Asymptotic Rejection of Nonlinear Periodic Disturbances in Linear Dynamic Systems

Zhengtao Ding

Control Systems Centre, School of Electrical and Electronic Eng., University of Manchester, PO Box 88, Manchester M60 1QD, UK (Tel: +44 161 3064663; e-mail: zhengtao.ding@manchester.ac.uk).

Abstract: This paper deals with asymptotic rejection of disturbances generated from nonlinear exosystems. The dynamic system is assumed to be linear. A new strategy for internal model design is proposed, based on a dynamic extension of the existing nonlinear observer design for the nonlinear exosystem. Additional filters are used to estimate the invariant manifold in the state space subject to the nonlinear disturbances generated from the exosystem. The proposed design for the internal model and control ensures that the state variable asymptotically converge to the invariant manifold, which implies that the designated output state asymptotically converge to zero.

1. INTRODUCTION

When the disturbances are generated from linear exosystems, the asymptotic rejection problem has been addressed in Bodson et al. [1994], Bodson and Douglas [1997], Ding [2001], Marino et al. [2003], Ding [2003, 2006a], even when the disturbance frequencies are completely unknown. The asymptotic rejction problem becomes much more difficult when the disturbances are generated from nonlinear exosystems. The difficulty is due to the internal model design and generation of invariant manifold. Limited successes have been reported in Ding [2006b], Priscoli et al. [2006], Xi and Ding [2007] where the nonlinear disturbances are assumed to enter the system via input channel. It remains a challenge to asymptotic reject a nonlinear disturbance when the match condition is not satisfied.

In this paper, we consider asymptotic rejection problem for disturbances generated from nonlinear exosystems without ristriction to the matched cases. To concentrate on the presentation of key results in the internal model design and the estimation of the invariant manifold, we assume the dynamic system is linear. The idea presented in this paper can be extended to nonlinear dynamic systems with an appropriate assumption of stabilization conditions.

When the regulation equation has an explicit solution, the desired feedforward term can be expressed explicitly in terms of the state of the exosystem. Furthermore, if there exists an immersion of the exosystem, an internal model may be designed to produce an estimate for the desired feedforward control design for disturbance rejection and output regulation. In general, an internal model is different from an observer for the state variables of the exosystem. When the exosystem is linear, it is relatively easy to design an internal model. For nonlinear exosystems, it is in general very difficult or even impossible to have an explicit expression of the desired feedforward term as an algebraic function of the state variable of the exosystem.

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We propose a new strategy for the internal model design. The nonlinear disturbance enters the dynamic system, and we view the dynamic system as a dynamic extension of the output measurement for the exosystem. Through this dynamic extension, we exploit observer design for the exosystem to estimate the state variables of the exosystem. Once the states of the exosystem are observed, they are used to generate estimates of the invariant manifold and the desired feedforward input. In this paper, this new strategy is presented for asymptotic rejection of nonlinear disturbances in linear dynamic systems. An example is included to demonstrate the proposed design.

2. PROBLEM FORMULATION

Consider a dynamic system

$$\begin{aligned} \dot{x} &= Ax + bu + d\mu(w) \\ \dot{w} &= s(w) \\ y &= c^T x \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$ is the state vector for the linear part, $u \in \mathbb{R}$ is the control, $y \in \mathbb{R}$ is the output to be regulated, $w \in \mathcal{W}$, a compact subset in $\mathbb{R}^s$, denotes the state variable of the nonlinear system, which is also referred to as the exosystem, $d \in \mathbb{R}^n$ is a vector, $\mu : \mathbb{R}^s \to \mathbb{R}$ is a smooth nonlinear function of w, denoting the nonlinear disturbance to the linear part, $s : \mathbb{R}^s \to \mathbb{R}^s$ is a smooth vector field, and $\{A, b\}$ is a controllable pair. Without losing generality, we write them as

$$A = \begin{bmatrix} -a_1 & 1 & 0 & \dots & 0 \\ -a_2 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -a_{n-1} & 0 & 0 & \dots & 1 \\ -a_n & 0 & 0 & \dots & 0 \end{bmatrix}, \quad b = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ b_\rho \\ \vdots \\ b_n \end{bmatrix},$$

$$c = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \quad d = \begin{bmatrix} d_1 \\ \vdots \\ d_n \end{bmatrix}$$

with $b_\rho \neq 0$.

Assumption 1. The flow of vector $s(w)$ is invariant in $\mathcal{W}$, and the converges to periodic solutions.

Assumption 2. For the nonlinear exosystem, there exists an observer

$$\dot{\xi} = f(\xi) + g(\mu(w) - \bar{\gamma}(\xi)) \quad (2)$$

with $\mu(T(\xi)) = \gamma(\xi)$,$g \in \mathbb{R}^s$, and $T : \mathbb{R}^s \to \mathbb{R}^s$ being a smooth function, and

$$\bar{\gamma}(\xi)) = \begin{cases} \gamma(\xi), & \text{for } |\gamma(\xi)| < M, \\ \text{sgn}(\gamma(\xi))M, & \text{for } |\gamma(\xi)| \geq M, \end{cases}$$

with $M = \max_{\xi \in \mathcal{X}} |\gamma(\xi)|$, and $\mathcal{X}$ being a compact subset in $\mathbb{R}^n$ and $\mathcal{X} \supset T^{-1}(\mathcal{W})$, such that there exists a Lyapunov function $V(\tilde{w})$, with $\tilde{w} = w - T(\xi)$ which satisfies, for $w \in \mathcal{W}$ and $\xi \in \mathcal{X}$, a compact subset in $\mathbb{R}^s$,

$$\begin{aligned} \alpha_1(\|\tilde{w}\|) &\leq V(\tilde{w}) \leq \alpha_2(\|\tilde{w}\|) \\ \dot{V} &\leq -\alpha_3(\|\tilde{w}\|) \end{aligned} \quad (3)$$

where α_1, α_2 and α_3 are $\mathcal{K}_\infty$ functions.

The disturbance rejection problem considered in this paper is to design a dynamic feedback control with the measurement x to completely rejection the disturbance and ensure the stability of the closed-loop system in the sense that $\lim_{t\to\infty} y(t) = 0$.

3. EQUIVALENT INPUT DISTURBANCE

To solve the equivalent input disturbance, we introduce a partial state transform for the system (1)

$$z = \begin{bmatrix} x_{\rho+1} \\ \vdots \\ x_n \end{bmatrix} - \sum_{i=1}^{\rho} B^{\rho-i}\bar{b}x_i \quad (4)$$

where

$$B = \begin{bmatrix} -b_{\rho+1}/b_\rho & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -b_{n-1}/b_\rho & 0 & \dots & 1 \\ -b_n/b_\rho & 0 & \dots & 0 \end{bmatrix}, \quad \bar{b} = \begin{bmatrix} b_{\rho+1}/b_\rho \\ \vdots \\ b_n/b_\rho \end{bmatrix}$$

The dynamics with the coordinates $(x_1, \dots, x_\rho, z)$ can be obtained as

$$\begin{aligned} \dot{x}_i &= x_{i+1} - a_i x_1 + d_i\mu(w), \quad i = 1, \dots, \rho - 1 \\ \dot{x}_\rho &= z_1 + \sum_{i=1}^{\rho} r_i x_i + a_\rho x_1 + d_\rho\mu(w) + b_\rho u \\ \dot{z} &= Bz - a_z x_1 + d_z\mu(w) \end{aligned} \quad (5)$$

where

$$r_i = [B^{\rho-i}\bar{b}]_1, \text{ for } i = 1, \dots, \rho$$

$$a_z = \begin{bmatrix} a_{\rho+1} \\ \vdots \\ a_n \end{bmatrix} - \sum_{i=1}^{\rho} B^{\rho-i}\bar{b}a_i - B^\rho\bar{b}$$

and

$$d_z = \begin{bmatrix} d_{\rho+1} \\ \vdots \\ d_n \end{bmatrix} - \sum_{i=1}^{\rho} B^{\rho-i}\bar{b}d_i$$

We will use the coordinates $(x_1, \dots, x_\rho, z)$ for the steady-state response of the system. We denote their corresponding steady state variables by $(\pi_1, \dots, \pi_\rho, \pi_z)$.

Let $\pi_1 = 0$. From the system dynamics, it can be obtained iteratively that, for $i = 1, \dots, \rho - 1$,

$$\pi_{i+1}(t) = \frac{d\pi_i(t)}{dt} - d_i\mu(w) \quad (6)$$

For π_z, we have

$$\dot{\pi}_z = B\pi_z + d_z\mu(w) \quad (7)$$

From the system dynamics, we have, for $i = 1, \dots, \rho - 1$,

$$\pi_{i+1}(t) = \frac{d\pi_i(t)}{dt} - d_i\mu(w) \quad (8)$$

Based on the state transformation introduced earlier, we can use its inverse transformation to obtain

$$\begin{bmatrix} \pi_{\rho+1} \\ \vdots \\ \pi_n \end{bmatrix} = \pi_z + \sum_{i=1}^{\rho} B^{\rho-i}\bar{b}\pi_i \quad (9)$$

Therefore the periodic trajectory, or the invariant manifold, in the state space is obtained as

$$\pi = [\pi_1, \dots, \pi_\rho, \pi_{\rho+1}, \dots, \pi_n]^T \quad (10)$$

Finally, the equivalent input disturbance α is given by

$$\alpha = \frac{1}{b_\rho}[\frac{d\pi_\rho(t)}{dt} - \sum_{i=1}^{\rho} \pi_i - c_z^T\pi_z(t) - d_\rho\mu(w)] \quad (11)$$

where $c_z = [1,\ 0, \dots,\ 0]^T \in \mathbb{R}^{n-\rho}$.

In general, it is difficult, if not impossible, to express π_z as a function of w, for nonlinear disturbances. However, we can express $\frac{d\pi_\rho(t)}{dt}$ as a function of w by invoking the equation (8). If we abuse the notation by denoting $\pi_i(t)$ by $\pi_i(w)$, we have, for $i = 1, \dots, \rho - 1$,

$$\pi_{i+1}(w) = \frac{\partial\pi_i(w)}{\partial w}s(w) - d_i\mu(w) \quad (12)$$

Therefore, we rewrite (11) as

$$\alpha = \frac{1}{b_\rho}[\sigma(w) - c_z^T\pi_z(t) - d_\rho\mu(w)] \quad (13)$$

where $\sigma(w) = \frac{\partial\pi_\rho(w)}{\partial w}s(w) + \sum_{i=1}^{\rho} r_i\pi(w)$.

4. INTERNAL MODEL DESIGN

When there is an explicit expression of α as an function of w, say $\alpha(w)$, we may be able to explore the internal model design for the case of linear exosystem with nonlinear terms as polynomials. For the expression of (13), we will obtain the desired input in three parts. Based on Assumption 2, we can design an extended observer design for w and $\mu(w)$. Let $h \in \mathbb{R}^n$ such that $h^T d = 1$. Define $p = h^T x$, and we have

$$\dot{p} = h^T Ax + h^T bu + \mu(w) \tag{14}$$

The observer is designed as

$$\begin{aligned} \dot{\xi} &= f(\xi) - gv \\ \dot{q} &= -\bar{c}q - \bar{c}p - h^T Ax - h^T bu - \bar{\gamma}(\xi) + v \\ v &= -k(p+q) \end{aligned} \tag{15}$$

where $\bar{c}$ and k are positive real design parameters.

Theorem 3.1 For the observer shown in (15), $T(\xi)$ is an asymptotic convergent estimate of w, and $\gamma(\xi)$ is an asymptotic estimate of $\mu(w)$.

Proof. Let $e = p + q$. We rewrite the observer (15) as

$$\begin{aligned} \dot{\xi} &= f(\xi) - gv \\ \dot{e} &= -\bar{c}e + \mu(w) - \bar{\gamma}(\xi) + v \\ v &= -ke \end{aligned} \tag{16}$$

From (16), it can be obtained that

$$\begin{aligned} |e(t)| &\le |e(0)| + \frac{1}{\bar{c}}[\max_{w\in\mathcal{W}} |\mu(w)| + M] \\ &\le |h^T x_0| + |q(0)| + \frac{1}{\bar{c}}[\max_{w\in\mathcal{W}} |\mu(w)| + M] \\ &:= \bar{e} \end{aligned}$$

Therefore e is bounded and $e \in \mathcal{E} = \{e|\ |e| \le \bar{e}\}$. Viewing e as the output of (16), the zero dynamics is exactly in the form described in (2) which is stable. Intuitively, the observer (16) can be viewed as a relative degree one system with stable zero dynamics. For such a system, a static high gain output feedback $v = -ke$ would stabilize the entire system. In fact, we can proceed our analysis in the traditional way.

Let $\eta = \xi + ge$. We have

$$\begin{aligned} \dot{\eta} &= f(\xi) + g(\mu(w) - \bar{\gamma}(\xi)) - \bar{c}ge \\ &= f(\eta - ge) + g(\mu(w) - \bar{\gamma}(\eta - ge)) - \bar{c}ge \end{aligned}$$

Hence, we write the observer in the coordinates η and e as

$$\begin{aligned} \dot{\eta} &= f(\eta) + g(\mu(w) - \gamma(\eta)) \\ &\quad + f_1(\eta, e)e - g\gamma_1(\eta, e)e - \bar{c}ge \\ \dot{e} &= -\bar{c}e + (\mu(w) - \gamma(\eta)) - \gamma_1(\eta, e)e - ke \end{aligned} \tag{17}$$

where

$$\begin{aligned} f_1(\eta, e)e &:= f(\eta - ge) - f(\eta), \\ \gamma_1(\eta, e)e &:= \bar{\gamma}(\eta - ge) - \bar{\gamma}(\eta). \end{aligned}$$

The zero dynamics with e as the output is given by

$$\dot{\eta} = f(\eta) + g(\mu(w) - \gamma(\eta))$$

which is indeed in the same form as in Assumption 2.

Let

$$W(\tilde{w}, e) = V(\tilde{w}) + \frac{1}{2}e^2 \tag{18}$$

where $\tilde{w} = w - T(\eta)$. Based on Assumption 2 and (16), we have

$$\begin{aligned} \dot{W} &= \frac{\partial W}{\partial \tilde{w}} s(w) - \frac{\partial W}{\partial \tilde{w}} \frac{\partial T}{\partial \eta}[f(\eta) + g(\mu(w) - \bar{\gamma}(\eta))] \\ &\quad -\bar{c}e^2 - ke^2 + \beta(w, \eta, e)e \\ &\le -\alpha_3(\tilde{w}) - \bar{c}e^2 - ke^2 + \beta(w, \eta, e)e \end{aligned} \tag{19}$$

where

$$\begin{aligned} \beta(w, \eta, e) &= -\frac{\partial W}{\partial \tilde{w}} \frac{\partial T}{\partial \eta}[f_1(\eta, e)e - g\gamma_1(\eta, e)e - \bar{c}ge] \\ &\quad +(\mu(w) - \gamma(\eta) - \gamma_1(\eta, e))e \end{aligned}$$

When $e = 0$, we have $\dot{W} \le -\alpha_3(\tilde{w})$. From the continuity of the functions involved in (19), there exists a neighbourhood $\mathcal{B} = \{e||e| \le d_e\}$ such that

$$\dot{W} < 0 \tag{20}$$

for $\tilde{w} \ne 0$. For $w \in \mathcal{W}$, $\eta \in \mathcal{X}$ and $e \in \mathcal{E}/\mathcal{B}$, we have

$$\dot{W} \le -\alpha_3(\tilde{w}) - \bar{c}e^2 - (k - \frac{1}{2})e^2 + \frac{1}{2}\beta^2(w, \eta, e) \tag{21}$$

Let

$$k \ge \frac{1}{2} + \frac{1}{2d_e^2} \max_{w\in\mathcal{W}, \eta\in\mathcal{X}, e\in\mathcal{E}/\mathcal{B}} \beta^2(w, \eta, e)$$

We have

$$\dot{W} \le -\alpha_3(\tilde{w}) - \bar{c}e^2 \tag{22}$$

Thus we have shown that $\dot{W} < 0$ for $w \in \mathcal{W}$, $\eta \in \mathcal{X}$ and $e \in \mathcal{B}$ when $\tilde{w} \ne 0$. From the invariant set theorem, we have $\lim_{t\to\infty} \tilde{w} = 0$ and $\lim_{t\to\infty} e = 0$. This completes the proof.

One problem for nonlinear exosystem is that we cannot have an explicit expression of π_z as a function of w. With an asymptotic estimation of w, we can further estimate π_z with

$$\dot{\hat{\pi}}_z = B\hat{\pi}_z + d_z\bar{\gamma}(\xi) \tag{23}$$

and use this estimate to compute the feedforward term for disturbance rejection. We put all equations for the feedforward term together as the following internal model

$$\begin{aligned} \dot{\xi} &= f(\xi) + gk(y + q) \\ \dot{q} &= -\bar{c}q - \bar{c}p - h^T Ax - h^T bu - \bar{\gamma}(\xi) - k(y + q) \\ \dot{\hat{\pi}}_z &= B\hat{\pi}_z + d_z\bar{\gamma}(\xi) \\ \hat{\alpha} &= \frac{1}{b_\rho}[\sigma(T(\xi)) - c_z^T\hat{\pi}_z - d_\rho\bar{\gamma}(\xi)] \end{aligned} \tag{24}$$

With the estimate $\hat{\pi}_z$, we have an estimate of the invariant manifold

$$\hat{\pi} = p_1(T(\xi)) + p_2\hat{\pi}_z \tag{25}$$

where $p_1 : \mathbb{R}^s \to \mathbb{R}^s$ is a known vector function, and $p_2 \in \mathbb{R}^s$ is a known vector.

5. CONTROL DESIGN

Considering the difference between the state variable and the invariant manifold $\tilde{x} = x - \pi$, we have

$$\dot{\tilde{x}}_z = A\tilde{x} + b(u - \alpha). \tag{26}$$

We design the control input as

$$u = -\bar{k}^T(x - \hat{\pi}) + \hat{\alpha} \tag{27}$$

where $\bar{k} \in \mathbb{R}^n$ such that $(A - b\bar{k}^T)$ is Hurwitz. For this control design, we have the stability result described in the following theorem.

Theorem 5.1 For a system (1) satisfies Assumptions 1 and 2, the control design shown in (27) with the internal model (24) ensures the asymptotic rejection of nonlinear disturbance in the sense that $\lim_{t\to\infty}(x - \pi) = 0$ that implies $\lim_{t\to\infty} x_1 = 0$

Proof. Based on the control design (27), the closed loop dynamics are described by

$$\dot{\tilde{x}}_z = (A - b\bar{k}^T)\tilde{x} + b\bar{k}^T(\tilde{\alpha} - \tilde{\pi}). \tag{28}$$

where $\tilde{\alpha} = \alpha - \hat{\alpha}$ and $\tilde{\pi} = \pi - \hat{\pi}$. It can be shown that $\tilde{\alpha}$ and $\tilde{\pi}$ are bounded from the internal model design, which implies the boundedness of $\tilde{x}$ from the stability of $(A - b\bar{k}^T)$. Furthermore, both $\tilde{\alpha}$ and $\tilde{\pi}$ can be shown to asymptotically converge to zero, which implies $\lim_{t\to\infty}(x-\pi) = 0$. This complete the proof.

6. EXAMPLE

Consider

$$\begin{aligned}
\dot{x}_1 &= -x_1 + x_2 + u + w_1 \\
\dot{x}_2 &= x_1 + u + 2w_1 \\
\dot{w}_1 &= w_2 + (w_1 - w_1^3) \\
\dot{w}_2 &= -w_1
\end{aligned} \tag{29}$$

Comparing with the structure shown in (1), we have

$$A = \begin{bmatrix} -1 & 1 \\ 1 & 0 \end{bmatrix},\ b = \begin{bmatrix} 1 \\ 1 \end{bmatrix},\ d = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

and $\mu(w) = w_1$. Note that $d \neq b$, which means the disturbances are unmatched. The exosystem

$$\begin{aligned}
\dot{w}_1 &= w_2 + (w_1 - w_1^3) \\
\dot{w}_2 &= -w_1
\end{aligned}$$

is a Van der Pol oscillator, and its solution converges to a limit cycle. For this exosystem, there exists an observer with $\gamma(\xi) = \xi_1$, $T = I$ and

$$f(\xi) = \begin{bmatrix} \xi_2 + \xi_1 - \xi_1^3 \\ -\xi_1 \end{bmatrix},\ g = \begin{bmatrix} g_1 \\ g_2 \end{bmatrix}$$

ie,

$$\begin{aligned}
\dot{\xi}_1 &= \xi_2 + \xi_1 - \xi_1^3 + g_1(w_1 - \xi_1) \\
\dot{\xi}_2 &= -\xi_1 + g_2(w_1 - \xi_1)
\end{aligned}$$

It can be shown that there exist g_1 and g_2 such that Assumption 2 is satisfied for $\xi \in \mathbb{R}^2$. For the invariant manifold, we define $z = x_2 - x_1$ and its dynamics is given by, together with the dynamics of x_1,

$$\begin{aligned}
\dot{z} &= -z + x_1 + 2w_1 \\
\dot{x}_1 &= z + u + w_1
\end{aligned}$$

Hence we have

$$\begin{aligned}
\pi_1 &= 0 \\
\pi_2 &= \pi_z \\
\dot{\pi}_z &= -\pi_z + 2w_1 \\
\alpha &= -\pi_z - w_1
\end{aligned}$$

Let $h = [-2,\ 1]^T$, and we have $p = x_2 - 2x_1$ and $h^T d = 1$. The dynamics of p is obtained as

$$\dot{p} = 3x_1 - 2x_2 - u + w_1$$

The internal model is designed as

$$\begin{aligned}
\dot{\xi}_1 &= \xi_2 + \xi_1 - \xi_1^3 - g_1\xi_1 - k(p + q) \\
\dot{\xi}_2 &= -\xi_1 - g_2\xi_1 \\
\dot{q} &= -\bar{c}q - \bar{c}p - (3x_1 - 2x_2 - u) - \xi_1 - k(p + q) \\
\dot{\hat{\pi}}_z &= -\hat{\pi}_z + 2\xi_1 \\
\hat{\alpha} &= -\hat{\pi}_z - \xi_1 \\
\hat{\pi} &= [0, \hat{\pi}_z]
\end{aligned}$$

The control input is designed as

$$u = -\bar{k}x_1 - \bar{k}_2(x_2 - \hat{\pi}_z) + \hat{\alpha}$$

In the simulation study, we set $g_1 = 2$, $g_2 = 2$, $c = 2$, $k = 10$, $\bar{k}_1 = 1$ and $\bar{k}_2 = 2$. Figure shows the state variable and the control input. The response of the exosystem and its estimates are shown in Figures 2 and 3, and their trajectories against time in Figure 4.

7. CONCLUSIONS

In this paper, we have presented a new strategy for internal model design for asymptotic rejection of the disturbances generated from nonlinear exosystems. The success of the new strategy is due to the dynamic extension for state observation of the exosystem. A set of filters are design for the dynamic extension and estimation of the invariant manifold. We present the result in this paper for disturbance rejection of linear dynamic systems with nonlinear exosystem to simplify the presentation. The new strategy applies to nonlinear systems with nonlinear exosystems as well.

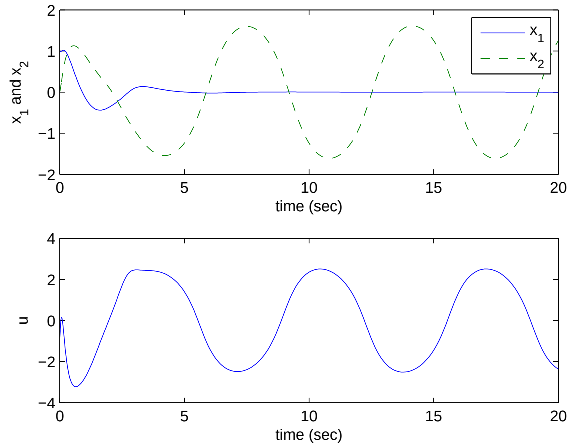

Fig. 1. The state variables and input

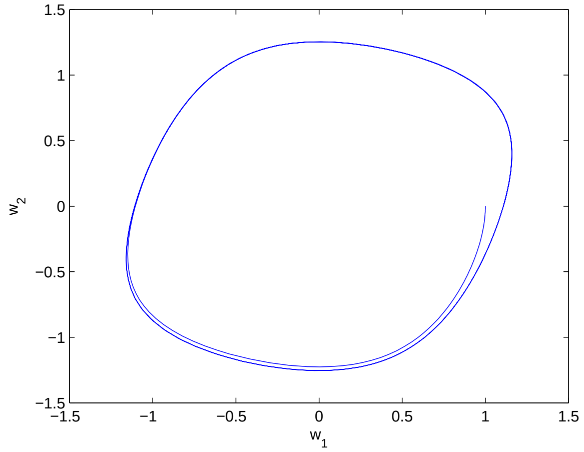

Fig. 2. Phase portrait of the exosystem

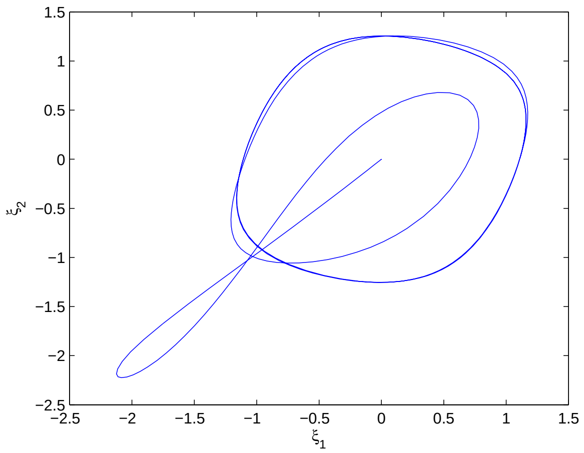

Fig. 3. Phase portrait of the observer for the exosystem

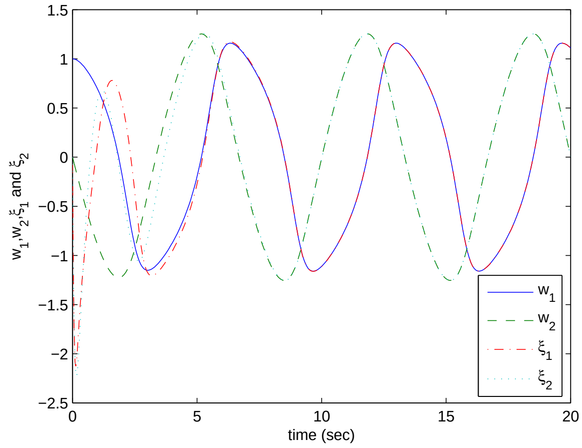

Fig. 4. Time responses of the exosystem and the observer

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