Study of Reduced-order and Non-linear Local Optimal Control Application to Aero Gas Turbines

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Abstract:

In this work second order linear model, reduced order (first order) linear model and second order non-linear model of a gas turbine engine have been obtained from the engine input-output data using evolutionary optimization technique. These three models have then been used in local optimal control design. The obtained controllers have been applied to the second order non-linear engine model and their performance has been compared.

Keywords: Local optimal controller, system identification, genetic algorithms.

1. INTRODUCTION

Simplified controllers have certain advantages over complex linear, as they are easier to understand and the computational requirements are less demanding [1]. That is why design of reduced order controllers for high order plants is used relatively often.

Reduced order controller can be obtained in three main ways [2]. The first approach defines a structure of reduced-order controller, and then search of the controller parameters is carried out by some optimisation technique. In the second approach, a high-order controller is first found and then some procedures are used to simplify it. These two approaches face frequency-weighted L_∞ approximation problem defined in [1], [2]**.** The third approach firstly approximates the plant by a reduced-order model. Then a reduced-order controller design is based on the plant approximation.

In this work the third approach is used. The disadvantage of this approach is related to the approximation of the process. The plant approximation usually requires *a priori* knowledge about the controller to be built [1].

Non-linear characteristics of a plant, e.g. gas turbine engine, change depending on the operating conditions. To control the non-linear model by linear controller, a piece-wise linear approximation is usually used. That can affect the controller performance due to the fact that a linear model is used in the controller design. This linear model differs from the real non-linear plant.

Using a non-linear controller, gas turbine engine control quality can be improved [3]. But as stated in [4], the nonlinear theory is not very often used in practical applications. This is because the linear solutions provide an acceptable approximation or it is extremely difficult to build a non-linear model, or properly identify its nonlinearities. In [4], authors check how good the linear solution is and come to the conclusion that solvability of the associated linear problem implies the sufficient solvability of the nonlinear problem.

Local optimal control introduced by Lyantsev et al. [3] can be used for design of linear as well as non-linear controllers. Mathematical programming is used to deal with constrains. Local optimal control can be used to control multivariable systems as well as single-input single-output systems [3], [5]. In this paper, the non-liner local optimal control is used to control a gas turbine engine and its performance compared with linear local optimal controller.

Model parameters obtained by genetic algorithm for different model structures are used for local optimal controller design. Different methods can be used for identification of the engine model. Genetic algorithms in comparison with usual methods of identification, generally used in industry (least squares and gradient methods), allows to obtain better results [5]. All parameters of the following models are obtained using genetic algorithm proposed in [6] based on the collected data from a real engine.

The gas turbine engine studied in this paper is a twin-shaft design. Thermodynamic models of the engine can be approximated by a series of linear models. These linear models have the same order as the number of engine shafts [7]. That is why second order linear model is used as the primary model in this paper. Reduced order (first order) linear model and modified second order non-linear model structures are used for comparison. Based on these different model structures the second order linear, second order non-linear, reduced order linear local optimal controllers are designed. These different controllers are applied to the second order non-linear model and their performances are compared.

The paper is organized as follows. Section 2 describes the gas turbine engine in the case studied. It introduces the different model structures used and the models' parameters obtained. Section 3 introduces the local optimal controller design for each model structure and represents the comparison between these different controllers. Section 4 introduces PI controller design based on local optimal controller. Finally, the concluding remarks are presented in section 5.

2. THE SYSTEM MODEL

In a gas turbine engine high velocity gases are produced in the combustion chamber when a mixture of compressed air and fuel are ignited. The air is compressed by compressors. The power required for these compressors is extracted from the high velocity gases when they pass through the engine turbines [8].
In the case studied a twin-shaft gas turbine engine is considered. It is required to control the high pressure shaft speed as the output *(y)* of the system using the fuel flow to the engine as the input *(u)* [8, 9]

Data collected from a real engine (Fig.1.) is used to create, investigate and compare proposed models.

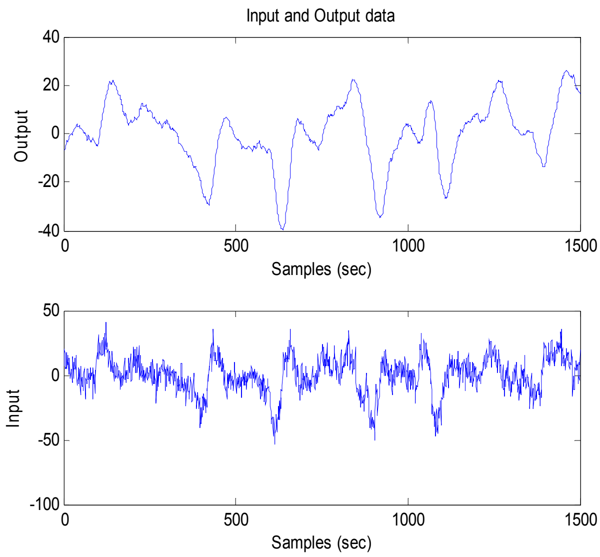

Fig.1. Input and output data

Estimation of the model parameters by genetic algorithm search is based on minimization of the model long term prediction error as shown in Fig.2.

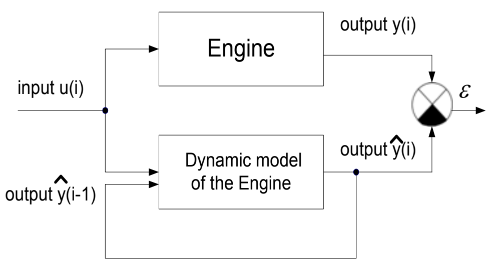

Fig.2. Long-term prediction error ε (for the first order model)

Second order linear model, reduced order (first order) linear model, and second order non-linear model are chosen for comparisons.
Increasing the order of the model might provide more accurate model of the plant. But at the same time it involves additional computation and increase the time of the search. It is known that introduction of the non-linear part can also enhance the accuracy of the model but also could affect the speed of the modelling, which is important for on-line applications.
Parameters of all listed models are obtained using the same data sets.

2.1 Linear model

The model structure for the second order linear model is given in (1).

$$y(i) = a_1 \cdot y(i-1) + a_2 \cdot y(i-2) + a_3 \cdot u(i-1) + a_4 \cdot u(i-2) \quad (1)$$

where u is the system input, y is the system output, a_1, a_2, a_3, a_4 are the model parameters.

The estimated parameters of the second order linear model are given in table 1.

Table 1. Parameters of the second order linear model

	a_1	a_2	a_3	a_4
M_1	1.2805	-0.2967	0.0093	0.0218

2.2 Reduced-order model

The reduced order (first order) linear model of the high order thermodynamic system is presented as follows:

$$y(i) = b_1 y(i-1) + b_2 u(i-1) \quad (2)$$

where u is the system input, y is the system output, b_1, b_2 are the model parameters.

The estimated parameters of the reduced order model are given in table 2.

Table 2. Parameters of the reduced order model

	b_1	b_2
M_2	0.9778	0.0435

2.3 Nonlinear model

Nonlinear parts of the model were selected to minimize the mean squared error between the engine data and the model output.

The model structure for the selected non-linear model is given in (3).

$$y(i) = c_1 \cdot y(i-1) + c_2 \cdot y(i-2) + c_3 \cdot u(i-1) + c_4 \cdot u(i-2) + c_5 \cdot u(i-2)^2 + c_6 \cdot y(i-2)^2 \quad (3)$$

where u is the system input, y is the system output, c_1, c_2, c_3, c_4, c_5, c_6 are the model parameters.

The estimated parameters of the non-linear model are given in table 3.

Table 3. Parameters of the second order non-linear model

	c_1	c_2	c_3	c_4	c_5	c_6
M_3	1.2788	-0.2950	0.0107	0.0211	$5.9199 \cdot 10^{-5}$	$-4.2324 \cdot 10^{-5}$

2.4 Models comparison

The mean squared error (MSE) between the engine data and model output for each model are listed in table 4.

Table 4. MSE for each model

	M_1	M_2	M_3
MSE	7.3174	9.1319	7.0646

As it was expected, the mean squared error between the engine data and the model output decreases as the order of the model increases and the non-linear part is introduced.

3. LOCAL OPTIMAL CONTROL DESIGN

In this section the local optimal controller is designed for each model structure represented in section 2. The design procedure for linear systems is described in [3, 9, 10]. The new approach for non-linear local optimal controller is described similarly in this section.

3.1 Linear local optimal controller

As stated the local optimal control approach for linear model is described in [3, 9, 10]. This can be summarized as follows for a system model represented in discrete-time form. Consider the system model as in (4).

$$A(z^{-1})y(t) = B(z^{-1})u(t-1) \tag{4}$$

where z^{-1} is the back shift operator, and

$$A(z^{-1}) = 1 + a_1 z^{-1} + a_2 z^{-2} + \cdots\cdots + a_{n_a} z^{-n_a}$$
$$B(z^{-1}) = b_o + b_1 z^{-1} + b_2 z^{-2} + \cdots\cdots + b_{n_b} z^{-n_b} \tag{5}$$

where n_a and n_b represent the order of the system model polynomials with $n_a > n_b$. A model of the system in (4) can be presented in compact form as:

$$y(t) = \varphi^T (t-1)\theta \tag{6}$$

where *y(t)* is the reference trajectory at sample *t*, θ is a vector of the identified system parameters:

$$\theta^T = \left[a_1, \cdots\cdots, a_{n_a}, b_0, b_1, \cdots\cdots, b_{n_b}\right] \tag{7}$$

and φ is a data vector includes the past measured values of inputs and outputs

$$\varphi^T (t-1) = \left[-y(t-1), \cdots\cdots, -y(t-n_a), u(t-1), \cdots\cdots, u(t-n_b-1)\right] \tag{8}$$

A modified form for the output deviation is derived using (6) as:

$$y(t+1) - y(t) = \Delta\varphi^T (t)\theta \tag{9}$$

where $\Delta\varphi(t)$ is the measured input and output data deviations between successive samples at sample *t* and can be defined as:

$$\Delta\varphi^T (t) = \left[-\Delta y(t), \cdots\cdots, -\Delta y(t-n_a+1), \Delta u(t), \cdots\cdots, \Delta u(t-n_b)\right] \tag{10}$$

$\Delta y(t) = y(t) - y(t-1)$ & $\Delta u(t) = u(t) - u(t-1)$,

where $\Delta u(t)$ is the control input rate of change at sample *t*. The model given in (6) presents an accurate description of the system. However, in this expression the vector of the system parameters θ is approximately identified. This leads to a residual error between the targeted trajectory, which can be defined as $y^*(t+1)$ and the actual related measured output *y(t)*. A weighting coefficient *h* is used to derive the reference trajectory deviation from (7), (8) and (9) as:

$$\Delta y^* (t+1) = y^*(t+1) - y(t) = h\left[-\sum_{i=1}^{n_a} a_i \Delta y(t-i+1) + \sum_{j=0}^{n_b} b_j \Delta u(t-j)\right] \tag{11}$$

The controller output rate of change $\Delta u(t)$ can be derived from (11) as:

$$\Delta u(t) = \frac{1}{b_0}\left[\frac{\Delta y^*(t+1)}{h} + \sum_{i=1}^{n_a} a_i \Delta y(t-i+1) - \sum_{j=1}^{n_b} b_j \Delta u(t-j)\right] \tag{12}$$

It's clear from (12) that controller output deviation is function of system parameters, past measured input and output data, reference trajectory, and the weighting coefficient *h*, *the only tuneable parameter*. For digital implementation, the controller output *u(t)* is derived from (12) using backward Euler discrete integration.

Fig.3. illustrates the chosen scheme of the local optimal controller [9].

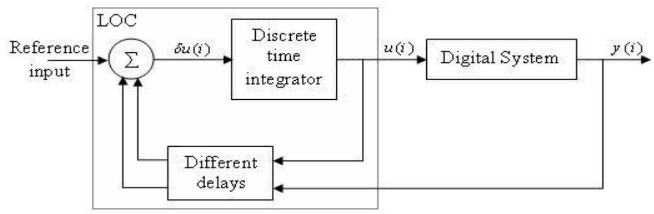

Fig.3. Block diagram of the chosen scheme of the local optimal controller

For the comparison of different local optimal controller performance the second order nonlinear model stated in (3) is used. This model is chosen because it provides the best model fit to the real data and can be considered as most accurate model of the real system. The controllers are compared by the speed of the response where no overshoot is allowed.

Second order linear local optimal controller design

The block diagram of the second order linear local optimal controller is shown in Fig.4. where $\Delta u(i)$ is given in (13).

$$\Delta u(i) = \frac{1}{a_3}\cdot\left(\frac{1}{h}\cdot\left[y(i+1) - y(i)\right] - a_1\cdot y(i) + (a_1 - a_2)\cdot y(i-1) + a_2\cdot y(i-2) + a_4 u(i-2) - a_4 u(i-1)\right) \tag{13}$$

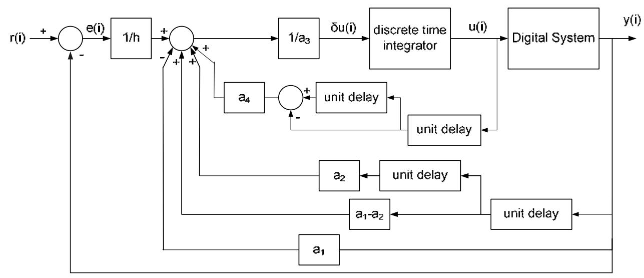

Fig.4. Block diagram of the second order linear local optimal controller

Fig.5. shows the response of the system given in (3) with the proposed second order linear local optimal controller for different values of the controller parameter *h (h=12,17,22).*

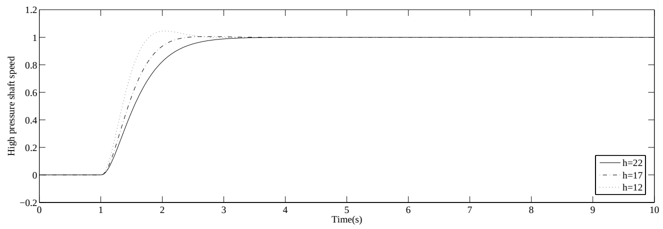

Fig.5. Simulated output response of second order linear local optimal controller for different h

The second order linear controller gives acceptable response at *h=22*. Time to reach the set point is about 3 sec.

The reduced order local optimal controller design

From the reduced order linear system model stated in (2) the block diagram of the first order (reduced order) local optimal controller is shown in Fig.6. where $\Delta u(i)$ is represented in (14).

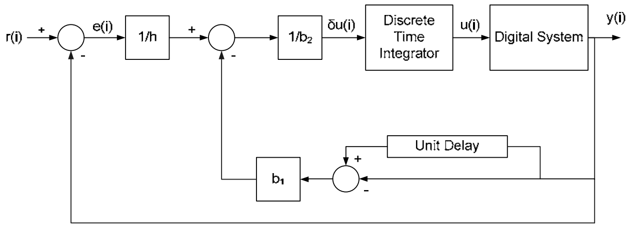

Fig.6. Block diagram of the first order linear local optimal controller

$$\Delta u(i) = \frac{1}{b_2}\cdot\left(\frac{1}{h}\cdot\left[y(i+1)-y(i)\right]-b_1\cdot y(i)+b_1\cdot y(i-1)\right) \quad (14)$$

Fig.7. shows the simulated response of the system given in (3) with the proposed reduced order local optimal controller for different values of the controller parameter *h (h=30,35,42).*

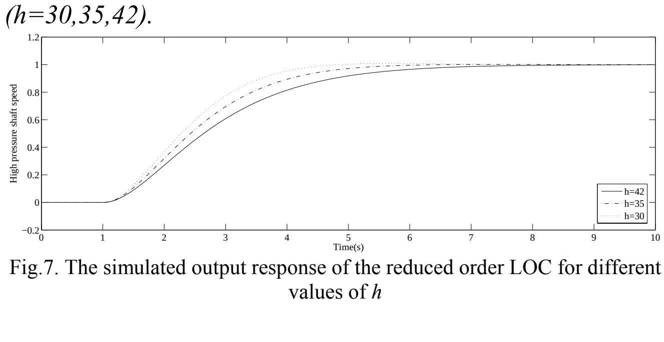

Fig.7. The simulated output response of the reduced order LOC for different values of *h*

The reduced order linear controller gives acceptable response at *h=42*. Time to reach the set point is about 8 sec.

3.2 Non-linear local optimal controller

Similar to (6) in section 3.1, the system model can be represented in compact form as in (15).

$$y(t) = \varphi^T(t-1)\theta \quad (15)$$

where *y(t)* is the reference trajectory at sample *t*, φ is a data vector that includes the past measured values of inputs and outputs as for linear systems beside all possible non-linear terms presented in (16).

$$\begin{aligned}\varphi^T(t-1) = [&-y(t-1),\cdots,-y(t-n_a),u(t-1),\cdots,u(t-n_b-1),\\ &y^2(t-1),\ldots,y^j(t-1),\ldots,y^2(t-n_a),\ldots,y^j(t-n_a),\\ &u^2(t-1),\ldots,u^k(t-1),\ldots,u^2(t-n_a),\ldots,u^k(t-n_a),\\ &y(t-1)u(t-1),\ldots,y(t-1)u^k(t-1),\ldots,\\ &y^j(t-1)u(t-1),\ldots,y^j(t-1)u^k(t-1),\ldots,\\ &y(t-n_a)u(t-1),\ldots,y(t-n_a)u^k(t-1),\ldots,\\ &y^j(t-n_a)u(t-1),\ldots,y^j(t-n_a)u^k(t-1),\ldots,\\ &y(t-1)u(t-n_b),\ldots,y^j(t-1)u(t-n_b),\ldots,\\ &y(t-1)u^k(t-n_b),\ldots,y^j(t-1)u^k(t-n_b),\ldots,\\ &y(t-n_a)u(t-n_b),\ldots,y^j(t-1)u(t-n_b),\ldots,\\ &y(t-n_a)u^k(t-n_b),\ldots,y^j(t-n_a)u^k(t-n_b)]\end{aligned} \quad (16)$$

θ is a vector of the identified system parameters for all linear and non-linear terms.

A modified form for the output deviation is derived using (15) as:

$$y(t+1)-y(t) = \Delta\varphi^T(t)\theta \quad (17)$$

In this expression the vector of system parameters θ is approximately identified. This leads to a residual error between the targeted trajectory, which can be defined as $y^*(t+1)$ and the actual related measured output *y(t)*. A weighting coefficient *h* is used to derive the reference trajectory deviation similar to (11)

The controller output rate of change $\Delta u(t)$ can be derived similar to (12):

Similar to (12), the controller output deviation is function of system parameters, past measured input and output data, reference trajectory, and the weighting coefficient *h*, the only tuneable parameter. For digital implementation, the controller output *u(t)* is derived using backward Euler discrete integration.

Non-linear local optimal controller design

According to the previous explanation the non-linear local optimal controller can be designed for the non-linear model given in (3). The non-linear local optimal controller block diagram for the model presented in (3) is shown in Fig.8., where $\Delta u(i)$ is represented in (18).

$$\Delta u(i)=\frac{1}{c_3}\cdot(\frac{1}{h}\cdot\left[y(i+1)-y(i)\right]-c_1\cdot y(i)+(c_1-c_2)\cdot y(i-1)$$
$$+c_2\cdot y(i-2)+c_4 u(i-2)-c_4 u(i-1)-c_5 u^2(i-1) \quad (18)$$
$$+c_5 u^2(i-2)-c_6\cdot y^2(i-1)+c_6\cdot y^2(i-2))$$

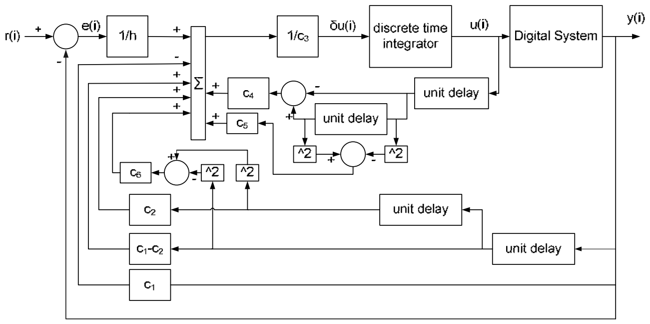

Fig.8. Block diagram of the second order non-linear local optimal controller

Fig.9. shows the response of the model stated in (3) with the proposed non-linear local optimal controller for different values of the controller parameter *h (h=14,19,24).*

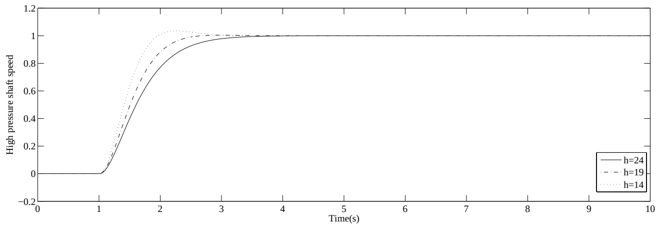

Fig.9. Simulated output response of second order non linear local optimal controller for different h

The non-linear controller gives acceptable response without overshoot at *h=24*. Time to reach the set point is about 4 sec.

3.3 Comparison between investigated controllers

In this section the responses of the system given in (3) with the three different controllers (second order linear local optimal controller, reduced order local optimal controller, and nonlinear local optimal controller) are compared. Simulated responses of the model are shown in Fig.10.

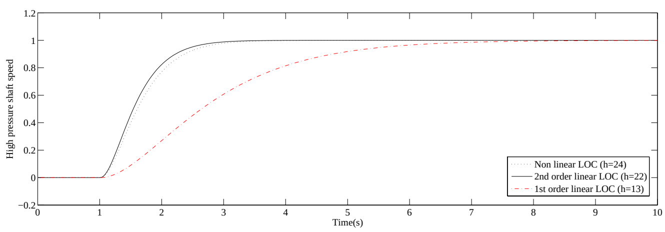

Fig.10. Simulated output responses using different controllers

This Fig.10. shows that the second order linear local optimal controller and the non-linear local optimal controller give very similar results. This is due to very low level of nonlinearity in the model obtained which appears in table 4 from the values of MSE. The input and output data collected for the system are around steady-state operating point, so it is approximately linear.

Also, it is clear that the reduced order linear local optimal controller still gives acceptable response without overshoot but very slow compared with the other two controllers.

4. PI CONTROLLER DESIGN BASED ON LOCAL OPTIMAL CONTROLLER

In this section, digital PI controller will be designed for the system based on the reduced first order local optimal controller. The relation between the reduced first order local optimal controller and the PI controller is given in [9] and the relations between the PI controller parameters and the model and controller parameters for first order local optimal controller can be represented in (19).

$$K_P=\frac{b_1 T_s}{b_2}, \qquad K_I=\frac{T_s}{b_2 h} \qquad (19)$$

where:
K_P, K_I are the PI controller parameters, T_s is the sampling period, b_1, b_2 are the first order model parameters in (2), h is the local optimal controller parameter (the tuneable parameter).

From this equation it is clear that the local optimal controller parameter h affects only K_I and has no effect on K_P. The K_P is only affected by the model parameters.
A modification has been done to the relations given in (19) by introducing a tuneable gain parameter K to be able to obtain faster responses. This modification can be represented in (20).

$$K_P=K\,\frac{b_1 T_s}{b_2}, \qquad K_I=K\,\frac{T_s}{b_2 h} \qquad (20)$$

As a result of this modification faster responses can be obtained with increasing the value of that tuneable gain *K.* Fig.11. shows the output response of the PI controlled system with different values of *K*. This Fig.11. shows also the PI controller designed using (19), which is equivalent to the PI controller designed using (20) when *K=1*.

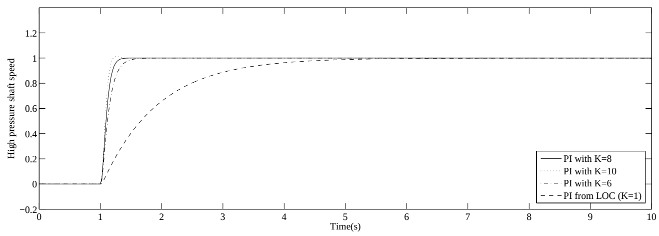

Fig.11. The output response of the system controlled by LOC tuned PI controller for different values of *K*

It is clear from this Fig.11. that a faster response without overshoot is obtained when *K=8.*

Fig.12. compares the output response of the PI controlled system using (20) when k=8 and that of the second order local optimal controller which is the fastest local optimal controller obtained in section 3. It is clear from this Fig.12. that the system with the PI controller obtained is faster than any local optimal controller obtained in section 3.

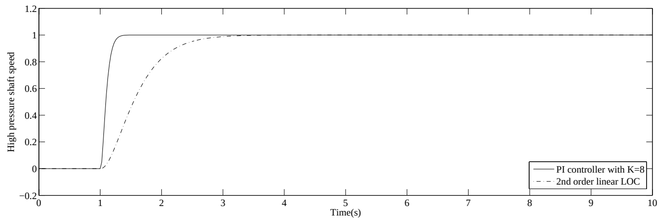

Fig.12. The output response of the system controlled by PI when K=8 for the second order linear local optimal controller

It should be noticed that; although the PI controller for the system under consideration gives faster response than the local optimal controller, local optimal controller provides good starting point for the PI design.

5. CONCLUSION

In this paper, a generalized version of local optimal controller approach dealing with the non-linear models as well as linear models is considered. Linear (full order and reduced order) local optimal controller is designed for the linear models obtained for the gas turbine engine. Non-linear local optimal controller is designed for the non-linear model obtained for the gas turbine engine. In the case studied, the output response of the system with these different controllers didn't show better response of the non-linear local optimal controller over the second order linear local optimal controller, although both of them was superior to the linear reduced order (first order) local optimal controller. The reason for this is very small non-linearity in the engine operating range considered (c_5 and c_6 in (3) are close to zero). Digital PI controller is designed based on the first order local optimal controller for tuning the controller parameters. This controller gives similar output response to the reduced order local optimal controller. So, a modification has been made for this approach to obtain faster response. The modified approach gives the fastest response obtained without overshoot.

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