Phi(s)$. Consider an ideal disturbance observer, $M(s)$ should be an identity matrix and $\Phi(s)$ is a zero matrix. In practice, $M(s)$ given by the high-gain observer is an identity matrix approximately, such that $\hat{d}_i(t)$ is dominantly determined by $d_i(t)$ alone and then regarded as the estimate of $d_i(t)$. Therefore, only the diagonal elements of $\Phi(s)$ is considered in calculating the time delay τ.

Substitute $s = \mathbf{j}\omega$ into $\Phi(s)$, the delay function $\tau_i(\omega)$ with respect to ω is given by

$$\tau_i(\omega) = -\frac{\Phi_{ii}(\mathbf{j}\omega)}{\omega} \tag{43}$$

where $\tau_i(\omega)$ is the delay between $\hat{d}_i(t)$ and $d_i(t)$ at frequency ω, and Φ_{ii} the i-th diagonal element of Φ.

Remark 1. As $F(s)$ is independent from model uncertainty $[\Delta A,\ \Delta B]$, $M(s)$ and $\Phi(s)$ are also independent from the model uncertainty. Hence, the time delay $\tau_i(\omega)$ does not change with respect to different value of $[\Delta A, \Delta B]$. Then, it is practical to compute the delay by assigning any value to $[\Delta A, \Delta B]$, and the result is applicable to any value of $[\Delta A, \Delta B]$.

Remark 2. $F(s)$ is readily obtained after the observer is designed by selecting $\bar{K}$, $\bar{L}$. One can compute the time delay $\tau_i(\omega)$ accurately from the phase response matrix $\Phi(s)$. Furthermore, a phase-shifter with unit magnitude response and $-\Phi(s)$ phase response can be properly designed to compensate the delay.

Remark 3. It can be found that the disturbance delay is a function of frequency ω. Generally, τ varies as the frequency increases from zero to infinity. For an easy delay compensation, it is suggested to construct an observer with an invariant delay over the frequency of interest.

Remark 4. Generally, a negative phase angle results in a positive τ so that $\hat{d}(t)$ lags behind $d(t)$; a positive phase angle results in a negative τ so that $\hat{d}(t)$ leads $d(t)$. Since $\hat{d}(t)$ is the estimate of $d(t)$, it is instinct that $\hat{d}(t)$ lags behind $d(t)$. Thus, $\tau > 0$ in the high-gain observer.

5. DELAY CALCULATION FOR A SERVO MOTOR SYSTEM

In this section, the proposed delay calculation approach is applied to a three-mass servo system. A high gain observer is first designed and the delay of the disturbance estimation is calculated. A reduced order model of a three-mass servo system can be expressed by (1) with the following coefficient matrices

$$A_0 = \begin{bmatrix} -1.0830 & -0.0453 \\ 0.1004 & 0.0014 \end{bmatrix}, B_0 = \begin{bmatrix} 0.9540 \\ 0.0140 \end{bmatrix} \qquad (44)$$

where the input $u(t)$ is the supply voltage to the servo motor, the output $y(t)$ is a two elements vector of the shaft speed and load speed, respectively. $u(t)$ and its spectrum are shown in Fig. 2. From the spectrum, it can be seen that $u(t)$ composes of a sum of sinusoids over a certain band $\Omega = \{0.4, 0.8, 1.2, 1.6, 2.0\}$ (Hz).

These coefficient matrices (44) are known in a priori by the handbook of the motor. And they are accurate when the motor is in good condition. In the meanwhile, due to the usage and degradation, ΔA and ΔB are non-zero matrices. These matrices are the parameters to be estimated and in this simulation, it is assumed that

$$\Delta A = \begin{bmatrix} 0.3 & 0.1 \\ 0.8 & -0.8 \end{bmatrix}, \Delta B == \begin{bmatrix} 0.1 \\ -0.3 \end{bmatrix}. \qquad (45)$$

5.1 High Gain Observer Design

Now a high gain observer is designed first. Construct the augmented system as (6), let $\alpha = 10$ and $M = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$, Select $\mu = 25$, the corresponding high gains $\bar{K}$ is computed as

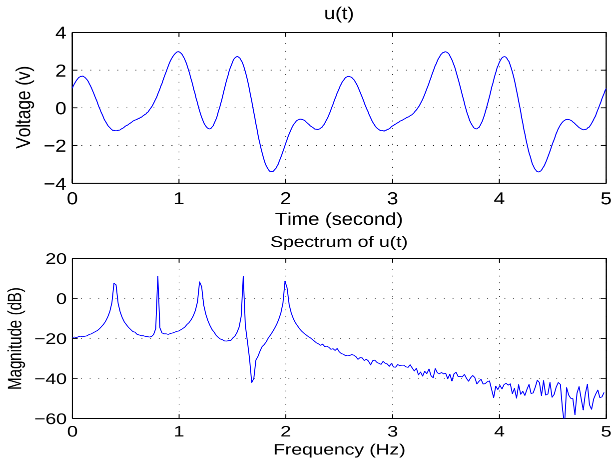

Fig. 2. The input $u(t)$ and its spectrum.

$$\bar{K} = \begin{bmatrix} 0.0722180144190 & 0.0000285599096 \\ 0.0000998298815 & 0.0749038031932 \\ 1.2204759747884 & 0.0024248585284 \\ -0.0012119322771 & 1.2475397787275 \\ 0.0014763389995 & 0.0000005184598 \\ 0.0000005184598 & 0.0014980303419 \end{bmatrix} \times 10^6 \quad (46)$$

5.2 Time Delay Analysis

The transfer function matrix $G_2(s)$, $G_3(s)$ and $F(s)$ from (18), (19), (32) are computed, respectively. The bode plots (magnitude response and phase response) of $F(s)$ are depicted in Fig. 3 and Fig. 4, respectively.

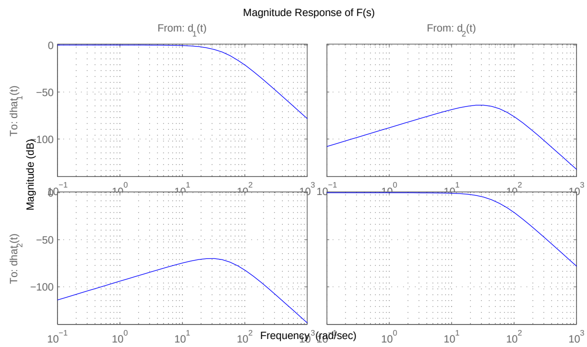

Fig. 3. Magnitude Response of $F(s)$

From Fig. 3, it can been seen that, the magnitude response of diagonal element $M_{ii}(\omega), i = 1, 2$ is 0 **dB** over [0,10] (rad/sec). It means the magnitude amplification of $\hat{d}_i(t)$ with respect to $d_i(t)$ is 1. Contrarily, the magnitude of non-diagonal element $M_{ij}(\omega), i \neq j$ is lower than -50 **dB** which is very very small compared to that of $M_{ii}(\omega)$. It means that the effects from $d_j(t)$ to $\hat{d}_i(t)$ is ignorable. Hence, it is verified that $M(\omega)$ can be treated as an identity matrix. Therefore, the $\hat{d}_i(t)$ is dominated by $d_i(t)$ and the time delay is determined by the diagonal element $\Phi_{ii}(\omega)$ alone.

According to the spectrum analysis in Fig. 2, the frequency range of $d(t)$ is in $\Omega = [1, 2]$ Hz. The resulting time delay

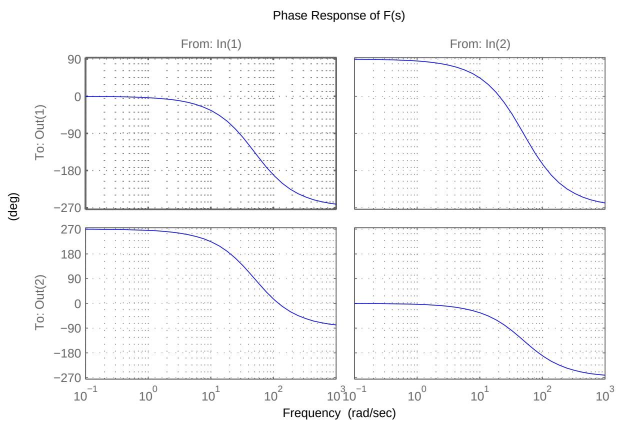

Fig. 4. Phase Response of $F(s)$

$\tau_i(\omega)$ (43) over the frequency range Ω is shown in Fig. 5. It can be found that, the time delay decreases as the

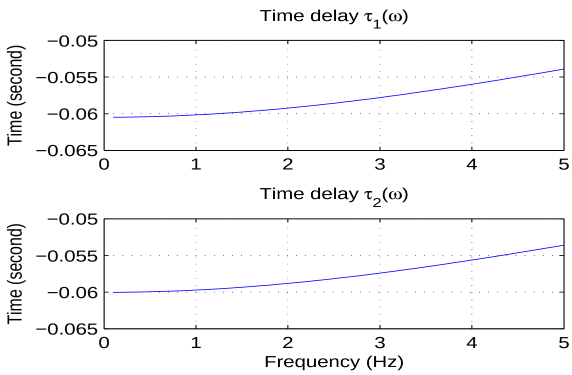

Fig. 5. Time delay $\tau_i(\omega)$ relating $d_i(t)$ to $\hat{d}_i(t)$, $i = 1, 2$.

frequency increases. Over the frequency $[0, 2]$ (Hz), both τ_1 and τ_2 are around 0.06 second and the variation is minor. Thus the high-gain observer is acceptable. For parameter identification, the time delay can be compensated by lagging the input $u(t)$ and $\hat{x}(t)$ by 0.06s.

The simulation of $d(t)$ and its estimate $\hat{d}(t)$ are shown in Fig. 6, from which, one can see that there is a time delay between $d_i(t)$ and $\hat{d}_i(t)$. The time delay read from the simulation results is around 0.06s, which agrees with the delay calculated from TFM $F(s)$.

6. CONCLUSION

In this paper, on the basis of the high-gain observer for model uncertainty identification, the properties of disturbance estimation is analysed. Because of the nonzero phase response, the time delay appears which challenges the application of high gain observer to parameter perturbation identification. This paper proves that the time delay in such a high gain observer is independent from the model uncertainty and can be computed in terms of TFMs. This has been verified by the simulation results on a servo motor. The main benefit of this approach is the time delay given by this algorithm is immune from both the external disturbance and the measurement noise. This technique also gives a new insight into the high-gain observer design.

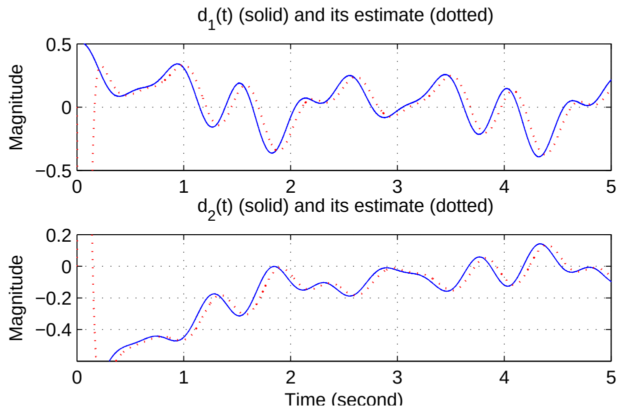

Fig. 6. Disturbance $d(t)$ and its estimate $\hat{d}(t)$ under $[\Delta A, \Delta B]$ is given as (45). The delay between $d(t)$ and $\hat{d}(t)$ can be seen clearly.

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