The Investigation of Multivariable Control Performance Assessment Techniques

Qiaolin Yuan and Barry Lennox

Abstract — **Much attention has been paid to Control Performance Assessment (CPA) since the Harris Index was first proposed. This paper argues that there are two fundamental requirements for any CPA algorithm. The first is that it should be able to detect any change in the performance of a control system and the second is that it should be able to identify the potential improvement that can be made to the performance of the control system by re-tuning or re-configuring it. The ability of current multivariable CPA techniques to address these two issues is investigated in this paper, and limitations with the currently available approaches are identified and concluded in brief. The benefits of addressing the two issues are demonstrated using a simulated multivariate system, and the results of a detailed study identify a CPA approach which is able to address both of these issues and also diagnose the root cause of any change in control performance.**

Keywords: Control Performance Assessment, Control Performance Benchmark, Control Performance Index.

I. INTRODUCTION

CONTROL systems are implemented with various design objectives. The performance of control systems directly influences the efficiency, quality, safety and asset utilization of production plants. Unfortunately, it is very common to find that the control systems applied to the process fail to satisfy their design objectives. It has been reported that as many as 60% of all industrial controllers have some kind of performance problem, and in many cases, the control system actually increases the process variability [2].

In the past few years there has been considerable commercial and academic interest in the development and application of methods for analyzing the performance of control systems. This field is commonly referred to as *Control Performance Assessment* (CPA). A highly comprehensive review of CPA is provided in [13].

CPA originated from the specifications for optimal controller design and therefore CPA techniques are based upon the use of autocorrelation to indicate how close the current controller performance is to that of the ideal controller performance. Conventionally, performance estimation procedures involve a comparison of the existing controller with that of an ideal benchmark. This comparison is termed the *Control Performance Index* (CPI) and is defined as:

$$\eta = \frac{J_{des}}{J_{act}} \tag{1}$$

where J_{des} is the ideal, optimal or desired value for a given performance criterion and J_{act} is the actual value of this criterion extracted from measured data. The index is ordinarily scaled to lie within the range $[0,1]$. When the value of the index is approaching to one it indicates that the current controller performance is similar to the ideal controller performance [13].

There are two important issues that need to be specified when applying a CPI: (i) the criteria for judging the performance, and (ii) the benchmark that the current control performance is to be compared with.

Traditionally, the criteria used when developing a control system were its response characteristics, such as rise time, settling time, overshoot, offset from set-point and integral on-stream time. More recently, the most widespread criterion considered for CPA is the variance of the difference between the output signal and its set-point, the variance of the output error. The reason for using this value is that the performance of a control loop might be deemed unacceptable if the variance of the output error, relative to a benchmark value, exceeds certain critical values. This paper focuses on the group of CPA approaches which monitor the variance of the output error as the assessment criterion, termed as *Variance-based Benchmarking* approaches.

The benchmark can be chosen as any ideal, optimal, desired or expected control performance. It can be a theoretically achievable lower bound, a user defined variable specification, or even a performance measure that was achieved historically. Fig.1 provides a summary of the various types of control loop performance monitoring metrics that have been proposed and how they are related.

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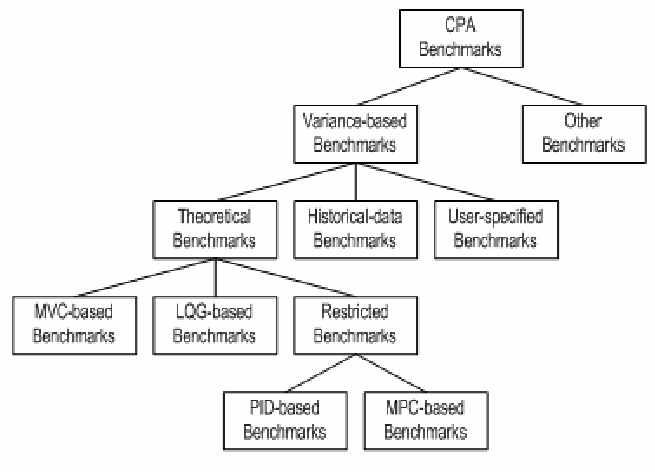

Fig.1: Variance-based benchmarks for CPA.

This paper argues that there are two fundamental requirements for any CPA algorithm. The first is that it should be able to detect any change in the performance of a control system and the second is that it should be able to identify the potential improvement that can be made to the performance of the control system by re-tuning or re-configuring it. Furthermore, the goal of the CPA techniques is that they should be used to assess the performance of the many hundreds or thousands of control loops in a large-scale process. Therefore the chosen controller diagnosis methodology must be reliable, computationally simple, and readily interpretable. Based on the above issues, the variance-based CPA benchmarking approaches that are believed to offer the most benefit in industrial applications are reviewed in Section II and summarized in Section III. The characteristics of the investigated multivariable CPA techniques are demonstrated using a simulated system in Section IV. Finally, conclusions from the work are drawn.

II. A Survey for the Multivariable CPA Techniques

Much attention has been paid to Control Performance Assessment (CPA) approaches since the Harris index was first proposed. The Harris index [6], expresses the single-loop control performance evaluation through closed-loop data using the Minimum Variance Control (MVC) as a benchmark. In other words, it indicates how far away the current output variance is to Minimum Variance (MV), and demonstrates the potential performance improvement compared with MVC. The Harris index became popular because of its simplicity. However, when the application is extended to multivariate systems, the requirements of *a-priori* knowledge and calculation burden are unavoidably increased due to the interactive effects among control loops.

The interactor matrix [21,20,10], which allows the feedback control-invariant term of the outputs to be extracted, is essential in the calculation of the MV for a delay involved multivariable system. Techniques for calculating the multivariable MVC benchmark that involve the interactor matrix directly are introduced in [7] and [11].

Both the concept and calculation of the interactor matrix are complicated and usually unrealistic. This has been an obstacle that has hindered the practical application of CPA to multivariable loops. The simplification or elimination of the interactor matrix when determining the performance of a multivariable control problem is therefore an important aim. [18] suggested the use of subspace approach to obtain the extended interactor matrix through the system state-space model. [17] and [16] estimated the outputs under minimum variance control by extracting the first few Markov parameters using open- or closed-loop data. However, the above techniques require detailed *a-priori* knowledge including the full system model or its relative information, which are normally challenging to obtain or estimate accurately. Plant tests introducing sufficient excitation, followed by considerable modeling effort has to be undertaken in order to get this information. This is the major difficulty for the application of multivariable MVC benchmark performance assessment algorithms.

Regarding the above issues, several studies have suggested using the upper and lower bounds of the MVC index to replace its true value in a multivariate system. [3] presented that a lower bound for the multivariable MVC index can be obtained from the performance of each loop in the control system using the minimum time delay unit for each individual output. Conversely, an upper bound for the multivariable MVC index can be calculated using the maximum time delay for each individual output. In a related article, [8] and [23] proposed a method for calculating the upper and lower bounds for the MVC index using routine, closed-loop operating data with only *a-priori* knowledge of the order of the interactor matrix or the I/O delay matrix.

A further limitation with the MVC approach to CPA is that it can often provide an unrealistic and undesired standard with which the current control performance is compared. MVC is rarely implemented industrially due to its poor robustness and need for excessive control action. The adoption of MVC as a benchmark does not imply that it is a required goal for the existing control system to achieve MVC performance [9]. To address this issue, several alternative benchmarks have been developed in recent years to assess the performance of multivariable control-loops, such as Linear Quadratic Gaussian (LQG) benchmark, user-specified benchmark, and the historical-data benchmark.

The LQG benchmark provides the lower limit of performance for any linear controller. It is based on the LQG *trade-off curve*, which displays the minimal achievable variance of the output errors versus the variance of the inputs. When the weights of the inputs are set to zero, the LQG benchmark is equal to the MVC benchmark. Approaches for obtaining the LQG trade-off curve are treated in [9] and [1]. In the work of [14], the Model Predictive Control (MPC) benchmark was applied as both a performance assessment indicator and an indicator of the potential improvement in performance. Based on this idea, the LQG benchmark can be applied in a similar way. The drawback of LQG benchmarking is that the knowledge of the full process and disturbance model, or their relative information such as Markov parameters, are required in order to calculate the trade-off curve. Once again, it is a real challenge to obtain this information which may cause practical problems.

For the monitoring of MPC systems, [12] commented that a major limitation with the LQG performance metric was that it represented an unattainable standard for commercial MPC algorithms that are typically derived from simplified disturbance models. [14] have since demonstrated that the MPC trade-off curve lies significantly above the LQG benchmark. This work showed that even when an MPC system is designed in the absence of a plant-model mismatch, it will never fall on the LQG curve unless the actual disturbance is a random walk. Recognising this limitation, several benchmark measures have been designed specifically for MPC systems [14] and [19]. Similar to the LQG benchmark, the current performance is compared with the original trade-off curve to provide a performance assessment, while the current performance compared with the updated trade-off curve highlights the potential improvement in performance.

As was stated earlier, MVC provides a global minimum benchmark, and the closed-loop impulse response coefficients are zero after the process dead time, however, it is almost impossible to achieve and may not be desired in practice. It is more realistic to compare the achieved performance with that of the performance in the design stage. An effective user-specified benchmark was derived by [9] based on the theory of MVC benchmark. For the user-specified benchmark, the closed-loop impulse response coefficients, which after the process dead time are not zero as with in MVC, and instead these terms are set as user desired closed-loop dynamic responses. Based on this theory, the approaches for calculating MVC benchmark can be used to calculate the user-specified benchmark. However, once again, the interactor matrix or its equivalent formation is required, which may cause practical problems. To avoid the calculation of the interactor matrix, [24] proposed two upper and lower bounds for the user-specified CPI, and suggested using the tight bounds to replace the true value of user-specified CPI.

An alternative benchmark can be to compare performance with that obtained from historical data collected when the process was performing well ([5], [22], and [4]). The advantage with the historical data benchmark is that it only requires closed-loop data collected under normal operation. The limitation with this technique is that it demonstrates performance changes only, and there is no any indication of potential improvement in control performance.

III. Conclusion of CPA Approaches

Based upon the reviews and developments of the multivariable CPA approaches in the last two sections, the characteristics of these approaches can be concluded as follows (denoting that η_{MVC}, η_{LQG}, η_{MPC}, η_{his}, and η_{user} are CPIs calculated based on MVC benchmark, LQG benchmark, MPC benchmark, historical-data benchmark, and user-specified benchmark):

1. η_{LQG} and η_{MPC} provide both a performance assessment indicator and an indicator of the potential improvement in performance. The current performance compared with the original trade-off curve, which is built through the old plant information, gives a performance assessment, while the current performance compared with the updated trade-off curve, which is built through the new plant information, highlights the potential improvement in performance.
2. η_{MVC} is a unique case of η_{LQG}, and η_{LQG} is a unique case of η_{MPC}. Without considering the influence of inputs, $\eta_{MVC} \le \eta_{LQG} \le \eta_{MPC}$. Among these three metrics, η_{LQG} and η_{MPC} are relatively complicated to calculate and require more *a-priori* knowledge than η_{MVC}. However, they can give more appropriate indices.
3. η_{MVC} and η_{user} are indicators of the potential improvement in performance. η_{MVC} is a unique case of η_{user}, which can be calculated from η_{user}, and $\eta_{MVC} \le \eta_{user}$. η_{user} gives a more appropriate index than η_{MVC}.
4. η_{his} is a very simple index that is easily understood and calculated, however, it does not provide the potential performance improvement possible.
5. Due to the difficulty and complexity when calculating η_{MVC} and η_{user}, the upper and lower bounds for η_{MVC} and η_{user} may be applied instead of the true value of η_{MVC} and η_{user}.

A summary of the techniques introduced in Sections II is listed in Table 1.

The understanding of the characteristics for each individual benchmark helps improve the effectiveness of their application. For example, the historical-data benchmark and user-specified benchmark bounds have the least calculation burden and require little *a-priori* knowledge. The combination of these two benchmarks is able to indicate both changes in the current performance of the control system and also the potential improvement that can be made to the performance through re-tuning or re-configuration of the control system. This approach has the further benefit that it avoids the complicated calculations associated with the LQG and MPC benchmarks. Furthermore, these two benchmarks can also provide the MVC benchmark bounds when setting the user-specified closed loop dynamics as zero.

TABLE1: SUMMARY OF CPA TECHNIQUES

Name of Benchmark	Type of Indicator	Constraints Handling	Calculation Burden	Required *a-priori* Knowledge (besides normal operating data)
MVC	Potential	No	****	plant model or its first few open-/closed-loop Markov parameters
MVC Bound	Potential	No	**	System time delay matrix or interactor matrix maximum and minimum order
LQG	Performance and Potential	Yes	*****	Both plant model and disturbance model or their open-loop Markov parameters
MPC	Performance and Potential	Yes	*****	Both plant model and disturbance model
Historical-data	Performance	Yes	*	///
User-specified	Potential	No	*****	plant model or its first few open-/closed-loop Markov parameters and user desired closed-loop dynamic functions
User-specified Bound	Potential	No	**	System time delay matrix or interactor matrix maximum and minimum order and user desired closed-loop dynamic functions

('Performance' means the performance of a control system; 'Potential' means the potential improvement that can be made to the performance of the control system by re-tuning or re-configuring it.)

IV. CASE STUDY FOR MULTIVARIABLE CPA APPROACHES

In this case study, a two-input two-output system is regulated by Ziegler-Nichols tuned PI controllers. The performance of this control system is then assessed under two different situations. The first is when the dynamics of the plant changes and the second is when there is a change in the structure of the disturbance that affects the plant. The original plant, original plant with changes in the dynamics, and the original plant with a change to the disturbance are referred to as *OS*, *PC*, and *DC*, respectively.

The CPA techniques defined in this section were then applied to data collected from these three systems to assess the perceived performance of the control system.

The original plant and disturbance dynamic are described by the following transfer functions [9]:

$$G_p = \begin{bmatrix} \dfrac{z^{-1}}{1-0.4z^{-1}} & \dfrac{z^{-2}}{1-0.1z^{-1}} \\ \dfrac{0.3z^{-1}}{1-0.1z^{-1}} & \dfrac{z^{-2}}{1-0.8z^{-1}} \end{bmatrix}$$

and

$$G_d = \begin{bmatrix} \dfrac{1}{1-0.5z^{-1}} & \dfrac{-0.6}{1-0.5z^{-1}} \\ \dfrac{0.5}{1-0.5z^{-1}} & \dfrac{1}{1-0.5z^{-1}} \end{bmatrix}$$

with disturbance variance $\Sigma_A = 0.0016I$, and sampling time $t_s = 1$ sec. After the change in plant dynamics, the plant model is

$$G_p = \begin{bmatrix} \dfrac{z^{-1}}{1-0.4z^{-1}} & \dfrac{16z^{-2}}{1-0.1z^{-1}} \\ \dfrac{0.3z^{-1}}{1-0.1z^{-1}} & \dfrac{z^{-2}}{1-0.8z^{-1}} \end{bmatrix}$$

and the disturbance change adjusts the disturbance dynamics to

$$G_d = \begin{bmatrix} \dfrac{1}{1-0.5z^{-1}} & \dfrac{-0.6}{1-0.5z^{-1}} \\ \dfrac{2}{1-0.5z^{-1}} & \dfrac{1}{1-0.5z^{-1}} \end{bmatrix}$$

The controllers for this process were specified as $K_p\left(1+\dfrac{t_s}{\tau_i\left(1-z^{-1}\right)}\right)$ with parameters $K_{p1} = 0.24$ and $\tau_{i1} = 1.60$ for the first loop, and $K_{p2} = 0.04$ and $\tau_{i2} = 2.67$ for the second loop.

With step changes on both set-points at time 100s, the response of the process for *OS*, *PC*, and *DC* are plotted in Fig.2. The performance of the control systems were then assessed using the benchmarks introduced above, and the results are listed in Tables 2-4. In Tables 2-4, 'UBD' Stands for 'Upper Bound', and 'LBD' stands for 'Lower Bound'.

Table 2 demonstrates the control performance assessment results for the original system. In this table, the CPI calculated based on the MVC benchmark is listed in the second column. This index lies between its upper and lower bounds, which are listed in the third column and is also lower than the LQG benchmark. The index in the fifth column in Table 2 demonstrates the CPI calculated using the historical-data benchmark. This figure indicates that, for the current system, the control performance is similar to that achieved historically, which implies that the original system is under normal operation. This benchmark cannot provide an indication of the potential for improving the performance of the control system. The CPI calculated using the user-specified benchmark is listed in the sixth column. This index approaches one, indicating that the current control system has achieved its design objective. This index is greater than the MVC performance metric, as was stated in the conclusion 3 in Section III. The upper and lower bounds for the user-specified performance metrics are listed in the last column, and they are greater and smaller than the user-specified performance metric respectively.

When the change in plant dynamics is introduced, Fig.2 shows a significant drop in the performance of the control system, this can be seen from both the variance of the output error and system response dynamics. This process is assessed in Table 3. The 'Performance Indices' in Table 3, both η_{LQG} and η_{his}, highlight this performance drop - their values decrease compared with those figures in Table 2. All the values of 'Potential indices' in Table 3 also decreased compared with those figures in Table 2. This implies that, the potential for improving the performance of the control system increases, which suggest that the controllers should be retuned after this change.

When the disturbance change occurs, Fig.2 displays performance deterioration with an increased variance. This process is assessed in Table 4. Both the 'Performance Indices' - η_{LQG} and η_{his} in Table 4, drop dramatically compared with similar figures in Table 2. However, all the 'Potential Indices' in Table 4 remain virtually unchanged. This implies that, although the performance of the controller deteriorates after the disturbance change, there is almost no change in the potential improvement in performance if the controller is re-tuned before or after the disturbance change. This is because retuning the controller cannot improve the poor performance of the system which is caused by the disturbance change.

This case study illustrates that if the values of both 'Performance Indicator' and 'Potential Indicator' change, it implies that the dynamics of the plant have changed, meaning that there might be a need to retune the controller. If the value of the 'Performance Indicators' change and the 'Potential Indicators' do not, this implies that there has been a disturbance change or a sensor/actuator failure, meaning that the performance can not be improved by simply retuning the controller. Based on these performances, a comprehensive CPA approach should be able to indicate: (i) whether or not the control performance has changed; (ii) whether or not the control performance can be improved by retuning the controller; and (iii) what caused this change.

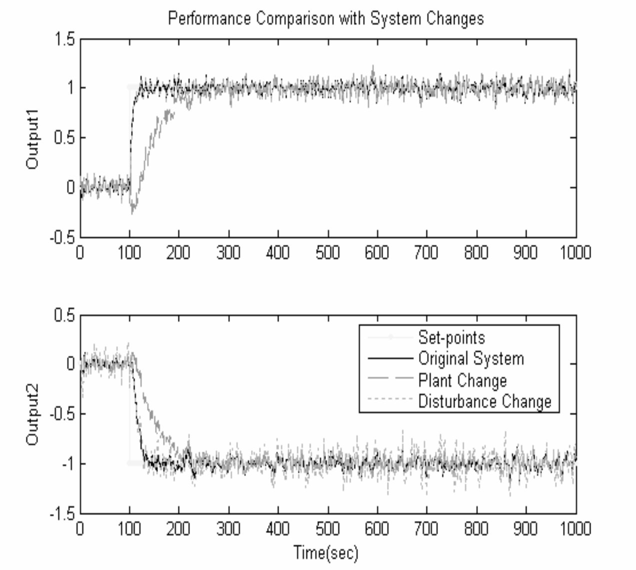

Fig.2: PI control performance on set-point step change with system changes.

TABLE 2: PERFORMANCE ASSESSMENT RESULTS FOR THE ORIGINAL SYSTEM

OS	η_{MVC}	η_{MVC} UBD / η_{MVC} LBD	η_{LQG}	η_{his}	η_{user}	η_{user} UBD / η_{user} LBD
Performance Indicator	N/A	N/A / N/A	0.889	1.000	N/A	N/A / N/A
Potential Indicator	0.846	0.847 / 0.739	0.889	N/A	1.038	1.049 / 1.013

TABLE 3: PERFORMANCE ASSESSMENT RESULTS FOR THE ORIGINAL SYSTEM WITH PLANT CHANGE.

PC	η_{MVC}	η_{MVC} UBD / η_{MVC} LBD	η_{LQG}	η_{his}	η_{user}	η_{user} UBD / η_{user} LBD
Performance Indicator	N/A	N/A / N/A	0.710	0.850	N/A	N/A / N/A
Potential Indicator	0.713	0.729 / 0.628	0.733	N/A	0.835	0.847 / 0.817

TABLE 4: PERFORMANCE ASSESSMENT RESULTS FOR THE ORIGINAL SYSTEM WITH DISTURBANCE CHANGE.

DC	η_{MVC}	η_{MVC} UBD	η_{LQG}	η_{his}	η_{user}	η_{user} UBD

<table>
<tr><th></th><th></th><th>η_{MVC} LBD</th><th></th><th></th><th></th><th>η_{user} LBD</th></tr>
<tr><td rowspan="2">Perfor-mance Indicator</td><td rowspan="2">N/A</td><td>N/A</td><td rowspan="2">0.348</td><td rowspan="2">0.459</td><td rowspan="2">N/A</td><td>N/A</td></tr>
<tr><td>N/A</td><td>N/A</td></tr>
<tr><td rowspan="2">Poten-tial Indicator</td><td rowspan="2">0.847</td><td>0.872</td><td rowspan="2">0.874</td><td rowspan="2">N/A</td><td rowspan="2">1.091</td><td>1.099</td></tr>
<tr><td>0.728</td><td>1.071</td></tr>
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V. CONCLUSION

In this paper, it is argued that there are two fundamental requirements for any CPA algorithm. One is that it provides an indication of changes in the current performance of the control system and the other is that it provides a measure of the potential improvement that can be made to the performance of the controller through its re-tuning or re-configuration. Based on these two indications, potential root causes of any controller problems can be determined. Focused on this concept, the current multivariable CPA approaches are reviewed and investigated through a simulated two-input two-output system. Limitations with current techniques are identified and a novel real-time CPA technique is proposed to address these issues.

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