Model Predictive Control of Substructured Systems

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Abstract: In this paper, we consider the control of multivariable substructured systems with input constraints. Model Predictive Control (MPC) is used to synchronize the interface between the physical and numerical substructures. As a case study, a quasi-motorcycle suspension system is converted into a multivariable substructured system. An MPC controller is developed for this system. Simulation results show the advantage of using an MPC controller to synchronize the substructured system.

Keywords: Predictive Control; Substructured systems; Multivariable control; Constrained control.

1. INTRODUCTION

In real-time experimental tests, some critical components of the dynamic system can be either too complicated to be numerically modelled, due to the presence of uncertainties and nonlinearities, or too difficult to be tested in a laboratory environment due to their size, for example, the testings of large-scale engineering structures such as bridges and dams. To circumvent this problem, the use of the dynamic substructuring concept in real-time experimental tests has become an appealing strategy in recent years (Nakashima et al., 1992). The principal idea of substructuring is to test the complex critical components of the system (represented as a physical substructure) in real-time and the remainder of the system as a numerical model (represented as a numerical substructure) simultaneously. This can be more advantageous than the existing testing methods such as full-size testing of the entire system, scale-model testing, pseudo-dynamic testing and purely numerical testing (Williams and Blakeborough, 2001).

An important issue of the substructuring method is the synchronization of the physical and numerical substructures, which significantly affects the testing accuracy of the entire system. This demands a high fidelity of control to reduce the error of the interface between the two substructures. However, dynamical interaction between the two substructures, together with the dynamics of the transfer system (and its associated actuators), will normally cause problems with synchronization. Successful control strategies that specifically take into account substructure interaction and transfer system dynamics include Linear Substructuring Control (LSC) and Minimal Control Synthesis (MCS) (Stoten and Benchoubane, 1990; Stoten and Hyde, 2006; Wagg and Stoten, 2001; Neild et al., 2005). However, the actuator saturation in the transfer system has not been explicitly part of the synthesis procedure so far.

In this paper, we aim to control the multivariable substructured system while explicitly considering the actuator constraints. To achieve this control objective, we use Model Predictive Control (MPC) on the multivariable substructuring framework extended from the one for SISO systems by Stoten and Hyde (2006).

MPC is an online optimization control strategy, which solves an optimization problem at each sampling instant, with the current state as the initial state. It is suitable for multivariable systems with constraints. However, since the time used for the computation of online optimization may increase with the order of the system and the length of prediction horizon, the initial applications of MPC were restricted to process control problems in chemical industries (Qin and Badgwell, 1997), such as oil refineries. Nevertheless, with the development of new, efficient optimization algorithms and the progress of computer computing ability, a large number of applications of MPC on fast systems have been found in areas such as aerospace, power plants and the automotive industry (Qin and Badgwell, 2003). For example, the application of MPC to an active structure using sampling rates up to $5kHz$ on a $200MHz$ DSP has been realized in Wills et al. (2008). The successful MPC application to rotor vibration suppression has also been reported in Bai and Ou (2002). These promising results motivated the implementation of MPC on substructured systems for real-time testing of electro-mechanical components, which always demand high sampling rates.

As a case study, a quasi-motorcycle suspension system is converted into a substructured framework. Two problems need to be considered: 1) it is a multivariable system when two or more wheels are taken into account; 2) actuators in the system limit the control action. Both of the problems can be coped with by an MPC controller in a systematic way. Although there exist some successful MPC applications on vehicle suspension control systems (Mehra et al., 1997; Chen and Scherer, 2004), the control

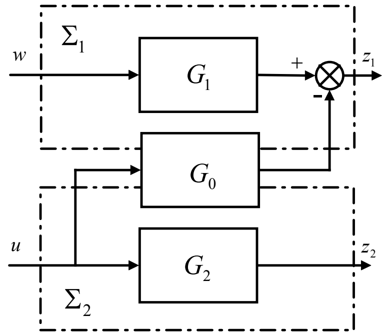

Fig. 1. The substructured system

objective in this paper is different. We aim to solve the synchronization problem between the physical and numerical substructures using an MPC control strategy when the vehicle suspension system is tested within a multivariable substructured system framework.

We first introduce the substructured framework proposed by Stoten and Hyde (2006) in section 2. Based on this framework, we develop the MPC control system in section 3. In section 4, the quasi-motorcycle suspension system is studied: we first convert the system into a two-input two-output substructured framework; then we use the numerical simulation to show the benefits of using MPC controller. Section 5 concludes the paper.

2. A BRIEF INTRODUCTION TO THE SUBSTRUCTURING FRAMEWORK

A general substructured dynamic system was proposed by Stoten and Hyde (2006) as shown in Fig. 1. The system can be expressed by

$$z_1 = G_1 w - G_0 u \tag{1}$$
$$z_2 = G_2 u \tag{2}$$

Transfer functions G_1 and G_2 represent the dynamics of the numerical and physical substructures, and G_0 the interaction dynamics between the two substructures. We use the generalized set $\{\Sigma_1, \Sigma_2\}$ to represent the numerical and physical substructures $\{\Sigma_N, \Sigma_P\}$ respectively, or conversely $\{\Sigma_P, \Sigma_N\}$. The control objective is to use a synchronizing control signal u to make the output z_2 of Σ_2 track the output z_1 of Σ_1, subject to the external disturbance (or testing signal) w. Note that this framework was originally proposed for SISO continuous systems; however, in this paper we extend this framework to MIMO and both continuous and discrete systems for MPC control.

3. MPC CONTROLLER DEVELOPMENT

When the synchronizing input signal is generated by an MPC controller, the system can be expressed as Fig. 2, where an observer is usually required to estimate the plant state at each sampling instant. Here $w(k)$ is assumed to be a measured disturbance.

Suppose that the discrete time transfer functions $G_0(z)$, $G_1(z)$ and $G_2(z)$ are strictly proper and their state space matrices are $G_0(z) \sim (A_0, B_0, C_0, 0)$, $G_1(z) \sim (A_1, B_1, C_1, 0)$ and $G_2(z) \sim (A_2, B_2, C_2, 0)$, then the state space realization for the whole system can be written as

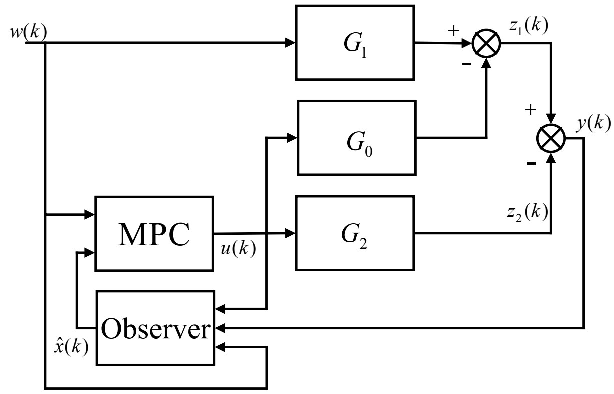

Fig. 2. The substructured system with MPC controller and observer

$$x(k+1) = Ax(k) + B_u u(k) + B_w w(k) \tag{3a}$$
$$y(k) = Cx(k) \tag{3b}$$

with

$$x(k) = \begin{bmatrix} x_0(k) \\ x_1(k) \\ x_2(k) \end{bmatrix} \quad A = \begin{bmatrix} A_0 & 0 & 0 \\ 0 & A_1 & 0 \\ 0 & 0 & A_2 \end{bmatrix} \quad B_u = \begin{bmatrix} B_0 \\ 0 \\ B_2 \end{bmatrix}$$
$$B_w = \begin{bmatrix} 0 \\ B_1 \\ 0 \end{bmatrix} \quad C = [-C_0 \; C_1 \; -C_2]$$

Here $x(k) \in \mathbb{R}^{n_x}$, $u(k)$ and $y(k) \in \mathbb{R}^{n_u}$.

The equations for the observer are

$$\hat{x}(k|k) = \hat{x}(k|k-1) + L'[y(k) - \hat{y}(k|k-1)] \tag{4a}$$
$$\hat{x}(k+1|k) = A\hat{x}(k|k) + Bu(k) + B_w w(k) \tag{4b}$$
$$\hat{y}(k|k-1) = C\hat{x}(k|k-1) \tag{4c}$$

Substituting (4a) and (4c) into (4b) yields

$$\hat{x}(k+1|k) = (A-LC)\hat{x}(k|k-1) + Bu(k) + B_w w(k) + Ly(k) \tag{5}$$

with $L = AL'$. Here $\hat{v}(i|j)$ denotes the value at instant i, which is estimated at instant j, and $\hat{v}(i) := \hat{v}(i|i)$. If (A, C) is observable, the eigenvalues of $A - LC$ can be arbitrarily assigned by choice of L. In this paper we do not consider the extra noises on state and output.

Suppose the cost function of the MPC controller is

$$J(\hat{x}(k+1|k), u(k)) = \|\hat{x}(k+N|k) - x_{ss}\|_P^2 + \sum_{i=1}^{N-1} \|\hat{x}(k+i|k) - x_{ss}\|_Q^2 + \sum_{i=0}^{N-1} \|u(k+i) - u_{ss}\|_R^2 \tag{6}$$

where N is the prediction horizon, Q the state weight, R the input weight, P the terminal weight for the terminal state x_N, and x_{ss}, u_{ss} are the desired steady-state values respectively.

Using straightforward manipulations (Maciejowski, 2002), (6) can be converted into a QP. Manipulations on (4b) leads to

$$X_k = \Lambda\hat{x}(k) + \Phi U_k + \Phi_w W_k \tag{7}$$

with vectors and matrices as

$$X_k = \begin{bmatrix} \hat{x}(k+1|k) \\ \hat{x}(k+2|k) \\ \vdots \\ \hat{x}(k+N|k) \end{bmatrix} \qquad U_k = \begin{bmatrix} \hat{u}(k|k) \\ \hat{u}(k+1|k) \\ \vdots \\ \hat{u}(k+N-1|k) \end{bmatrix}$$

$$W_k = \begin{bmatrix} \hat{w}(k|k) \\ \hat{w}(k+1|k) \\ \vdots \\ \hat{w}(k+N-1|k) \end{bmatrix} \qquad \Lambda = \begin{bmatrix} A \\ A^2 \\ \vdots \\ A^N \end{bmatrix}$$

$$\Phi = \begin{bmatrix} B_u & & & \\ AB_u & B_u & & \\ \vdots & \vdots & \ddots & \\ A^{N-1}B_u & A^{N-2}B_u & \cdots & B_u \end{bmatrix}$$

$$\Phi_w = \begin{bmatrix} B_w & & & \\ AB_w & B_w & & \\ \vdots & \vdots & \ddots & \\ A^{N-1}B_w & A^{N-2}B_w & \cdots & B_w \end{bmatrix}$$

Here the disturbance $w(k)$ is measured at the same time as the measurement of $y(k)$. The future estimation of $\hat{w}(k+i|k)$ is influenced by the knowledge of the behaviour of the disturbance. The common assumption on the estimation of $\hat{w}(k+i|k)$ is to assume it to be constant, i.e. $w(k) = \hat{w}(k+1|k) = \ldots \hat{w}(k+N-1|k)$ (Maciejowski, 2002). In this case, we can replace W_k by $w(k)$ in equation (7) and Φ_w by

$$\Phi_w = \begin{bmatrix} B_w \\ AB_w + B_w \\ \vdots \\ A^{N-1}B_w + A^{N-2}B_w + \cdots + B_w \end{bmatrix} \tag{8}$$

Using the vectors defined in (7), the cost function (6) can be written concisely in matrix form as

$$J(k) = \|X_k - X_{ss}\|^2_{\mathcal{Q}} + \|U_k - U_{ss}\|^2_{\mathcal{R}} \tag{9}$$

where

$$X_{ss} = I_x x_{ss} = [I \cdots I]^T x_{ss} \quad \text{with} \quad I_x \in \mathbb{R}^{n_x \times N n_x}$$
$$U_{ss} = I_u u_{ss} = [I \cdots I]^T u_{ss} \quad \text{with} \quad I_u \in \mathbb{R}^{n_u \times N n_u}$$

$$\mathcal{Q} = \begin{bmatrix} Q & & & \\ & \ddots & & \\ & & Q & \\ & & & P \end{bmatrix} \qquad \mathcal{R} = \begin{bmatrix} R & & \\ & \ddots & \\ & & R \end{bmatrix}$$

where P is the terminal weight calculated from discrete algebraic Riccati equation (DARE):

$$P = A^T[P - PB(B^TPB + R)^{-1}B^TP]A + Q$$

Note that we can calculate the LQR feedback gain by

$$K_{LQR} = (B^TPB + R)^{-1}B^TPA \tag{10}$$

so that the input control signal is determined by $u(k) = -K_{LQR}\hat{x}(k)$.

Substituting (7) into (9) gives

$$J(k) = U_k^T H U_k + 2U_k^T \left[F_x \hat{x}(k) + F_w w_k - f_{ss}\right] + c \tag{11}$$

with

$$H = \Phi^T \mathcal{Q} \Phi + \mathcal{R} \qquad F_x = \Phi^T \mathcal{Q} \Lambda$$
$$F_w = \Phi^T \mathcal{Q} \Phi_w \qquad f_{ss} = -\Phi^T \mathcal{Q} I_x x_{ss} - \mathcal{R} I_u u_{ss}$$

Here the constant c can be ignored without influencing the optimization, and u_{ss} is calculated by a separate QP:

$$\begin{aligned} u_{ss} &= \arg\min_u \left\| C(I-A)^{-1}Bu + \hat{d} - r \right\|^2_{Q_{ss}} \\ &= \arg\min_u \; u^T H_{ss} u + 2u^T F_{ss}(C_{aw}w_k - r) + c \qquad (12) \\ &\text{s.t. } u \in \mathbb{U} \text{ and } (I-A)^{-1}Bu \in \mathbb{X} \end{aligned}$$

with

$$H_{ss} = B^T (I-A)^{-T} C^T Q_{ss} C (I-A)^{-1} B \tag{13}$$
$$F_{ss} = B^T (I-A)^{-T} C^T Q_{ss} \tag{14}$$
$$C_{aw} = C(I-A)^{-1}B_w \tag{15}$$

and x_{ss} is determined by

$$x_{ss} = (I-A)^{-1} B u_{ss} \tag{16}$$

Substituting (16) into (11) yields

$$J(k) = U_k^T H U_k + 2U_k^T f_k + c \tag{17}$$

with

$$f_k = F_x \hat{x}(k|k+1) + F_w w(k) + 2F_s u_{ss}$$
$$F_s = -\frac{1}{2}\left(\Phi^T \mathcal{Q} I_x (I-A)^{-1} B + \mathcal{R} I_u\right)$$

Furthermore, suppose that the system is subject to input constraints, which can be represented by a set of equality and inequality constraints. Then the MPC controller can be expressed as

$$\begin{aligned} U_k^* = \phi(f_k) = \arg\min_{U_k} \frac{1}{2} U_k^T H U_k + U_k^T f_k \\ \text{subject to } LU_k \preceq b \\ \text{and } MU_k = 0 \end{aligned} \tag{18}$$

with the constant vector $b \succeq 0$. $u(k)^* = \bar{E}U_k^*$ is fed into the plant as the input. Here

$$\bar{E} = [I, 0, \ldots, 0] \in \mathbb{R}^{n_u \times N n_u} \tag{19}$$

with the identity matrix $I \in \mathbb{R}^{n_u}$ and the zero matrix $0 \in \mathbb{R}^{n_u}$.

4. CASE STUDY

We consider a quasi-motorcycle suspension system currently being developed at the University of Bristol. In this case study, we separate the system into three parts: the quasi-motorcycle body with two suspension struts containing physical parameters, front and rear wheels modelled numerically, as shown in Fig. 3. We call this a *single mode* substructure. We can also model one wheel numerically and the other physically, or two wheels physically and the body with two suspension struts numerically, depending on the problems that we are interested in. The control objective is to synchronize the physical and numerical substructures by minimizing the displacement errors between the front/rear suspension struts and front/rear wheel hubs, subject to disturbances and actuator constraints. In the following, we first convert this single mode quasi-motorcycle suspension system into the standard substructured framework. Then numerical simulations are presented to show the advantage of using MPC controller to synchronize this substructured system over an LQR controller. LQR has also been used for vehicle suspension control, for example Martinus et al. (1996), although the control objective is different. The QP in the

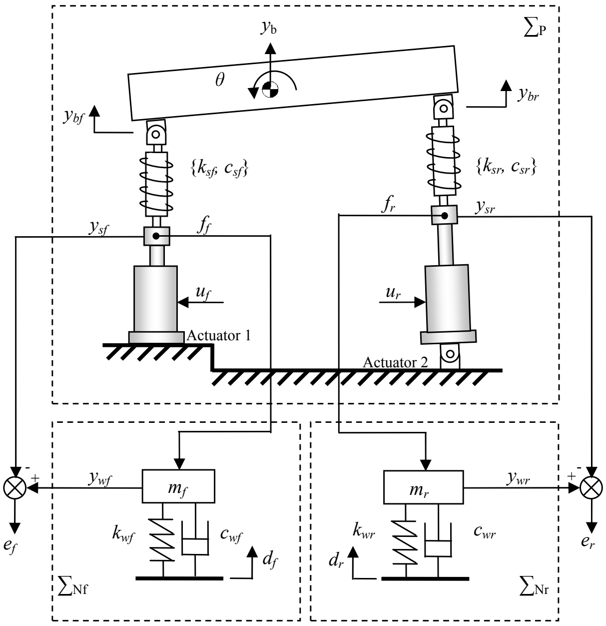

Fig. 3. The development of the substructured systems

MPC controller is solved using the routines developed by Wills (2007), which can guarantee the computing speed (Wills et al., 2008). All of the notation for the variables and parameters, as well as the values of parameters are listed in Appendix.

4.1 Model establishment for quasi-motorcycle suspension system

The dynamic equations of the quasi-motorcycle body are

$$m_b\ddot{y}_b = f_f + f_r - m_b g \tag{20}$$

$$J\ddot{\theta} = L_r f_r - L_f f_f \tag{21}$$

with $\theta = (y_{br} - y_{bf})/L$ and $y_b = (L_r y_{bf} + L_f y_{br})/L$. From (20) and (21), the interaction forces can be represented by

$$f_f(s) =(m_b L_r^2/L^2 + J/L^2)s^2 y_{bf}(s) + (m_b L_r^2/L^2 + J/L^2)s^2 y_{br}(s) \tag{22}$$

$$f_r(s) =(m_b L_f L_r/L^2 - J/L^2)s^2 y_{bf}(s) + (m_b L_f^2/L^2 + J/L^2)s^2 y_{br}(s) \tag{23}$$

The dynamic equations for the front and rear ends of the quasi-motorcycle body are

$$\frac{L_r}{L} m_b \ddot{y}_{bf} = k_{sf}(y_{sf} - y_{bf}) + c_{sf}(\dot{y}_{sf} - \dot{y}_{bf}) - \frac{L_r}{L} m_b g$$

$$\frac{L_f}{L} m_b \ddot{y}_{br} = k_{sr}(y_{sr} - y_{br}) + c_{sr}(\dot{y}_{sr} - \dot{y}_{br}) - \frac{L_f}{L} m_b g$$

The corresponding Laplace transforms are

$$y_{bf}(s) =\frac{(c_{sf}s + k_{sf})y_{sf}(s) - \frac{L_r}{L} m_b g}{\frac{L_r}{L} m_b s^2 + c_{sf}s + k_{sf}} \tag{24}$$

$$y_{br}(s) =\frac{(c_{sr}s + k_{sr})y_{sr}(s) - \frac{L_f}{L} m_b g}{\frac{L_f}{L} m_b s^2 + c_{sr}s + k_{sr}} \tag{25}$$

The dynamic equations for the front and rear wheels are

$$m_{wf}\ddot{y}_{wf} =k_{wf}(d_f - y_{wf}) + c_{wf}(\dot{d}_f - \dot{y}_f) - f_f - m_{wf} g$$

$$m_{wr}\ddot{y}_{wr} =k_{wr}(d_r - y_{wr}) + c_{wr}(\dot{d}_r - \dot{y}_r) - f_r - m_{wr} g$$

and the corresponding response transforms are

$$y_{wf}(s) = \frac{c_{wf}s + (k_{wf})d_f(s) - f_f(s) - m_{wf}g}{m_{wf}s^2 + c_{wf}s + k_{wf}} \tag{26a}$$

$$y_{wr}(s) = \frac{c_{wr}s + (k_{wr})d_r(s) - f_r(s) - m_{wr}g}{m_{wr}s^2 + c_{wr}s + k_{wr}} \tag{26b}$$

The response transforms for the two inner-loop controlled actuators are approximately given by

$$y_{sf} = G_{2f} u_f \tag{27a}$$

$$y_{rf} = G_{2r} u_r \tag{27b}$$

with

$$G_{2f} = \frac{b_f}{s + a_f} \qquad G_{2r} = \frac{b_r}{s + a_r}$$

By straightforward substitutions, we have

$$y_{wf}(s) = G_{1f}(s)d_f(s) - G_{0ff}(s)u_f(s) - G_{0fr}u_r(s) \tag{28a}$$

$$y_{wr}(s) = G_{1r}(s)d_r(s) - G_{0rf}(s)u_f(s) - G_{0rr}u_r(s) \tag{28b}$$

with

$$G_{1f} =\frac{c_{wf}s + k_{wf}}{m_f s^2 + c_{wf}s + k_{wf}}$$

$$G_{0ff} =\frac{c_{sf}s^3 + k_{sf}s^2}{(\frac{L_r}{L} m_b s^2 + c_{sf}s + k_{sf})(m_f s^2 + c_{wf}s + k_{wf})} \times \left(\frac{m_b L_r^2}{L^2} + \frac{J}{L^2}\right) \times G_{2f}$$

$$G_{0fr} =\frac{c_{sr}s^3 + k_{sr}s^2}{(\frac{L_f}{L} m_b s^2 + c_{sr}s + k_{sr})(m_f s^2 + c_{wf}s + k_{wf})} \times \left(\frac{m_b L_f L_r}{L^2} - \frac{J}{L^2}\right) \times G_{2r}$$

$$G_{1r} =\frac{c_{wr}s + k_{wr}}{m_r s^2 + c_{wr}s + k_{wr}}$$

$$G_{0rf} =\frac{c_{sf}s^3 + k_{sf}s^2}{(\frac{L_r}{L} m_b s^2 + c_{sf}s + k_{sf})(m_r s^2 + c_{wr}s + k_{wr})} \times \left(\frac{m_b L_f L_r}{L^2} - \frac{J}{L^2}\right) \times G_{2f}$$

$$G_{0rr} =\frac{c_{sr}s^3 + k_{sr}s^2}{(\frac{L_f}{L} m_b s^2 + c_{sr}s + k_{sr})(m_r s^2 + c_{wr}s + k_{wr})} \times \left(\frac{m_b L_f^2}{L^2} + \frac{J}{L^2}\right) \times G_{2r}$$

Note that gravity only affects the initial states of the system. Hence we set the gravity constant to zero for simplicity, without influencing the resulting controller design.

Define

$$z_1 = \begin{bmatrix} y_{wf} \\ y_{wr} \end{bmatrix} \quad z_2 = \begin{bmatrix} y_{sf} \\ y_{sr} \end{bmatrix} \quad u = \begin{bmatrix} u_f \\ u_r \end{bmatrix} \quad w = \begin{bmatrix} d_f \\ d_r \end{bmatrix}$$

and

$$G_1(s) = \begin{bmatrix} G_{1f}(s) & 0 \\ 0 & G_{1r}(s) \end{bmatrix} \qquad G_2(s) = \begin{bmatrix} G_{2f}(s) & 0 \\ 0 & G_{2r}(s) \end{bmatrix}$$

$$G_0(s) = \begin{bmatrix} G_{0ff}(s) & G_{0fr}(s) \\ G_{0rf}(s) & G_{0rr}(s) \end{bmatrix}$$

Then (27a), (27b), (28a) and (28b) can be written in the standard substructured framework as

$$z_1(s) = G_1(s)w(s) - G_0(s)u(s) \tag{29a}$$

$$z_2(s) = G_2(s)u(s) \tag{29b}$$

Table 1. Disturbances on the quasi-motorcycle system

k	1	2	21	22	41	42	61	62	81	82
d_f(cm)	1	0	-1	0	1	0	-1	0	1	0
d_r (cm)	0	1	0	-1	0	1	0	-1	0	1

Note that this is a coupled multivariable substructured system: G_1 contains the numerical substructure parameters, G_2 the physical substructure parameters and G_0 both numerical and physical substructure parameters.

4.2 Simulation

The system is simulated with an internal perturbation to the parameters of the physical substructure G_2, which is composed of two servo-hydraulic actuators. We suppose the real values of a_f, a_r, b_f and b_r for the plant model are all 0.83; however, we use 8.3 as their values in the controller formulation. The parameter values chosen are not intended to be representative of real vehicle condition; they are used merely to test the control performance subject to parameter variations in actuators.

To demonstrate the disturbance rejection performance and the advantage of using MPC to cope with saturation, we make a comparison between the MPC and LQR controllers. The simulation duration is 5s with a sampling frequency 20 Hz. The disturbances acting on the two wheels at sampling instants are shown in Table 1, which is based on the assumption that the vehicle speed is around 120 km/h. Because of the actuator saturation, the two inputs are constrained within $-0.02 \sim 0.02$ m. Furthermore, we do not want the output errors to exceed 0.01 m.

For the MPC controller we choose the prediction horizon $N = 10$, input weight $R = 4I_u$ with the identity matrix $I_u \in \mathbb{R}^{n_u}$ and state weight $Q = I_x$ with the identity matrix $I_x \in \mathbb{R}^{n_x}$. In Fig. 4, outputs 1 and 2 correspond to the front and rear displacement errors between the suspension struts and the wheel hubs, and inputs 1 and 2 are the front and rear actuator inputs. This result shows that the MPC controller can guarantee that the outputs are within $-0.01 \sim 0.01$ m, while the input saturations are also satisfied.

We also use LQR controller with the same conditions and choose the same input and output weights as the MPC controller to calculate the constant feedback gain by (10). The input signals are clipped within the saturation limits. We use the sum of square of output and input signals to represent the input and output energies of the system controlled by MPC and LQR controllers as in Figs. 5 and 6, which show that MPC outperforms saturated LQR.

5. CONCLUSION

We have developed an MPC controller to synchronize multivariable substructured systems subject to input constraints. In the study case, we converted a quasi-motorcycle system into a multivariable substructured framework and then applied MPC controller on this system. The numerical simulation shows the advantage of using MPC to cope with the actuator saturation over conventional LQR optimal control. Real-time experiments on a test rig are currently the subject of research work.

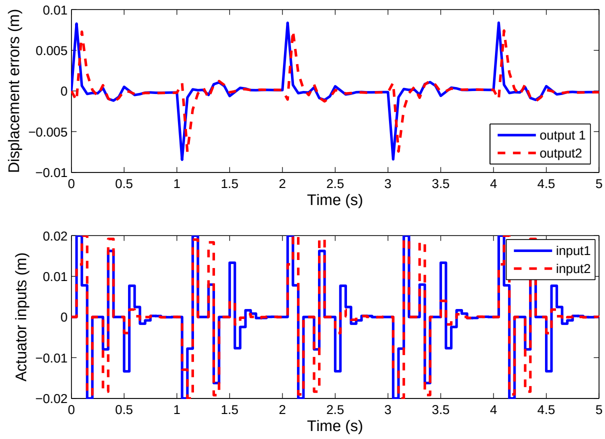

Fig. 4. Displacement errors and saturation inputs of the substructured system with MPC controller

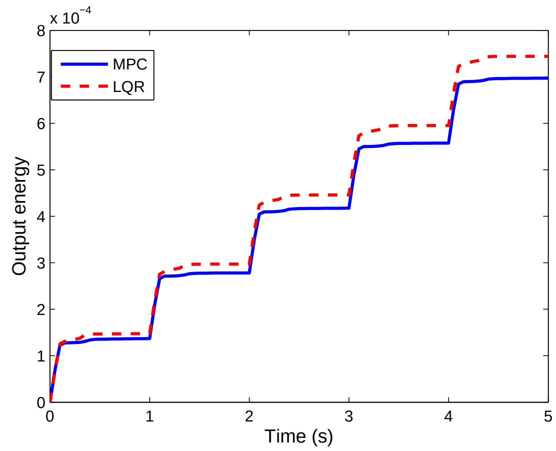

Fig. 5. Comparison of the output energy between MPC and saturated LQR

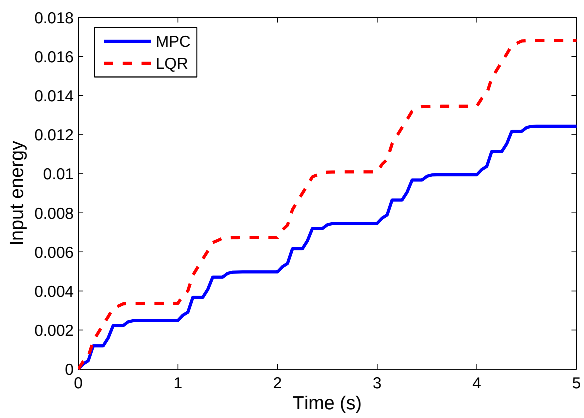

Fig. 6. Comparison of the input energy between MPC and saturated LQR

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Appendix A. NOTATION LIST FOR THE HALF-CAR BODY SYSTEM

A.1 Parameters

Notation	**Description**	**Values**
	Quasi-motorcycle body:	
m_b	Mass	160 kg
J	Moment of inertia	60 kg m^2
L	Body length	2 m
L_f, L_r	Lengths from front/rear end to mass center	1.2, 0.8 m
	Front/rear suspension:	
k_{sf}, k_{sr}	Stiffness.	7600, 8000 Nm^{-1}
c_{sf}, c_{sr}	Damping	1020, 1120 Nsm^{-1}
	Front/rear wheels:	
m_f, m_r	Mass	15 kg
k_{wf}, k_{wr}	Stiffness	6800, 7000 Nm^{-1}
c_{wf}, c_{wr}	Dampness	420, 454 Nsm^{-1}
	Front/rear actuators:	
$\tilde{a}_f$, $\tilde{b}_f$	Parameters for controller	8.3 s^{-1}, 8.3
$\tilde{a}_r$, $\tilde{b}_r$	Parameters for controller	8.3 s^{-1}, 8.3
a_f, b_f	Parameters for plant	2.2 s^{-1}, 2.2
a_r, b_r	Parameters for plant	2.2 s^{-1}, 2.2

A.2 Variables

Notation	**Description**
y_{wf}, y_{wr}	Front/rear wheel displacements.
y_b	Body center of mass displacement.
θ	Pitch of the body.
y_{bf}, y_{br}	Front/rear ends of body displacements.
y_{sf}, y_{sr}	Front/rear suspension base displacements.
u_f, u_r	Inputs of the front/rear actuators.
f_f, f_r	Interaction forces.
d_f, d_r	Disturbances on the front/rear wheels.
y_{af}, y_{ar}	Front/rear wheel actuators displacements.