Sub-Optimal Control Based on Passivity for Euler-Lagrange Systems.

J. Patricio Ordaz-Oliver, * **Omar A. Domínguez-Ramírez** ** **and Filiberto Muñoz-Palacios** *

* *Universidad Politécnica de Pachuca, Mechatronics Faculty, Ex. Hacienda Sta. Bárbara, Municipio de Zempoala, Hgo., México, (Tel: +52 771-5477510; e-mail: patricio@upp.edu.mx, mupafi@upp.edu.mx).*
** *Universidad Autónoma del Estado de Hidalgo, Instituo de Ciencias Básicas e Ingeniería, Centro de Investigación en Tecnologías de Información y Sistemas Carr. Pachuca-Tulancingo, 42090; Km. 4.5, México (Tel: +52 771 7172000 ext 6734 y 6738 e-mail: omar@uaeh.edu.mx)*

Abstract: This paper, present a class of nonlinear control, the feedback error scheme is proposed for trajectory tracking of an Euler-Lagrange system. The controller in this paper has the advantage of global stability and robustness, moreover, we provide a passivity based on stability analysis which suggest that the system has a condition of strictly semi-definite positive realness of tracking error dynamics, this is a necessary condition for a global stability, to this end the explicit solution of the Hamilton-Jacobi-Bellman principle found by solving the Lyapunov function. In order to demonstrate the control approach, we present a simulation using a 3-DOF robot, to this case we use Phantom Haptic device dynamical model.

1. INTRODUCTION.

Some methodologies have been proposed for the control of nonlinear systems, the classic control that are used in robotics does not allow to compensate the dynamics performance. On the literature review, is well known that the PD plus gravity compensation controller can globally stabilize a manipulators [10], and for the parametric uncertainty an adaptive version of PD controller has introduced [11], the main drawback of this approach is the gravity regressor matrix has to be known. In the case when the system presents the uncertainty of the potential energy, the PI^{2}D control has introduced in [14], but this control has not high performance on trajectory tracking.

On the industrial robots is well known that the simple PID control which does not required the dynamical model in to its control law, this simple linear control with an appropriate control gains achieves the desired position, this is a main raison why the PID controllers are still used in industrial robots. The local asymptotic stability of the linear PID controller in a closed loop with robot manipulator is proved in [12]. The proof, it can be seen that the cubic term in the derivative of the Lyapunov function hampers the global asymptotic stability, this is the reason to believe that linear PID control is inadequate to nonlinear systems like robot manipulators. And the trajectory tracking the computed torque has introduced in [3], this control technique is required the dynamical model, and the computed torque can be approximate the computed torque plus gravity compensation [13].

Motivated by the results of the control design in [4, 5, 16], that is a control design via Lyapunov theory, for n-DOF manipulator, in this case we consider when the manipulator required to follow a desired trajectory. The primary goal of motion control in joint space is to make the robot joint positions q track a given timevarying desired joint position q_d. Rigorously, the motion control objective in joint space is achieved provided that

$$\lim_{t\to\infty} s(t) = \lim_{t\to\infty} \left[\dot{q}(t) - \dot{q}_d(t) + \alpha q(t) - \alpha q_q(t)\right] = 0 \quad (1)$$

Where $\dot{q}(t) \in R^n$ is the articular velocity, $\dot{q}_d(t) \in R^n$ is the desired articular velocity, $q(t) \in R^n$ is the articular position, $q_d(t) \in R^n$ is the desired articular position, and $\alpha \in R^{n\times n}$ is a strictly positive definite matrix gain.

In the recent years the control analysis has allowed to design many tools for robot regulation and trajectory tracking [2, 7, 13]. The first control based on the dynamic error which ensures global asymptotic stability is proposed in [16], and many authors that based on this approach, introducing a sliding mode for compensate the nonlinearities [1, 17], but the problem of this analysis is the chattering problem, this control techniques, does not compensate the disturbance performance. Since that an optimal control approach is given to solve a Hamilton-Jacobi-Bellman equation and present feedback solution to non linear systems, to this end this paper we added the error via dynamic extension, use the dynamic properties, and the Hamilton-Jacobi-Bellman principle for present the analytical solution with minimization of the applied torque.

2. DYNAMIC PROPERTIES.

The dynamical model for this case, we introduce the Euler-Lagrange equations [13].

$$\frac{d}{dt}\left[\frac{\partial \mathcal{L}(q,\dot{q}(t))}{\partial \dot{q}(t)}\right]-\frac{\partial \mathcal{L}(q,\dot{q}(t))}{\partial q(t)}=\tau, \tag{2}$$

From (2) we obtain the generalized Euler-lagrange equation for manipulator robots as:

$$D(q)\ddot{q}+C(q,\dot{q})\dot{q}+G(q)=\tau. \tag{3}$$

The position coordinates $q \in R^n$ with associated velocities $\dot{q}$ and accelerations $\ddot{q}$ are controlled with the driving forces $\tau \in R^n$. The generalized moment of inertia $D(q)$, the coriolis and centripetal forces $C(q,\dot{q})\dot{q}$, and the gravitational forces $G(q)$ all vary along the trajectories, where τ are the control forces, and n indicate the degree of freedom (DOF). Form the Euler-Lagrange formulation one can obtain the mathematical model as:

$$\frac{d}{dt}\begin{bmatrix} q \\ \dot{q}\end{bmatrix}=\begin{bmatrix}\dot{q} \\ D(q)^{-1}\left[\tau - C(q,\dot{q})\dot{q}-G(q)\right]\end{bmatrix}. \tag{4}$$

Observe that (4) can be generally written as follows:

$$\dot{x}=f(x)+g(x)\tau \tag{5}$$

where $x \in M \subseteq R^{2n}$ is the system state, $\tau \in R^m$ is the control imput. Since our problem formulation is given for desired trajectory track, x_d, then nonlinear system (5) is stated as follows:

$$\dot{\tilde{x}}=f(\tilde{x})+g(\tilde{x})\tau \tag{6}$$

where $\tilde{x}=x-x_d$, and stand the error.

2.1 Properties of Euler-Lagrange Systems.

The dynamic equation for manipulator robots (3) have intrusting follow properties [13]:

- There exist some positive constant α such that
  $$D(q)\geq \alpha I \quad \forall q \in R^n$$
  where I denotes the $n\times n$ identity matrix. The $D(q)^{-1}$ exist and this is positive definite.
- The matrix $C(q,\dot{q})$ have a relationship with the inertial matrix as:
  $$\dot{q}^T\left[\frac{1}{2}\dot{D}(q)-C(q,\dot{q})\right]\dot{q}=0 \quad \forall q,\dot{q}\ \in R^n.$$
- The Euler-Lagrange system have the total energy as:
  $$\mathcal{E}=\mathcal{K}+\mathcal{U} \tag{7}$$
  where $\mathcal{K}$ are kinetic sum for each coordinate (8), and $\mathcal{U}$ are the potential energy respectively (9).

  The kinetic energy $\mathcal{K}$ is obtained by:
  $$\mathcal{K}=\frac{1}{2}\sum_{i=1}^{n} m_i v_i^2=\frac{1}{2}\dot{q}^T D(q)\dot{q} \tag{8}$$
  where $m_i \in R$ are the i mass of the i link, $v_i \in R^n$ are the i velocity of the i link.

  The potential energy $\mathcal{U}$ is obtained by:
  $$\mathcal{U}=\sum_{i=1}^{n} m_i h_i g \tag{9}$$
  where $m_i \in R$ are the i mass of the i link, $h_i \in R^n$ are the i height of the i link respect to mass center and g is a gravitational constant. By differentiating (7) we obtain
  $$\begin{aligned}\dot{\mathcal{E}}&=\dot{q}^T D(q)\ddot{q}+\frac{1}{2}\dot{q}^T\dot{D}(q)\dot{q}+\dot{q}^T G(q)\\ &=\dot{q}^T\left(-C(q,\dot{q})-G(q)+\tau\right)+\frac{1}{2}\dot{q}^T\dot{D}(q)\dot{q}+\dot{q}^T G(q)\\ &=\dot{q}^T\tau.\end{aligned} \tag{10}$$
- From the passivity property we have that:
  $$V(x)-V(x_0)\leq \int_0^t y^T(s)u(s)ds, \tag{11}$$
  where $V(x)$ is a storage function, $y(s)$ is the output, and $u(s)$ is the input of the system, and s is a variable change. Using (10) for the Euler-Lagrange system, energy function $\mathcal{E}$ as the storage function, and we have the passivity property as:
  $$\mathcal{E}(t)-\mathcal{E}(0)\leq \int_0^t \dot{q}^T\tau dt \tag{12}$$
  where $\dot{q}$ is the output, and τ is the input of the system.

2.2 Passivity of the error dynamics.

The passivity property described present an interesting property in the regulation problem on Euler-Lagrange systems, however this property can be extended for the trajectory tracking problem solution, [15] drive this passive error dynamic property, as follow for:

$$D(q)\dot{s}+\left[C(q,\dot{q})+K_d(q,\dot{q})\right]s=0 \tag{13}$$

where s denotes an error signal that we want to drive to zero, $K_d(q,\dot{q})=K_d(q,\dot{q})^T>0$ is a damping injection matrix, and $C(q,\dot{q})$ is the centripetal and Coriolis matrix forces. Based in this property, and the skew-symmetric property we follow to next lemma:

Lemma 1: The differential equation:

$$D(q)\dot{s}+\left[C(q,\dot{q})+K_D(q,\dot{q})\right]s=\Psi \tag{14}$$

where $D(q)$ and K_d are positive definite matrix and $C(q,\dot{q})$ satisfies (13) defines an output strictly passive operator $\sum_d : \Psi \mapsto s$. Consequently, if $\Psi \equiv 0$ we have $s \in \mathcal{L}_2$ [15].

With the **lemma 1**, and the dynamics properties, we can follow a control law design, in this paper we obtain the control technique via Lyapunov theory, dynamical properties, and passive properties.

3. HAMILTON-JACOBI-BELLMAN EQUATION.

Define the Hamiltonian of optimization [19] as

$$H\left(\left.\frac{dV\left(\tilde{x},\tau\right)}{dt}\right|_{(5)}\right)=\left(\left.\frac{dV\left(\tilde{x},\tau\right)}{d\tilde{x}}\right|_{(5)}\right)^T\dot{\tilde{x}}+f_0\left(\tilde{x},\tau\right) \tag{15}$$

where $f_0(\cdot)$ is an positive definite specified function. The control τ^* is called the optimal control, and where V satisfies the partial differential equation

$$-\left.\frac{dV\left(\tilde{x},\tau\right)}{dt}\right|_{(5)}=\left(\left.\frac{dV\left(\tilde{x},\tau\right)}{d\tilde{x}}\right|_{(5)}\right)^T\dot{\tilde{x}}+f_0\left(\tilde{x},\tau\right)$$

A necessary and sufficient condition for optimality, is to choose a value function V that satisfies the Hamilton-Jacobi-Equation

$$\min_{\tau^*} H\left(\left.\frac{dV(\tilde{x},\tau^*)}{dt}\right|_{(5)} + f_0(\tilde{x},\tau^*)\right) = 0 \quad (16)$$

This minimum is attained for the optimal control $\tau = \tau^*(t)$ and the Hamiltonian [18].

In this paper, we want to find an admissible control $\tau^*(t)$ which achieves that (6) follows a trajectory $\tilde{x}$ by minimizing a performance index, written in terms of the following performance index

$$J = \int_0^\infty [f_0(\tilde{x},\tau)]dt, \quad (17)$$

Equation (16) is the well known Hamilton-Jacobi-Bellman (HJB) equation which immediately leads to an optimal control in feedback form. The following classical result states sufficient conditions for a local minimum for a scalar function.

Theorem: [19] Let $L(x,\tau)$ a scalar single valued function of the variables x and τ. Let $\frac{\partial^2 L(x,\tau^*)}{\partial u^2}$ exists and be bounded and continuous. Also assume that

$$\frac{\partial L(x,\tau^*)}{\partial u} = 0 \quad (18)$$

and

$$\frac{\partial^2 L(x,\tau^*)}{\partial \tau^2} > 0. \quad (19)$$

Then u^* is a local minimum.

4. NONLINEAR SUB-OPTIMAL CONTROL.

In this section we propose a nonlinear control based on the optimality Hamilton-Jacobi-Bellman principle, based on Lyapunov analysis, for this approach we consider, the Lyapunov function:

$$V(\tilde{x}) = \frac{1}{2}\tilde{x}^T\Lambda\tilde{x}, \quad \Lambda = \begin{bmatrix} D(q) & k \\ k & D(q)\end{bmatrix}, \quad (20)$$

where $k \in R^{n\times n}$ is a positive definite constant matrix, $k = k^T$ with the property $\lambda \max(k) \leq \lambda \min(D(q))$, then $V(\tilde{x}) \geq 0$, for the control approach we present required the desired trajectory $q_d(t)$, the control design problem is derive a from Lyapunov stability analysis such that the manipulator output $q(t)$ closely the desired trajectory, and we define the error function as:

$$\tilde{x} = x - x_d = \begin{pmatrix} s \\ \dot{\tilde{q}} \end{pmatrix} \quad (21)$$

where $s = [s_1, s_2, ..., s_n]^T$ denotes the dynamic error $s = \dot{q} - \dot{q}_r$, $\dot{q}_r = \dot{q}_d - \alpha\Delta q$ and Δq is an error function, where $\Delta q = q - q_d$, q_d is the desired position, $\dot{\tilde{q}} = \dot{q} - \dot{q}_d$. The error dynamics of the manipulator may be obtained from (3) and (21) as a state-space description, where the derivative of $\tilde{x}$ is

$$\dot{\tilde{x}} = \begin{bmatrix} \dot{s} \\ \ddot{\tilde{q}} \end{bmatrix} = \begin{bmatrix} D^{-1}(q)\left(\tau - Y_r\Phi - C(q,\dot{q})s - K_d s\right) \\ D^{-1}(q)\left(\tau - C(q,\dot{q})\dot{q} - G(q)\right)\end{bmatrix} \quad (22)$$

as saw in (22) we applied lemma 1, and use the follow result:

$$D(q)\dot{s} = \Psi - [C(q,\dot{q}) + K_d]\, s \quad (23)$$

where Ψ are the system dynamics:

$$\begin{aligned} \Psi &= \tau - (D(q)\ddot{q}_r + C(q,\dot{q})\dot{q}_r + g(q)) \\ &= \tau - Y_r(q,\dot{q},\dot{q}_r,\ddot{q}_r)\Phi = \tau - Y_r\Phi \end{aligned} \quad (24)$$

where $\dot{q}_r$ and $\ddot{q}_r$ are the nominal references.

When use dynamic programm (16) we required derivative (20), and obtain the follow result:

$$\begin{aligned} \left.\frac{dV(\tilde{x})}{dt}\right|_{(5)} &= \frac{1}{2}\tilde{x}^T \begin{bmatrix} D(q) & k \\ k & D(q)\end{bmatrix}\dot{\tilde{x}} \\ &+ \frac{1}{2}\dot{\tilde{x}}^T \begin{bmatrix} D(q) & k \\ k & D(q)\end{bmatrix}\tilde{x} + \frac{1}{2}\tilde{x}^T \begin{bmatrix} \dot{D}(q) & 0 \\ 0 & \dot{D}(q)\end{bmatrix}\tilde{x} \end{aligned}$$

this is:

$$\left.\frac{dV(\tilde{x})}{dt}\right|_{(5)} = \tilde{x}^T \begin{bmatrix} D(q) & k \\ k & D(q)\end{bmatrix}\dot{\tilde{x}} + \frac{1}{2}\tilde{x}^T \begin{bmatrix} \dot{D}(q) & 0 \\ 0 & \dot{D}(q)\end{bmatrix}\tilde{x}$$

introducing (21) and (22) in the above we obtain the follow equation:

$$\begin{aligned} \left.\frac{dV(\tilde{x})}{dt}\right|_{(5)} &= \begin{bmatrix} s^T \\ \dot{\tilde{q}}^T \end{bmatrix}^T \begin{bmatrix} D(q) & k \\ k & D(q)\end{bmatrix}\begin{bmatrix} \dot{s} \\ \ddot{\tilde{q}} \end{bmatrix} \\ &+ \frac{1}{2}\begin{bmatrix} s^T \\ \dot{\tilde{q}}^T \end{bmatrix}^T \begin{bmatrix} \dot{D}(q) & 0 \\ 0 & \dot{D}(q)\end{bmatrix}\begin{bmatrix} s \\ \dot{\tilde{q}} \end{bmatrix} \end{aligned}$$

and this is:

$$\begin{aligned} \left.\frac{dV(\tilde{x})}{dt}\right|_{(5)} &= s^T D(q)\dot{s} + \dot{\tilde{q}}^T k\dot{s} + s^T k\ddot{\tilde{q}} + \dot{\tilde{q}}^T D(q)\ddot{\tilde{q}} \\ &+ \frac{1}{2}s^T\dot{D}(q)s + \frac{1}{2}\dot{\tilde{q}}^T\dot{D}(q)\dot{\tilde{q}} \end{aligned} \quad (25)$$

by properties of Euler-Lagrange systems and the passivity of the error dynamics we have that:

$$\begin{aligned} D(q)\ddot{\tilde{q}} &= \tau - C(q,\dot{q})\dot{\tilde{q}} - G(q) \\ D(q)\dot{s} &= \tau - Y_r\Phi - C(q,\dot{q})s - K_d s \end{aligned}$$

then (25) can de rewrite as:

$$\begin{aligned} \dot{V}(\tilde{x}) &= s^T\left[\tau - Y_r\Phi - C(q,\dot{q})s - K_d s\right] \\ &+ \dot{\tilde{q}}^T kD^{-1}(q)\left(\tau - Y_r\Phi - C(q,\dot{q})s - K_d s\right) \\ &+ s^T kD^{-1}(q)\left(\tau - C(q,\dot{q})\dot{q} - G(q)\right) \\ &+ \dot{\tilde{q}}^T\left[\tau - C(q,\dot{q})\dot{\tilde{q}} - G(q)\right] + \frac{1}{2}s^T\dot{D}(q)s + \frac{1}{2}\dot{\tilde{q}}^T\dot{D}(q)\dot{\tilde{q}} \end{aligned}$$

this is:

$$\begin{aligned} \dot{V}(\tilde{x}) &= s^T\left[\tau - Y_r\Phi + \left(\frac{1}{2}\dot{D}(q) - C(q,\dot{q})\right)s - K_d s\right] \\ &+ \dot{\tilde{q}}^T\left[\tau + \left(\frac{1}{2}\dot{D}(q) - C(q,\dot{q})\right)\dot{\tilde{q}} - G(q)\right] \\ &+ \dot{\tilde{q}}^T kD^{-1}(q)\left(\tau - Y_r\Phi - C(q,\dot{q})s - K_d s\right) \\ &+ s^T kD^{-1}(q)\left(\tau - C(q,\dot{q})\dot{q} - G(q)\right) \end{aligned}$$

$$\begin{aligned} \left.\frac{dV(\tilde{x})}{dt}\right|_{(5)} &= s^T\left[\tau - Y_r\Phi - K_d s\right] + \dot{\tilde{q}}^T\left[\tau - G(q)\right] \\ &+ \dot{\tilde{q}}^T kD^{-1}(q)\left(\tau - Y_r\Phi - C(q,\dot{q})s - K_d s\right) \\ &+ s^T kD^{-1}(q)\left(\tau - C(q,\dot{q})\dot{q} - G(q)\right) \end{aligned} \quad (26)$$

by applying the Hamilton-Jacobi-Bellman principle (15):

$$\frac{\partial}{\partial \tau}\left\{ \left.\frac{dV(\tilde{x})}{dt}\right|_{(5)} + f_0(\tilde{x},\tau)\right\} = 0$$

with the follow performance index:

$$J = \int_0^\infty \overbrace{\left(\tau^T R \tau\right)}^{f_0(\tilde{x},u)} dt$$

where $R \in R^{n\times n}$ is a definite positive matrix, $R = R^T$, we obtain the follow expression:

$$s^T + \dot{\tilde{q}}^T + \dot{\tilde{q}}^T kD^{-1}(q) + s^T kD^{-1}(q) + \tau^T R = 0$$

by apply the follow property, $x^T y = y^T x$, and the symmetric matrix properties, and we obtain:

$$s + \dot{\tilde{q}} + kD^{-1}(q)\dot{\tilde{q}} + kD^{-1}(q)s + R\tau = 0$$

then the control law is:

$$\tau = -R^{-1}\left[s + \dot{\tilde{q}} + kD^{-1}(q)\dot{\tilde{q}} + kD^{-1}(q)s\right] \tag{27}$$

and reducing above equation we obtain:

$$\tau = -R^{-1}\left[\left(I + kD^{-1}(q)\right)s_p\right] \tag{28}$$

where $s_p = [s_{p1}, s_{p2}, ..., s_{pn}]^T$ denotes an extension of the dynamic error $s_p = \dot{q} - \dot{q}_r + \dot{\tilde{q}} = 2\dot{q} - \dot{q}_{rd}$, $\dot{q}_{rd} = 2\dot{q}_d - \alpha\Delta q$ and Δq is an error function, where $\Delta q = q - q_d$, q_d is the desired position, $\dot{\tilde{q}} = \dot{q} - \dot{q}_d$.

Note: By differentiating once again (28), the condition (19) holds [1] for Euler-Lagrange systems trajectory track.

Proposition: Consider the Euler-Lagrange system (3). Takin the Lyapunov function (20), with strictly positive matrix Λ, then the solution of the close loop system with the control law (28) is asymptotic stable in the trajectory $\dot{q} - \dot{q}_d = \varepsilon$, where $|\varepsilon| < \varepsilon^\star$ and $\varepsilon^\star$ is arbitrary small.

Proof: In order to demonstrate the stability we required (27) in closed loop with (3), and we obtain:

$$\begin{array}{l} \dot{V}(x) = -\begin{bmatrix} \dot{\tilde{q}}^T \\ s^T \end{bmatrix}^T \overbrace{\begin{bmatrix} Q_1 & Q_{12} \\ Q_{12} & Q_2 \end{bmatrix}}^{Q(x)} \begin{bmatrix} \dot{\tilde{q}} \\ s \end{bmatrix} - \\ \left.\begin{array}{l} \left(s^T Y_r\Phi + \dot{\tilde{q}}^T G(q) + \dot{\tilde{q}}^T kD^{-1}(q)K_d s\right. \\ +s^T kD^{-1}(q)C(q,\dot{q})\dot{q} + s^T kD^{-1}(q)G(q) \\ \left.+\dot{\tilde{q}}^T kD^{-1}(q)Y_r\Phi + \dot{\tilde{q}}^T kD^{-1}(q)C(q,\dot{q})s\right) \end{array}\right\} \varphi(x) \end{array} \tag{29}$$

where:

$$\begin{array}{l} Q_1 = R^{-1} + R^{-1}kD^{-1}(q) + kD^{-1}(q)R^{-1}kD^{-1}(q) \\ Q_{12} = Q_1 \\ Q_2 = Q_1 + K_d \end{array}$$

since that $\det(Q(x)) = K_d$, then the sufficient condition for $Q(x) > 0$ is $K_d > 0$, the norm of the function $Y_r\Phi$ in (29) is upper bounded and we have that, some positive scalars $\beta_i (i = 0, 1..., 5)$ such that [13]:

$$\begin{array}{r} \|D(q)\| \geq \lambda_m\left(D(q)\right) > \beta_0 > 0 \\ \|D(q)\| \leq \lambda_M\left(D(q)\right) < \beta_1 < \infty \\ \|C(q,\dot{q})\| \leq \beta_2\|\dot{q}\| \\ \|G(q)\| \leq \beta_3 \end{array} \tag{30}$$

according to the follow derivations [9]:

$$\begin{array}{l} Y_r\Phi \leq \|D(q)\|\,\|\ddot{q}_r\| + \|\{K_d + C(q,\dot{q})\}\,\dot{q}_r\| + \|G(q)\| \\ \quad \leq \beta_1\alpha\left\|\dot{\tilde{q}}\right\| + \lambda_M\left(K_d\right) + \beta_2\|\dot{q}\|\left(\alpha\left\|\dot{\tilde{q}}\right\| + \beta_4 + \right. \\ \quad \left.+\gamma\|\sigma\|\right) + \bar{\beta}_3 \\ \quad \leq \eta\left(\tilde{q},\dot{\tilde{q}},\sigma,\beta_i\right) \end{array}$$

where $\bar{\beta}_3 = \beta_1\beta_5 + \beta_3$, and $\eta\left(\tilde{q},\dot{\tilde{q}},\sigma,\beta_i\right)$ is a scalar. According with above equation and 30 we obtain:

$$\begin{array}{l} \varphi(x) \leq \|s\|\,\eta\left(\tilde{q},\dot{\tilde{q}},\sigma,\beta_i\right) + \left\|\dot{\tilde{q}}\right\|\beta_3 + \left\|\dot{\tilde{q}}\right\|\|s\|\,\lambda_M k\beta_0 \\ +\|q\|\,\|s\|\,\lambda_M k\beta_0\beta_2 + \left\|\dot{\tilde{q}}\right\|\lambda_M k\beta_0\eta\left(\tilde{q},\dot{\tilde{q}},\sigma,\beta_i\right) \\ +\|s\|\,\lambda_M k\beta_0\beta_3 + \left\|\dot{\tilde{q}}\right\|\|s\|\,\lambda_M k\beta_2 \\ \leq \|s\|\,\eta_2\left(\tilde{q},\dot{\tilde{q}},\sigma,\beta_i,\lambda_M k\right) + \left\|\dot{\tilde{q}}\right\|\eta_3\left(\tilde{q},\dot{\tilde{q}},\sigma,\beta_i,\lambda_M k\right) \end{array}$$

since that

$$\|s\|\,\eta_2\left(\tilde{q},\dot{\tilde{q}},\sigma,\beta_i,\lambda_M k\right) > \left\|\dot{\tilde{q}}\right\|\eta_3\left(\tilde{q},\dot{\tilde{q}},\sigma,\beta_i,\lambda_M k\right)$$

then

$$\varphi(x) \leq \|s\|\,\eta_2\left(\tilde{q},\dot{\tilde{q}},\sigma,\beta_i,\lambda_M k\right) \tag{31}$$

and we have that $\beta_i > 0$ and $\lambda_M K_d > 0$, this implies:

$$\dot{V}(x) = -\|s\|^2\lambda_M K_d + \|s\|\,\eta_2\left(\tilde{q},\dot{\tilde{q}},\sigma,\beta_i,\lambda_M k\right) \tag{32}$$

then K_d are bigger than $\eta_2\left(\tilde{q},\dot{\tilde{q}},\sigma,\beta_i,\lambda_M k\right)$, and this is sufficient conditions for conclude that the system belongs to compact set and this involving the Lyapunov arguments $\lambda_M k < \lambda_m D(q)$ large enough such that $\tilde{x}$ converges into neighborhood $\varepsilon > 0$ with radius $r > 0$.

5. SIMULATION RESULTS.

For better illustrate the control law (28), in this section we present an simulation results applied on 3-DOF robot platform. We applied this approach in the study case PHANToM 1.0 [8]. In order to illustrate the performance of the proposed control law based on the passivity, we have performed some simulations, and we propose the matrix α, R and k as follow:

$$\begin{array}{c} R = \alpha = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}, \\ k = diag\left(\lambda\min\left(D(q)\right)\right) \end{array} \tag{33}$$

We start the simulation with the follow initial conditions:

$$\begin{array}{ll} q_1 = 0.7854, & \dot{q}_1 = 0 \\ q_2 = -0.7854, & \dot{q}_2 = 0 \\ q_3 = -0.7854, & \dot{q}_3 = 0 \end{array}$$

For this simulation we consider the follow trajectories:

$$x(t) = \rho(t)\cos(\varphi(t)) + h \tag{34}$$

$$y(t) = \rho(t)\sin(\varphi(t)) + k \tag{35}$$

and

$$\varphi(t) = \lambda t \tag{36}$$

[1] i.e. taking $\frac{\partial^2}{\partial^2\tau}\left(\left.\frac{dV}{dt}\right|_{(5)} + f_0\right)$.

$$\rho(t) = r\cos(n\varphi(t)) \tag{37}$$

where $n = 3$, $h = 0.3m$, $k = 0.2m$, $r = 0.1$, $\lambda = 1$, $\omega = 2\pi/5$, this is a trajectory of an rouse.

In order to see the performance of the control law (28) we propose a comparative control law as the Slotine-Li approach, i.e. $\tau = Yr\Phi - KS$, where $K = R$ [16], this approach have the property of global asymptotic stability, and we propose a disturb on the first link at time t=5 seconds in order to see the robustness of own approach, the main result can see in the work space figure 1 and 4 own approach is robust at the disturbance and the Slotine-Li approach generate an error, the dynamic error goes to zero, i.e. $s_1 \to 0$ and $s_2 \to 0$, $s_3 \to 0$ as shown in the Fig 2, meanwhile the performance of the error is constant in the control law (28), obviously the performance of the error is stationary in comparison with Slotine-Li approach, this performance can be see to comparison Fig 3 with Fig 6.

6. CONCLUDING REMARKS.

In this paper has been presented a class of global stable control for Euler-Lagrange Systems. We use the passivity injection based on the Euler-Lagrange dynamical properties, a passivity based error and stability of Lyapunov's direct method for guarantee the asymptotic convergence, the control design is approach via solution of Hamilton-Jacobi-Bellman equations with performance index (17).

This approach is called sub-optimal control because we find the sufficient condition for conclude the stability proof.

The future work is find the optimal control based on trajectory track for Euler-Lagrange systems and results applied to 3-DOF experimental robot platform.

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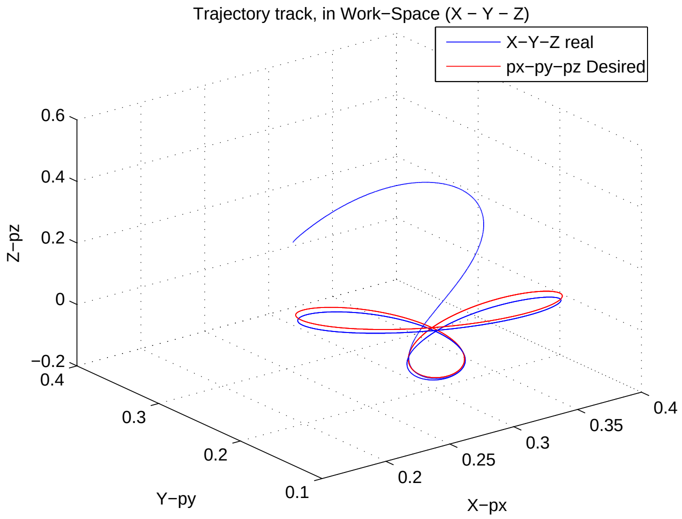

Fig. 1. Trajectory track, X-Y-Z for Slotine-Li Control.

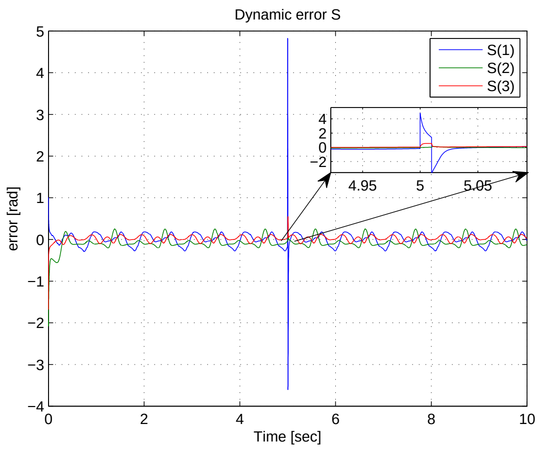

Fig. 2. Dynamic error, for Slotine-Li Control.

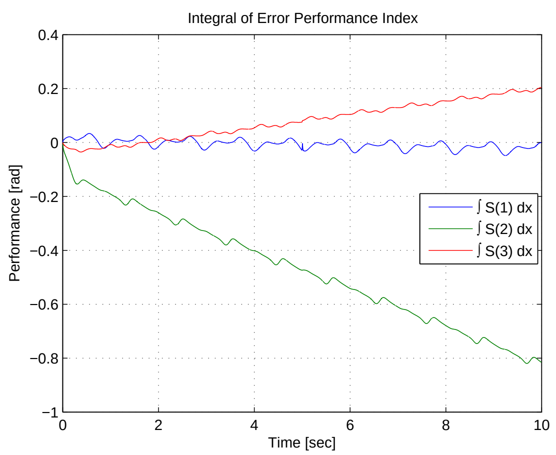

Fig. 3. Integral of error performance index for Slotine-Li control.

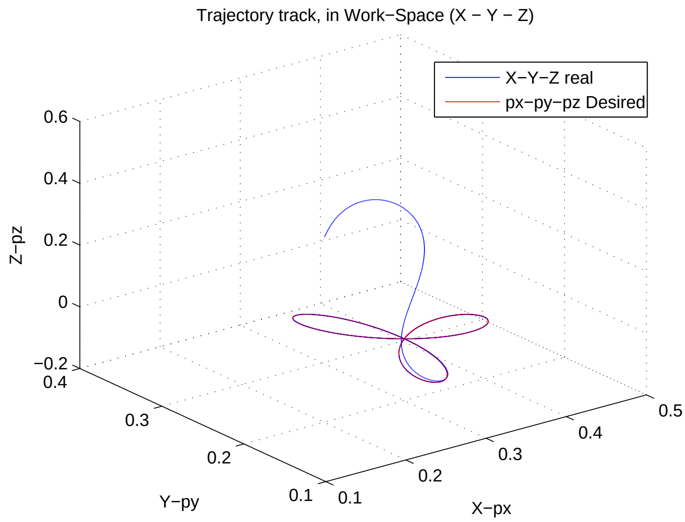

Fig. 4. Trajectory track, X-Y-Z, for sub-optimal control.

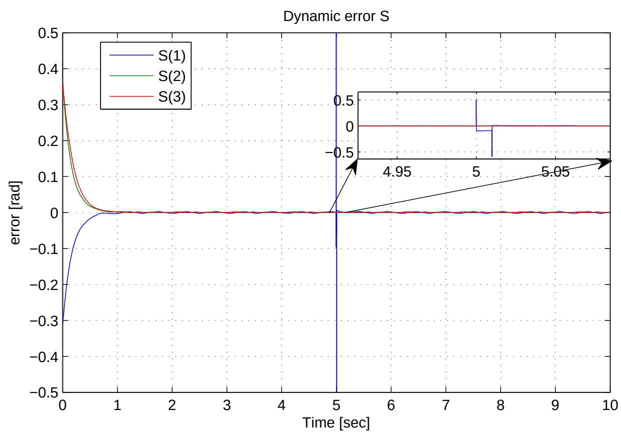

Fig. 5. Dynamic error function for sub-optimal control.

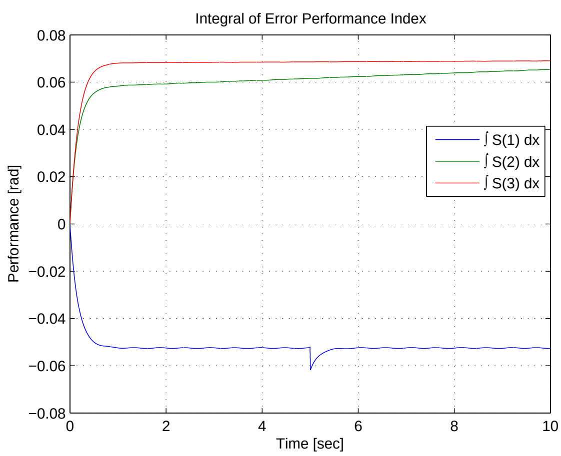

Fig. 6. Integral of error performance index for sub-optimal control.