Fuzzy Sliding Mode Controllers for Vehicle Active Suspensions

Ali J. Koshkouei and Keith J. Burnham

Control Theory and Applications Centre, Coventry University, Coventry CV1 5FB

Abstract: In this paper a method using sliding mode control and fuzzy logic techniques is presented for controlling a quarter-vehicle hydraulic active suspension. In this method a set of linear systems is considered which approximately describes the behaviour of the nonlinear suspension model. For each subsystem a suitable sliding mode controller is designed and then based on the Takagi-Sugeno fuzzy method an overall sliding mode controller for the Takagi-Sugeno model is designed. The proposed method considers two phases. In the first phase, the suspension dynamics is controlled via the actuator between the sprung and unsprung masses. Then the spool valve displacement dynamics is considered to control the current of the servo valve. Since there is an unknown parameter in the system an adaptation law is proposed to yield an appropriate estimate.

1. INTRODUCTION

A car suspension system is designed to reduce the vibrations resulting from road roughness and load on board when a vehicle moves along a road. Suspension system design relies on mathematical models of a vehicle. Suspension performance depends upon the type of suspension system and control strategy. Various active, semi-active and passive suspension systems have been designed in recent years. The passive or semi-active suspension systems have been widely used to reduce the effect of the external disturbance including road disturbance, improve the manoeuvrability for various riding conditions and comfortability of passengers. A passive suspension system, which has usually fixed parameters, stores the energy using one or two springs and dissipates this via a damper. Its parameters are selected based upon a trade-off between road holding, load carrying (passengers) and comfort. To improve the suspension performance, active suspensions are used. An actuating force within an active suspension system enables the system to suppress the effect of the disturbance on the vehicle body and passengers. A hydraulic active suspension is essentially a nonlinear system.

The aim of designing a suspension control is to improve the ride comfort and compromises between ride comfort and road holding. A suitable vehicle suspension system should be able to enhance road holding and passenger comfort. Compromise between these two issues is a major problem for designing and controlling suspension systems. In general, vehicle body vibrations depend on the speed of the vehicle, the road condition, the weight of the vehicle and load. Some vehicles have stiff suspensions with poor passenger comfort and many others have softer suspensions with poor road holding capabilities. Passive suspension systems provide only fixed rates for spring and damping coefficients. Therefore, they are effectively not able to compromise between ride comfort and road holding. Comfortability of a vehicle's passengers depends on the vertical motion and angular motion of a vehicle. If a force is the only control input which is applied between the sprung and unsprung masses, the vibration of the sprung mass at the wheel frequency modes cannot be reduced. Active suspension systems are used to reduce vibration of the sprung mass, and to provide an appropriate response depending on many factors including road conditions and vehicle speed. The forces in the damper are generated by modulating its orifice for fluid flow and the performance of a damper is controlled via this modulation. Therefore, it can only dissipate or store energy (Alanoly and Sankar, 1988). In fact, an active suspension system has the ability to store, dissipate and to introduce energy to the system and consists of a forcing element with a spring and a damper. Its parameters may vary depending upon operating conditions and can have knowledge other than the strut deflection the passive system is limited to. The force is varied by the forcing elements based upon a control law.

Rajamani and Hedrick (1995) developed an adaptive observer for the parameter identification of an active suspension system. Chantranuwathana and Peng (1999) designed an adaptive robust force controller to tackle the actuator uncertainties of active suspension systems. Fukao *et al.* (1999) used a H_∞ controller to tackle the road surface disturbance and an adaptive backstepping control method to deal with the actuator nonlinearities. Smith and Wang (2002) derived a parameterised stable controller with a fixed prespecified closed-loop transfer function for a vehicle active suspension system. Fialho and Balas (2002) combined a linear parameter-varying control and a nonlinear backstepping technique to design a road adaptive active suspension system. The control performance of the adaptive control scheme fully depends on the accuracy of the dynamic model.

Design of controllers using the fuzzy logic methods including Mamdani and Takagi-Sugeno (T-S) models have considered in recent years and are able to approximate a wide class of nonlinear dynamical systems (Takagi and Sugeno, 1985; Tanaka *et al*. 1996; Tanaka and Wang, 2001, Erbatur and Kaynak,, 2001). A fuzzy representation of a nonlinear system is described by a number of if-then rules including plant and control rules, and introducing appropriate membership functions. Stability analysis is carried out using the Lyapunov direct method and linear matrix inequalities (Wang *et al*., 1996). The triangular, trapezoidal or Gaussian are normally used as membership functions which are selected depending on the nature of the nonlinear system. A Takagi-Sugeno-Kang type fuzzy neural-network control scheme for a robot manipulator by considering a five-layer structure has been studied in Wai and Chen (2004) to achieve high-precision position tracking. A design method using hierarchical fuzzy sliding mode control (SMC) has been presented in Lin and Mon (2005) to stabilise a class of nonlinear systems by decoupling a nonlinear system into several subsystems and the state response of each subsystem can then be designed to be governed by a corresponding sliding surface. Then the entire system is controlled by a hierarchical SMC. Various SMC algorithms and controllers have been proposed to control active suspension systems with random disturbances (Sam, *et al.,* 2004; Kim *et al.*, 1999)

The control of an active suspension system using a fuzzy SMC is addressed in this paper. The suspension model is nonlinear with road disturbances. In this model there are four dynamics including the displacement of the car body, the displacement of the wheel, the pressure drop across the piston (or the hydraulic actuator) and the displacement of the spool valve. The actual control is obtained after three different processes.

The paper is organised as follows: The mathematical model and system description are presented in Section 2. Section 3 deals with the design of a fuzzy SMC and tracking the command signal actuator. The simulation results are given in Section 4. Conclusions are expressed in Section 5.

2. VEHICLE ACTIVE SUSPENSION

Consider the nonlinear dynamics of a quarter-car suspension system

$$\begin{aligned} M_s\ddot{x}_1 &= K_a^l(x_1-x_2)+C_a^l(\dot{x}_2-\dot{x}_1)+C_a^s\left|\dot{x}_2-\dot{x}_1\right|+ \\ &\quad C_a^n\sqrt{\left|\dot{x}_2-\dot{x}_1\right|}\,\mathrm{sgn}\left(\dot{x}_2-\dot{x}_1\right)+K_a^n(x_1-x_2)^3+F \\ M_u\ddot{x}_2 &= -K_a^l(x_1-x_2)-C_a^l(\dot{x}_2-\dot{x}_1)-C_a^s\left|\dot{x}_2-\dot{x}_1\right|- \\ &\quad C_a^n\sqrt{\left|\dot{x}_2-\dot{x}_1\right|}\,\mathrm{sgn}\left(\dot{x}_2-\dot{x}_1\right)-K_a^n(x_1-x_2)^3-K_t(x_2-r)+F \end{aligned} \tag{1}$$

where

M_s : The sprung mass

M_u : The unsprung mass

r : The road elevation profile:

x_1 : Displacement of the car body

x_2 : Displacement of the wheel

K_a^l : The linear part of the spring coefficient

K_a^n : The nonlinear part of the spring coefficient

K_t : The stiffness of the unsprung element

C_a^l : The linear part of the damper coefficient

C_a^n : The nonlinear part of the damper coefficient

C_a^s : The coefficient related to symmetric behaviour of the damper

F : The force from the hydraulic actuator

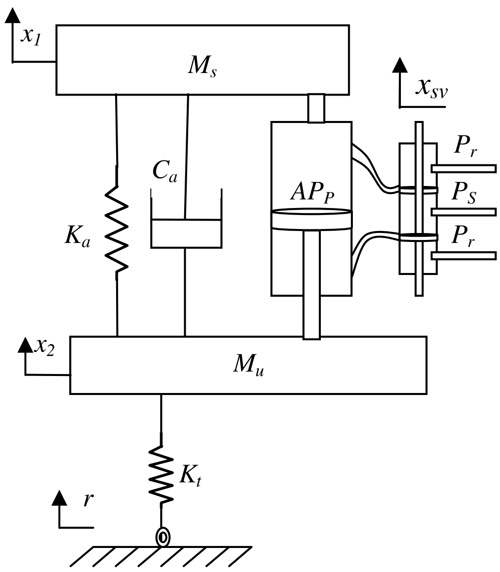

Fig. 1. A suspension model of a quarter of a vehicle with electro-hydraulic actuator

The hydraulic actuator is a four valve-piston system which is mounted parallel to the passive suspension system and is controlled by electro-hydraulic servo-valves. Therefore, the forces between sprung and unsprung masses can be generated. The actuator force F is given by

$$F = AP_p$$

where A is the area of the piston and P_p is the pressure drop across the piston with respect to the front and rear of the suspensions. The dynamics of P_p may be expressed as

$$\frac{V_t}{4\beta_m}\dot{P}_p = -C_{il}P_p - A\left(x_2 - x_4\right) + H_{lf} \tag{2}$$

with

$$H_{lf} = C_d S\ x_{sv}\sqrt{\tfrac{1}{\rho}\left|P_s - \mathrm{sgn}(x_{sv})P_p\right|}\,\mathrm{sgn}\left(P_s - \mathrm{sgn}(x_{sv})P_p\right)$$

where C_d , S , x_{sv} , P_s and ρ are, respectively, the discharge coefficient, the spool valve area gradient, the displacement of the spool valve, the supply pressure and the density of the hydraulic fluid. H_{lf} is the hydraulic load flow. The displacement of the spool valve x_{sv} is controlled by the servo-valve current u and its dynamics is approximately given by

$$\dot{x}_{sv} = \tfrac{1}{\tau}\left(-x_{sv} + ku\right) \tag{3}$$

where τ is the time constant and k is a gain. See Fig. 1 in which P_r is the return pressure out of the spool valve.

Consider $\dot{x}_1 = x_3$, $\dot{x}_2 = x_4, x_5 = P_p$, $x_6 = x_{sv}$. Then the state space representation of the system excluding the actuator force, outer-loop controller, is

$$\begin{aligned}
\dot{x}_1 &= x_3 \\
\dot{x}_2 &= x_4 \\
\dot{x}_3 &= \tfrac{1}{M_s}\Big(K_a^l(x_1 - x_2) + C_a^l(x_4 - x_3) + C_a^s\left|x_4 - x_3\right| + \\
&\qquad C_a^n\sqrt{\left|x_4 - x_3\right|}\,\mathrm{sgn}\left(x_4 - x_3\right) + K_a^n(x_1 - x_2)^3 + Av_c\Big) \\
\dot{x}_4 &= \tfrac{1}{M_u}\Big(-K_a^l(x_1 - x_2) - C_a^l(x_4 - x_3) - C_a^s\left|x_4 - x_3\right| - \\
&\qquad C_a^n\sqrt{\left|x_4 - x_3\right|}\,\mathrm{sgn}\left(x_4 - x_3\right) - K_a^n(x_1 - x_2)^3 - K_t(x_2 - r) - Av_c\Big)
\end{aligned} \tag{4}$$

The inner-loop controller can be expressed as

$$\begin{aligned}
\dot{x}_5 &= \theta\Big(-\beta x_5 - 4A\left(x_3 - x_4\right) + \gamma x_6\sqrt{\left|P_S - \mathrm{sgn}(x_6)x_5\right|}\,\mathrm{sgn}\left(P_S - \mathrm{sgn}(x_6)x_5\right)\Big) \\
\dot{x}_6 &= \tfrac{1}{\tau}\left(-x_6 + ku\right)
\end{aligned} \tag{5}$$

with $\theta = \dfrac{\beta_m}{V_t}$, $\beta = 4C_{il}$ and $\gamma = 4C_d S\sqrt{\tfrac{1}{\rho}}$. Note that $\theta = \dfrac{\beta_m}{V_t}$ is an unknown parameter.

An appropriate control is designed to improve the vehicle stability and to maximise the passenger comfort by controlling the suspension system under road disturbances. The road disturbance magnitude does not reach to the suspension travel limits at any time of travel. In addition, a suitable control is expected to minimise the car body accelerations under the road disturbances. The control of suspension is achieved in two main phases. Using the system (4) the force command v is designed to achieve the performance requirement and reduce the affect of the disturbance including the road disturbances. A suitable SMC is able to reduce the disturbances even if the disturbances are relatively large. Using the force command v_c and inner-loop (5) the servo-valve command u is determined. Note that the suspension travel x_1 is physically constrained, i.e. for any t , $\left|x_1\right| \le \bar{x}_1$.

The disturbance for a smooth road is small and usually is neglected. The road disturbances may be considered as a piecewise continuous function such as

$$r = \begin{cases} a_k\left(1 - \sin(b_k \pi t)\right) & \text{if } t_k^d \le t \le t_{k+1}^d \\ 0 & \text{Otherwise} \end{cases} \tag{6}$$

where t_k^d and t_{k+1}^d are the times that road disturbance is nonzero, a_k and b_k are related to the disturbance magnitude and amplitude, respectively.

3. T-S FUZZY MODELS

An SMC is a robust control which is designed for a system with uncertainties. SMCs are able to reject matched disturbances completely and reduce the effect of unmatched disturbances on the system. This important property is the main reason for the wide use of SMCs. In this section an SMC is designed for stabilising the system (4). Assume that $\left|r\right| \le h$ where h is a known real function or a constant.

The stability performance for a complex nonlinear system may not be straightforwardly achieved. However, it is possible to introduce many subsystems which express the behaviour of the nonlinear system within specified regions. These subsystems are preferably linear. These subsystems are used for constructing a model and designing a controller for obtained model. The T-S approach is a suitable method for designing an appropriate control using this set of linear systems and is considered in this section. A Takagi-Sugeno model is represented by a set of fuzzy 'If-Then' rules comprising of plant rules and control rules. If all system states are not available, a set of observer rules is given to estimate the unavailable states. For designing a fuzzy SMC, a set of sliding surfaces rules may be required. Assume that the linear dynamics of the nonlinear system (4) is locally described by k linear systems which are represented the system dynamics under some conditions and about particular points. All conditions are considered for introducing the plant and control rules.

The i-th ($i = 1, \cdots, k$) plant rule is:

If x_1 is M_{i1} and x_2 is M_{i2} and $\cdots$ x_n is M_{in} then

$$x = A_i x + B_i u + D_i \varsigma$$

where $x \in R^n$ is the state, $u \in R^m$ is control, $A \in R^{n\times n}$, D_i, $B_i \in R^{n\times m}$, $\varsigma \in R^m$ is the unknown function and M_{ij}, $(i = 1, \cdots, k; j = 1, \cdots, n)$ are input fuzzy sets. The fuzzy weights are defined as

$$w_i(x) = \prod_{j=1}^{n} \mu_{ij}(x)$$

where $\mu_{ij}(x)$ is membership function of the j-th fuzzy set in the i-th rule. The fuzzy convex coefficients are also defined as

$$\lambda_1 = \frac{w_i(x)}{\sum_{i=1}^{k} w_i(x)}$$

So $\sum_{i=1}^{k} \lambda_1 = 1$.Then using product inference and weighted average deffuzifier, the aggregated fuzzy system is

$$x = \sum_{i=1}^{k} \lambda_i \left(A_i x + B_i u + D_i \varsigma \right)$$

If the outputs of the subsystems are $y = C_i x$ then the output of aggregated system is $y = \sum_{i=1}^{k} \lambda_i C_i x$.

4. FUZZY SMC DESIGN

The control may be obtained using the SMC techniques including the traditional SMC (Utkin, 1992) or higher-order SMC (Lavant, 2003). For the *i*-th subsystem, the sliding surface is defined $s_i = \tilde{C}_i x$. The i-th SMC rule is defined as

If x_1 is M_{i1} and x_2 is M_{i2} and $\cdots$ x_n is M_{in} then

$$u = -\left(\tilde{C}_i B_i\right)^{-1} \left(\tilde{C}_i A_i x - K_i \ \operatorname{sgn}\left(s_i\right)\right)$$

where $K_i \in R^{m \times m}$ is a positive-definite matrix, preferably symmetric. The aggregated SMC is

$$u = -\sum_{i=1}^{k} \lambda_i \left(\tilde{C}_i B_i\right)^{-1} \left(\tilde{C}_i A_i x - K_i \ \operatorname{sgn}\left(s_i\right)\right)$$

So the overall sliding mode system is

$$\dot{x} = \sum_{i=1}^{k} \sum_{j=1}^{k} \lambda_j \lambda_i \left(\left(A_i - \left(\tilde{C}_j B_j\right)^{-1} \tilde{C}_j A_j \right) x - K_j \ \operatorname{sgn}\left(s_j\right) \right)$$

The overall reduced-order system

$$\dot{x} = \sum_{i=1}^{k} \sum_{j=1}^{k} \lambda_j \lambda_i \left(A_i - \left(\tilde{C}_j B_j\right)^{-1} \tilde{C}_j A_j \right) x$$

Is asymptotically stable if there exists a semi-positive definite matrix with at least $n-m$ nonzero eigenvalues such that

$$\tilde{A}_i^T P - P\tilde{A}_j \le 0, \qquad i = 1, \cdots, k$$

$$F_{ij}^T P - PAF_{ij} \le 0, \qquad \text{j<i<k}$$

where

$$\tilde{A}_i = A_i - \left(\tilde{C}_j B_j\right)^{-1} \tilde{C}_j A_j$$

$$F_{ij} = A_i - \left(\tilde{C}_j B_j\right)^{-1} \tilde{C}_j A_j + A_j - \left(\tilde{C}_i B_i\right)^{-1} \tilde{C}_i A_i$$

Therefore, all $n-m$ eigenvalues of $\tilde{A}_i$ and $n-m$ eigenvalues of F_{ij} should be in the left half-plane and the remaining may be zero. If the *i*-th subsystem is the sliding mode then there exists a finite time t_{s_i} such that $s_i = 0$ for $t \le t_{s_i}$ after a finite time $t_s = \max_{1 \le i \le k} t_{s_i}$, $s = \sum_{i=1}^{k} \lambda_i s_i$ and $\dot{s} = \sum_{i=1}^{k} \lambda_i s_i = 0$ which guarantees the existence of the sliding mode of the overall system. However, to guarantee the stability of the sliding mode for overall system in general, an extra sufficient condition, say for all $i = 1, \cdots k$ and $j = 1, \cdots, k$ $s_i s_j < 0$ may be required.

Now an SMC is designed for an approximation of the system (4) which is obtained by linearising around $x_1 = x_2 = 0$ and neglecting the higher-order terms $x_3 - x_4$. The sliding surface is defined as

$$s = c_1 x_1 + c_2 x_2 + c_3 x_3 + x_4 = 0 \qquad (7)$$

The coefficients c_1, c_2 and c_3 are selected so that the reduced-order system (system in the sliding mode) is asymptotically stable. The selection of the sliding function depends essentially on the nature of the system model. For a particular problem, various methods based on SMC may be designed. Dynamic, integral and proportional-integral (PI) SMC approaches are a variety of SMCs. The most important problem is to select an appropriate SMC approach for the system. The control v is designed so that the sliding mode reaching condition $\dot{s}s < 0$ is fulfilled.

Consider the control

$$v_c = \frac{-1}{A}\left[\left(K_a^l (x_1 - x_2) + C_a^l (x_4 - x_3) + C_a^s \left|x_4 - x_3\right| + \frac{M_u M_s}{(c_3 M_u - M_s)}\left(\frac{-K_t}{M_u} x_2 + c_1 x_3 + c_2 x_4 + W \operatorname{sgn}(s)\right)\right)\right] \qquad (8)$$

where $W > K_t h$. The condition $W > K_t h$ is vital for ensuring the stability of the sliding mode, and without this condition the sliding mode stability may not be achieved. Using the derivative of the sliding function s defined as in (7) and substituting the control (8) yields

$$\begin{aligned} s\dot{s} &= s\left(\dot{x}_4 + c_1 x_3 + c_2 x_4 + c_3 \dot{x}_3\right) \\ &= s\Big[c_1 x_3 + c_2 x_4 + \tfrac{c_3}{M_s}\Big(K_a^l (x_1 - x_2) + C_a^l (x_4 - x_3) + \\ &\quad C_a^s \left|x_4 - x_3\right| + Av\Big) + \tfrac{1}{M_u}\Big(-K_a^l (x_1 - x_2) - \\ &\quad C_a^l (x_4 - x_3) - C_a^s \left|x_4 - x_3\right| - K_t (x_2 - r) - Av\Big) \\ &\le (-W + K_t h)\left|s\right| \end{aligned} \qquad (9)$$

Therefore the control (8) enforces the state trajectories onto the sliding surface (7). The main problem is now to prove the asymptotic stability of the system. During the sliding mode $s = 0$ and $\dot{s} = 0$. From $\dot{s} = 0$ the equivalent control is obtained

$$v_c^{eq} = \frac{-1}{A}\left[\left(K_a^l (x_1 - x_2) + C_a^l (x_4 - x_3) + C_a^s \left|x_4 - x_3\right| + \frac{M_u M_s}{\left(c_3 M_u - M_s\right)}\left(\frac{-K_t}{M_u}\left(x_2 - r\right) + c_1 x_3 + c_2 x_4\right)\right)\right] \qquad (10)$$

Note that the equivalent control is not usually accessible because the disturbance r is not measurable. This control is normally used for analysing the system in the sliding mode. Substituting (10) into (4) implies the sliding mode system

$$\begin{aligned} \dot{x}_1 &= x_3 \\ \dot{x}_2 &= x_4 \\ \dot{x}_3 &= \frac{-M_u}{(c_3 M_u - M_s)}\left(\frac{-K_t}{M_u}(x_2 - r) + c_1 x_3 + c_2 x_4\right) \\ \dot{x}_4 &= \frac{M_s}{(c_3 M_u - M_s)}\left(c_1 x_3 + c_2 x_4 - \frac{c_3 K_t}{M_s}\left(x_2 - r\right)\right) \end{aligned} \qquad (11)$$

Now the objective is to find the coefficients of the function

$$s = c_1 x_1 + c_2 x_2 + c_3 x_3 + x_4 \tag{12}$$

c_1, c_2 and c_3 are selected such that the polynomial

$$\begin{aligned}(c_3 M_u - M_s) s^3 + (M_u c_1 - c_2 M_s) s^2 + \\ c_3 K_t s + c_1 K_t (c_3 M_u - M_s) = 0\end{aligned} \tag{13}$$

to be Hurwitz. Then for the system in the sliding mode, the reduced order system, is

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ -c_1 & -c_2 & -c_3 \\ \dfrac{c_1 c_2 M_u}{c_3 M_u - M_s} & \dfrac{c_2^2 M_u - K_t}{c_3 M_u - M_s} & \dfrac{a(c_2 c_3 - c_1) M_u}{c_3 M_u - M_s} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dfrac{-K_t}{c3 M_u - M_s} \end{bmatrix} r \tag{14}$$

The condition on (13) guarantees the stability of the system (14) in the absence of disturbance. However, the design parameters, may be selected such that the effect of the disturbance is reduced.

Now the tracking problem of the actuator is considered. Select

$$w = x_6 \sqrt{|P_S - \mathrm{sgn}(x_6) v|}\, \mathrm{sgn}\left(P_S - \mathrm{sgn}(x_6) v\right) \tag{15}$$

Then the inner-loop controller is governed by

$$\begin{aligned}\dot{v} &= \theta\left(-\beta v - 4A(x_3 - x_4) + \gamma w\right) \\ \dot{x}_6 &= \frac{1}{\tau}(-x_6 + ku)\end{aligned} \tag{16}$$

It is required to design the new control w such that the actuator force v converges to the command signal v_c. Then the servo-valve current u is obtained using (16). θ is an unknown parameter and should be estimated. Let $\hat{\theta}$ be an estimate of θ. Consider the Lyapunov function

$$V = \frac{1}{2}(v - v_c)^2 + \frac{\rho}{2}\left(\theta - \hat{\theta}\right)^2 \tag{17}$$

where $\rho > 0$, then

$$\begin{aligned}\dot{V} &= (v - v_c)(\dot{v} - \dot{v}_c) - \rho\left(\theta - \hat{\theta}\right)\dot{\hat{\theta}} \\ &= \left[\hat{\theta}\left(-\beta v - 4A(x_3 - x_4) + \gamma w\right) - \dot{v}_c\right](v - v_c) \\ &\quad + \rho\left(\theta - \hat{\theta}\right)\left(\left(-\beta v - 4A(x_3 - x_4) + \gamma w\right)(v - v_c) - \dot{\hat{\theta}}\right)\end{aligned}$$

The adaptation law is given by

$$\dot{\hat{\theta}} = \left(-\beta v - 4A(x_3 - x_4) + \gamma w\right)(v - v_c) \tag{18}$$

Select

$$v = v_c - \varepsilon\hat{\theta}\left(-\beta v - 4A(x_3 - x_4) + \gamma w\right) - \dot{v}_c \tag{19}$$

where $\varepsilon > 0$. Then

$$\dot{V} = -\varepsilon (v - v_c)^2$$

In fact, from (16)

$$\dot{v} - \dot{v}_c = -\varepsilon (v - v_c)$$

which guarantees $v \to v_c$ exponentially.

Now to complete the design procedure, the actual control u should be designed. From (15) and (16) the servo-valve current is obtained. An appropriate servo-valve current is

$$u = \frac{w\,\mathrm{sgn}\left(P_S - \mathrm{sgn}(w) v\right)}{k\sqrt{|P_S - \mathrm{sgn}(w) v|}} \tag{20}$$

(Chantranuwathana and Peng, 1999). Since $v \to v_c$ exponentially, one may consider v_c as an estimate of v for evaluating u defined as in (20).

4. SIMULATION RESULTS

Fig. 2. shows the membership function that has been considered for the model which is in terms of the vehicle body displacement.

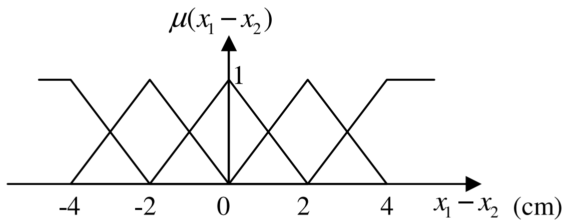

Fig. 2. Membership functions for the displacement between the vehicle body and the wheel.

Consider the following values for simulation:

$M_s = 443$ kg, $M_u = 40$ kg, $K_a^l = 14671$ N/m, $C_a^l = 100$ Ns/m, $C_a^n = 41400$ Ns/m, $C_a^s = 220$ Ns/m, $K_a^n = 100000$ N/m, $x_b = 0.08$ m, $K_t = 124660$ N/m, $\gamma = 1.545\times10^9$ (N/m$^{5/2}$kg$^{1/2}$), $A = 3.35\times10^{-4}$ m^2.

Also the road disturbance r is considered as

$$r = \begin{cases} a_0\left(1 - \sin(b_0 \pi t)\right) & \text{if } 0.2 < t < 0.75 \text{ or } 1.5 < t < 2.1 \\ 0 & \text{Otherwise} \end{cases}$$

with $a_0 = 0.02$ and $b_0 = 0.3$. The sliding surface for all subsystems is defined as

$$s = x_1 + 6x_2 + 8x_3 + x_4 = 0$$

The SMC is given by (8) with $W = 1$. It is assumed that the suspension travel limits are ±8 cm and spool valve displacement limits are ±1 cm. The simulation results are shown in Fig. 3. Fig. 3 shows that the displacement of the car body is between −2 and 7 (cm). The chattering is related to the discontinuous controller and the nature of the systems. For each plot the time axis is 10 seconds. To reduce the chatting one may use the continuous approximation of discontinuous terms and/or higher order SMC.

5. CONCLUSIONS

A quarter-car hydraulic active suspension system has been controlled using fuzzy logic and SMC techniques. The system is nonlinear and several subsystems have been considered to establish the behaviour of the active suspension system. The proposed fuzzy SMC is based on the Takagi-Sugeno method by using designing appropriate SMC control for these subsystems. The suspension dynamics has been controlled using the actuator between the sprung and unsprung masses. The spool valve displacement dynamics has been considered to control the current of the servo valve. An adaptation law has been presented for estimation of a system with an unknown parameter.

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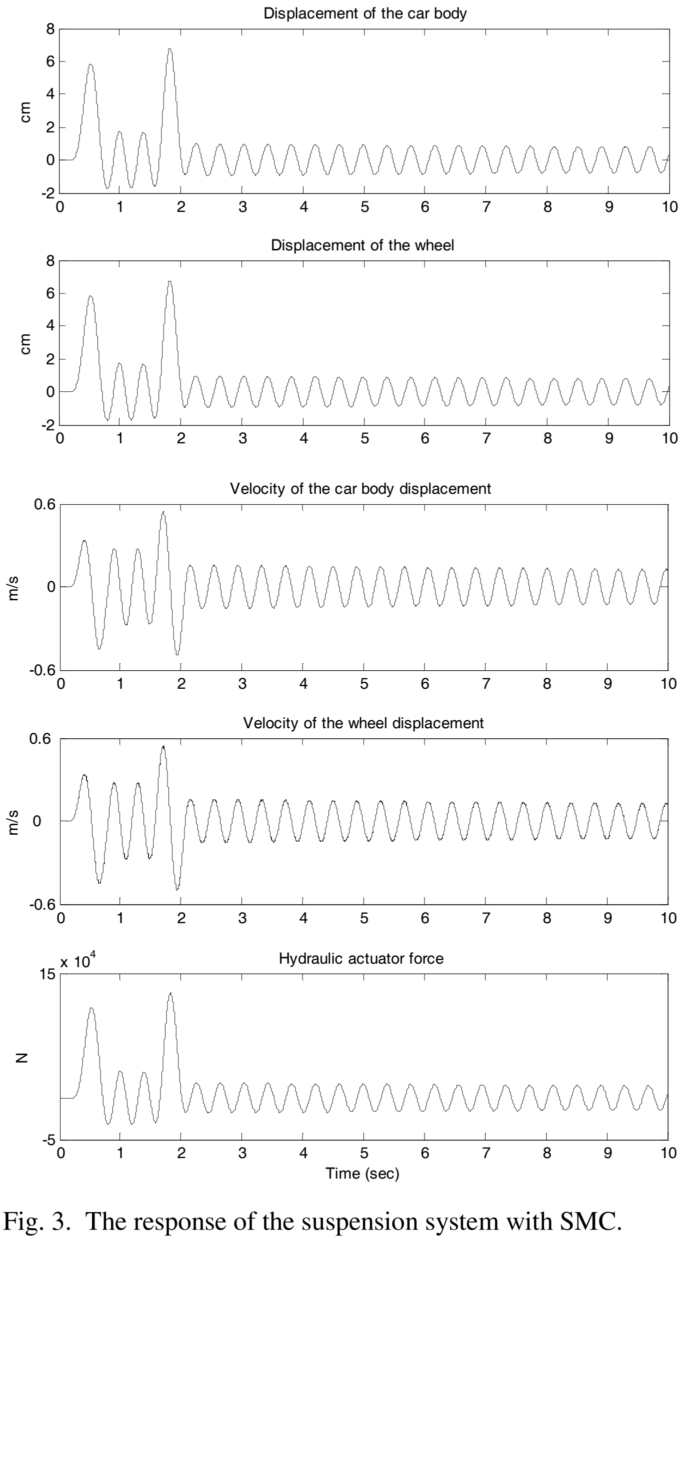

Fig. 3. The response of the suspension system with SMC.