

Design and Implementation of a Time Varying Local Optimal Controller Based on RLS Algorithm for Multivariable Systems

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Abstract: Since the local optimal controller is a model based controller, the controller parameters can be updated with the on-line parameter tuning. Recursive least squared algorithm is used for on-line closed-loop identification of the model parameters. In this paper, the local optimal controller is designed for multivariable system and its parameters are updated on-line. The time varying local optimal controller is implemented on a lab-based test rig. In addition to its computational efficiency and structure simplicity, the experimental results confirm the effectiveness of this controller especially when the parameters of the system are time-variant.

Keywords: Multivariable local optimal controller, system identification.

1. INTRODUCTION

Due to the increasing complexity of process control systems, multivariable process control has been of considerable attention over the past decades and numerous theoretical and practical works have been proposed in this area of research [1]. Among the existing methods of multivariable control, the Local Optimal Control (LOC) emerged as a new method for controlling multivariable systems as well as single-input single-output systems. This new method was first introduced by Lyantsev, et al. in 2004 [2]. However the existing LOC approach is incapable of dealing with non-minimum phase systems. Therefore, a modification has been done to the existing algorithm to extend the idea to the non-minimum phase systems [3].

Since the local optimal controller is a model based controller, the controller parameters can be updated with the on-line parameter tuning. Recursive Least Squared (RLS) is the algorithm used to identify the model parameters on-line. The parameters of the time varying LOC are updated on-line based on the RLS algorithm. This time varying LOC is designed and implemented on a lab-based test rig. The experimental results confirm the effectiveness of the proposed controller especially when the parameters of the system are time-variant.

This paper is organized as follows. Section 2 is a brief description about the test rig to be controlled. Section 3 introduces the parameters identification off-line as well as on-line of the system under consideration. It also introduced the RLS algorithm used for the on-line identification. The model parameters that obtained on-line are compared with those obtained off-line. Section 4 represents the LOC and the modifications done to obtain the time varying LOC. It also represents the advantage of that time varying LOC when

dealing with systems of time-variant model parameters. Concluding remarks are summarized in section 5.

2. THE TEST RIG PROCESS DESCRIPTION

The test rig is Bytronic Process Control Unit (PCU) which is based around a fluid flow process, where flow rate and temperature of that fluid can be controlled. This reflects a typical process control problem commonly seen in the food and beverages manufacturing and petrochemical industry [4].

As shown in Fig.1, the PCU consists of a sump (fluid reservoir), a pump, a flow meter, a cooler (radiator), and a process tank. The process tank contains an electrical heating element, a temperature sensor, a stirrer, and a high-level safety switch.



Fig.1. Bytronic process control unit (PCU)

The fluid is pumped in a closed-loop path starting from the sump through a radiator to a process tank where the fluid can be heated and then is drained back again to the sump.

The PCU is connected to a power supply unit that provides the input power for each element in the PCU. Also, the process inputs and outputs are connected to a computer control module that works as an interface between the PCU and the PC-based controller. The computer control module is shown in fig.2.



Fig.2. Computer control module

The system was modified by replacing the existing I/O interface card with a NI PCI-DIO-96 digital I/O card to enable working in Matlab environment [5]. As such, the computer control module is connected to the new digital I/O card through the connectors shown in fig.3.

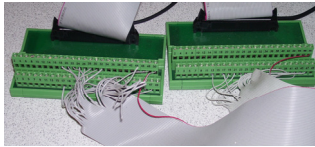


Fig.3. The connectors between the computer control module and the new digital I/O card

The overall block diagram of the I/O interface system is illustrated in fig. 4.

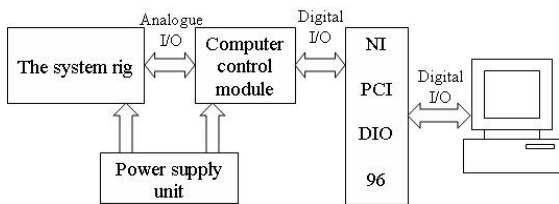


Fig.4. The overall block diagram of the I/O interface system

3. SYSTEM IDENTIFICATION

The PCU is considered as two-input, two-output system with the inputs as the DC voltage to the pump and the power to the heater, and the outputs as the fluid flow rate and the fluid temperature in the process tank. The input power to the heater element is computer-controlled using Pulse Width Modulation (PWM) technique. In fact the corresponding control signal manipulates the mark/space ratio (duty factor)

of PWM signal which in turn controls the average current applied to the heater.

From fluid dynamics point of view; the fluid flow rate depends on the pump speed, meaning that the input power to the heater has no effect on the fluid flow rate. However, the fluid temperature varies with both of the pump speed and the heating power.

There are two different approaches available for system identification. In the first approach, the plant is considered as two sub-systems according to number of outputs, one of which is Single-Input Single-Output (SISO) system and the other is two-input single-output system. Then each sub-system is identified separately [3], [6]. In the second approach the plant is identified as Multi-Input Multi-Output (MIMO) system (two-input two-output system). Both approaches have been considered and compared in [3]. As it was found, the first approach is more efficient than the second one. So, it is used in this paper.

3.1 Off-line system identification

In this section, off-line open-loop system identification for each sub-system is considered. According to the characteristics of each sub-system, the excitation input is designed. Then the model structure and the model parameters are determined to be compared with that obtained from on-line system identification.

First sub-system

For this subsystem the input is the DC voltage to the pump and the output is the fluid flow rate on the pipes. Prior to the system identification several initial open loop tests must be performed to determine the characteristics of the system and design the excitation signal [7], based on which the system's time constant is 700 ms and its cut-off frequency is 6 rad/s . As such, the sampling time is chosen as 125 ms and the frequency band for the excitation signal is chosen accordingly. The input to the pump can vary between 0 and 12 V . The input-output characteristic curve for this subsystem shows that the system can be considered as linear when working between 3 and 9 V .

According to the previous discussion, the excitation signal must vary between $3\text{-}9\text{ V}$ in amplitude and its power spectrum must be flat for frequencies from $0\text{-}6\text{ rad/s}$. Chirp and multi-sine signals can be considered as the excitation signals satisfying the mentioned requirements [3], [7].

In this paper the multi-sine excitation input is considered. The multi-sine input is a sum of sinusoids as given in (1).

$$u(t) = \sum_{k=1}^d a_k \cos(\omega_k t + \phi_k) \quad (1)$$

where ϕ_k is chosen by Schroeder phase choice [7].

In the case studied a sum of 13 sinusoids is used where ω_k is spaced each time by 0.5 rad/s . The amplitude a_k in the case studied is calculated to be 0.4706 .

This excitation signal is applied for the input of the pump for 550 s and the fluid flow rate is recorded during that time. Using the input-output data obtained the ARX model of the system is deduced [8] as in (2).

$$y(i) = -a_1 y(i-1) - a_2 y(i-2) + b_1 u(i-1) + b_2 u(i-2) \quad (2)$$

where:

$$a_1 = -1.067, \quad a_2 = 0.313, \quad b_1 = 0.043, \quad b_2 = 0.0098$$

The obtained model is validated using two different quantities. The first one is the Mean Squared Error (MSE) expressed as in (3). The second one is the fitness (FIT) described in (4).

$$MSE = \frac{1}{N} \sum_{i=1}^N (y(i) - \hat{y}(i))^2 \quad (3)$$

$$FIT = \left(1 - \left(\frac{\sum_{i=1}^N (y(i) - \hat{y}(i))^2}{\sum_{i=1}^N (y(i) - \bar{y})^2} \right) \right) \times 100\% \quad (4)$$

where \bar{y} is the mean value of the measured output.

Using k -step ahead predictors \hat{y}_k , the two extreme predictors $\hat{y}_1, \hat{y}_\infty$ are used for fitness.

For this model it is found that the MSE is 0.000174 , and the FIT is 86.06% , and 93.61% , for $k=1$ and $k=\infty$, respectively.

Second sub-system

This sub-system is two-input single output system. The two inputs are the input voltage to the pump (u_1) and the input power to the heater (u_2). The output is the fluid temperature in the process tank. It must be noted that while such inputs can directly contribute to increasing the temperature, decreasing temperature will be associated with the temperature difference between the fluid and the surrounding air, and conductivity of the cavity. Therefore, to identify this subsystem two models must be taken in account; one for increasing temperature, and the other for decreasing temperature. In this paper the first model will be considered. Open loop tests result in time constant about 300 s which is too high compared with the first sub-system (700 ms).

The excitation signals for this sub-system are chosen to be multi-step signals to cope with the slow response [3]. These excitation signals are given in (5). The sub-system output for these inputs is shown in fig.5.

$$u_1 = \begin{cases} 4.7059 & 0 \leq t < 600 \\ 6.5882 & 600 \leq t < 1000 \\ 8.0000 & 1000 \leq t < 1400 \end{cases} \quad (5-a)$$

$$u_2 = \begin{cases} 0 & 0 \leq t < 200 \\ 29.4176 & 200 \leq t < 800 \\ 58.8235 & 800 \leq t < 1200 \\ 88.2353 & 1200 \leq t < 1400 \end{cases} \quad (5-b)$$

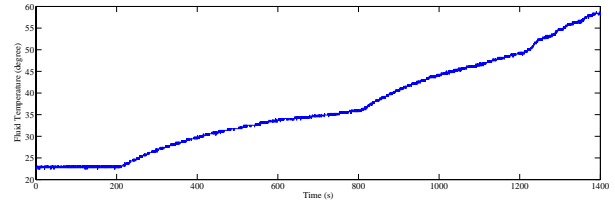


Fig.5. Output of the second sub-system

From these input-output data, the ARX model of the sub-system is obtained as in (6).

$$y(i) = -\sum_{n=1}^4 a_n y(i-n) + \sum_{m=1}^4 b_{1m} u_1(i-m) + \sum_{k=1}^4 b_{2k} u_2(i-k) \quad (6)$$

where:

$$a_1 = -0.2505, \quad a_2 = -0.2571, \quad a_3 = -0.2429, \quad a_4 = -0.2484 \\ b_{11} = 0.0400, \quad b_{12} = -0.0828, \quad b_{13} = 0.0660, \quad b_{14} = -0.0183 \\ b_{21} = 0.0032, \quad b_{22} = -0.0011, \quad b_{23} = 0.0044, \quad b_{24} = -0.0061$$

For this model it is found that the MSE is 0.0298 , and the fitness for $k=\infty$, $k=1$, is 85.18% , 98.35% , respectively.

3.2 On-line system identification

In this section, experiments are performed to find the discrete time model that best describes the system using Recursive Least Squared (RLS) method.

Introduction to RLS method

In this method, let the system model be represented as in (7).

$$A(Z^{-1})y(t) = B(Z^{-1})u(t) \quad (7)$$

where Z^{-1} is the back shift operator, and

$$A(Z^{-1}) = 1 + a_1 Z^{-1} + a_2 Z^{-2} + \dots + a_{n_a} Z^{-n_a} \quad (8)$$

$$B(Z^{-1}) = b_1 Z^{-1} + b_2 Z^{-2} + \dots + b_{n_b} Z^{-n_b}$$

The model given in (7) can be represented in an alternative form as.

$$y(t) = \phi^T(t) \theta \quad (9)$$

where θ is the vector of model parameters represented in (10), and ϕ is the vector of regression that consists of the measured values of inputs and outputs as in (11).

$$\theta = [a_1 \ a_2 \ \dots \ a_{n_a} \ b_1 \ \dots \ b_{n_b}]^T \quad (10)$$

$$\phi^T(t) = [-y(t-1) \ \dots \ -y(t-n_a) \ u(t-1) \ \dots \ u(t-n_b)] \quad (11)$$

For the accurate model given in (9), the vector of model parameters θ is unknown. It is important to determine it using input-output data available. For that purpose a model of the system is supposed to be:

$$\hat{y}(t) = \phi^T(t) \hat{\theta}(t-1) \quad (12)$$

For the RLS algorithm to be able to update the parameters at each sample time, it is necessary to define an error. The model prediction error $\varepsilon(t)$ is a key variable in RLS algorithm and is defined as in (13).

$$\varepsilon(t) = y(t) - \hat{y}(t) = y(t) - \phi^T(t) \hat{\theta}(t-1) \quad (13)$$

The error $\varepsilon(t)$ is the difference between the system output

and the estimated model output. This model prediction error is used to update the parameter estimates as in (14).

$$\hat{\theta}(t) = \hat{\theta}(t-1) + P(t)\phi(t)\varepsilon(t) \quad (14)$$

where the estimator covariance matrix $P(t)$ is calculated as given in (15).

$$P(t) = \frac{1}{\lambda} P(t-1) \left[I_p - \frac{\phi(t)\phi^T(t)P(t-1)}{\lambda + \phi^T(t)P(t-1)\phi(t)} \right] \quad (15)$$

whilst the subscript p is the dimension of identity matrix, $P = n_a + n_b$. Also λ is the forgetting factor ($0 < \lambda \leq 1$).

Forgetting factor λ controls the convergence speed of the estimated model parameters. When $\lambda = 1$, it gives the slowest convergence speed but the best robustness against noise. When $\lambda < 1$, it results in increasing the convergence speed of the estimated model parameters. In general, choosing $0.98 < \lambda < 0.995$ gives a good balance between convergence speed and noise susceptibility [9].

Application of RLS method demands supposition of the initial values of $P(t)$ and $\hat{\theta}(t)$. There is no unique way to initialize them. However the standard choice of $P(0)$ is the identity matrix scaled by a positive scalar α , (i.e. $P(0) = \alpha I_p$), where α is recommended to be chosen in the interval $[1 \ 1000]$ depending on the existence of prior knowledge about the system parameters [10].

Closed-loop system identification

In this section the RLS method described above is used to identify the model parameters of the multivariable system in a closed-loop structure. Direct method is used for the closed-loop system identification. A Local Optimal Controller (LOC) that provides a stable closed-loop for the multivariable system [3] is implemented using real time windows target toolbox [5].

The reference inputs for the multivariable system in that closed-loop with the LOC is given in (16) and (17).

$$\text{flow rate (l/min)} = \begin{cases} 1 & 0 < t \leq 800 \\ 1.4 & 800 < t \leq 1300 \\ 1.6 & 1300 < t \leq 1600 \end{cases} \quad (16)$$

$$\text{fluid temperature (}^\circ\text{C)} = \begin{cases} 35 & 0 < t \leq 500 \\ 45 & 500 < t \leq 1000 \\ 55 & 1000 < t \leq 1600 \end{cases} \quad (17)$$

For the first subsystem, the estimated model parameters of the model structure given in (2) are shown in fig.6.

For the second subsystem, the estimated model parameters of the model structure given in (6) are shown in fig.7.

From these figures, it can be seen that the model parameters are converge to certain values. These values are listed in table 1 for the first sub-system and in table 2 for the second sub-system. These two tables also represented the off-line model parameters for comparison.

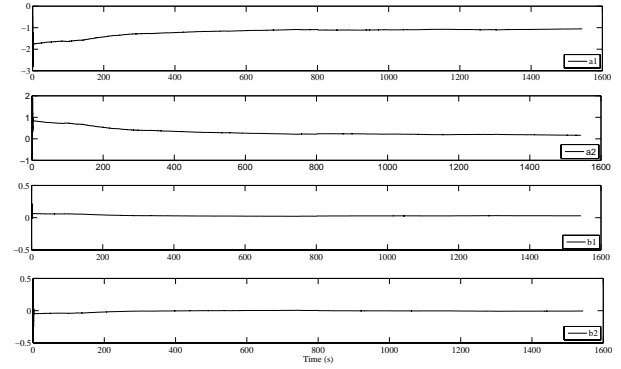


Fig.6. The model parameters for the first sub-system

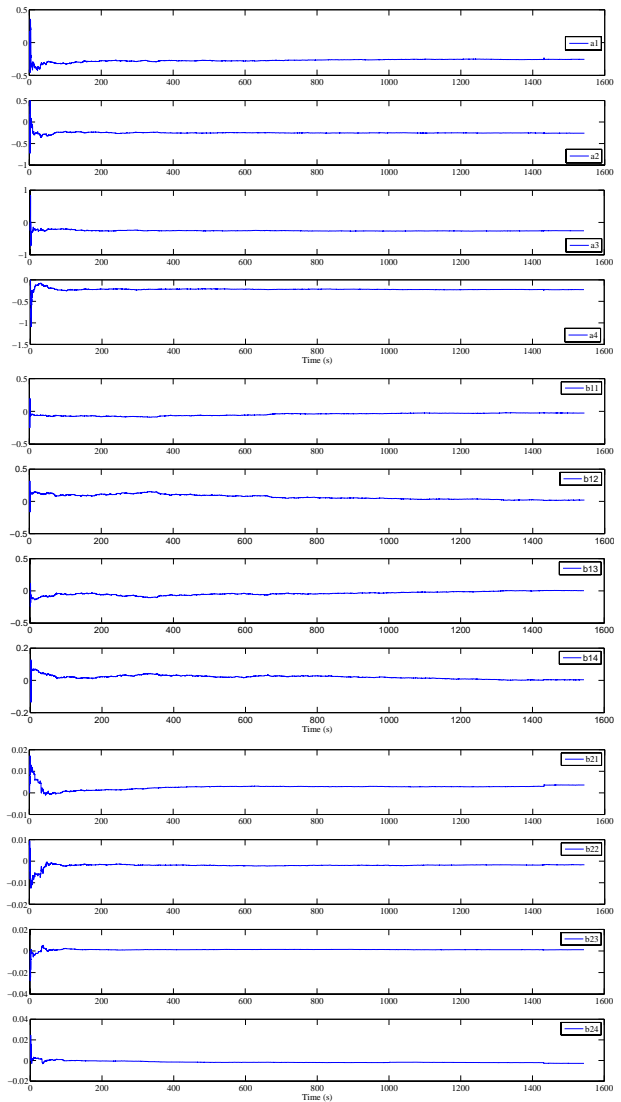


Fig.7. The model parameters for the second sub-system

Table 1 model parameters for first sub-system

	Off-line open-loop	On-line closed-loop
$a1$	-1.067	-1.0567
$a2$	0.313	0.165
$b1$	0.043	0.0293
$b2$	0.0098	-0.0059

Table 2 model parameters for second subsystem

	Off-line open-loop	On-line closed-loop
$a1$	-0.2505	-0.2554
$a2$	-0.2571	-0.2594
$a3$	-0.2429	-0.2578
$a4$	-0.2484	-0.2270
$b11$	0.0400	-0.0261
$b12$	-0.0828	0.0203
$b13$	0.0660	0.0032
$b14$	-0.0183	0.0040
$b21$	0.0032	0.0036
$b22$	-0.0011	-0.0017
$b23$	0.0044	0.0012
$b24$	0.0061	-0.0028

4. LOCAL OPTIMAL CONTROLLER

The main purpose of this section is to design and implement a time varying LOC for the multivariable system (PCU). Since the LOC is a model dependent controller, its parameters can be updated based on the on-line identified parameters of the system model using RLS algorithm.

4.1 Design of LOC

The local optimal controller is designed according to the method discussed in [2] and the models obtained for each output in (2) and (6). Therefore, (18) and (19) can be obtained for the local optimal controller of this multivariable system [3].

$$y_1(i+1)-y_1(i)=h_1[\alpha_1\delta y_1(i)+\alpha_2\delta y_1(i-1)+\beta_1\delta u_1(i)+\beta_2\delta u_1(i-1)] \quad (18)$$

$$y_2(i+1)-y_2(i)=h_2[\sum_{n=0}^3\alpha_n\delta y_2(i-n)+\sum_{m=0}^3\beta_m\delta u_1(i-m)+\sum_{k=0}^3\gamma_k\delta u_2(i-k)] \quad (19)$$

where:

- h_1 and h_2 are weighting coefficients indicating the level of uncertainty involved in the plant dynamics [2],
- $\delta y_j(i) = y_j(i) - y_j(i-1)$,
- $\delta u_j(i) = u_j(i) - u_j(i-1)$,
- $y_1(i+1)$ and $y_2(i+1)$ are the two reference inputs required for the multivariable system,
- $\alpha_{ij}, \beta_{ij}, \gamma_{ij}$ are scalar coefficients depend on the model parameters given in (2) and (6).

By solving (18) and (19) for $\delta u_1(i)$ and $\delta u_2(i)$, the LOC can be constructed as in fig.8 which is the modified LOC represented in [3]. This modified LOC can deal with the non-minimum phase systems as well as the minimum phase systems [3].

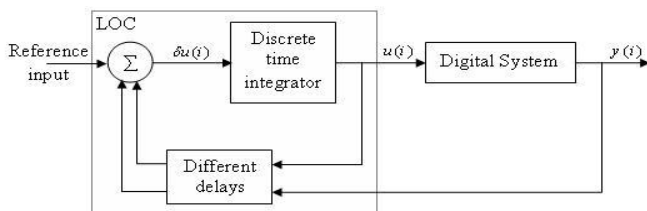


Fig.8. Modified block diagram of the LOC

This algorithm for designing LOC was applied to the PCU as a multivariable system in [3]. The LOC parameters in this case was fixed (time-invariant) and the results obtained was satisfactory.

4.2 Design of time varying LOC

In this section, a time varying LOC is designed and implemented for the PCU based on the RLS algorithm. As the LOC is a model based controller, its parameters can be updated through updating the parameters of the system model. RLS algorithm with direct method is used for on-line closed-loop system identification. Therefore, the model parameters are updated every sample and the controller parameters are updated every n samples ($n \geq 1$). In the case studied, the controller parameters are updated every 10 samples. Fig.9 and fig.10 show the response of the two outputs of the multivariable system (PCU) when it is subjected to the reference inputs represented in (16) and (17).

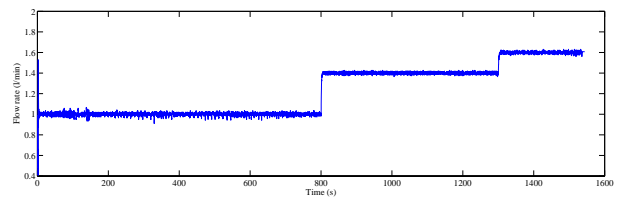


Fig.9. Flow rate (first output)

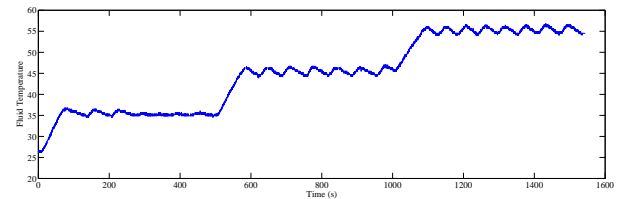


Fig.10. Fluid temperature (second output)

From these figures, it is clear that, the outputs track the reference inputs. However the flow rate output is faster than the fluid temperature output. Also, it is clear that there no coupling between the outputs. It must be noted that, the oscillations appear in figure 10 are due to the temperature sensor sensitivity.

In this part the results obtained are similar to that obtained in [3] where the LOC parameters were fixed (time-invariant). This is the case when the system parameters are time-invariant. However, the case is different when the system parameters are varying with the time and the need for the time varying LOC is appeared.

4.3 Time-varying LOC with time-variant model parameters

while In this part the system is supposed to have a leakage in its pipes. So, of course the flow rate will be affected. This can be done practically in the PCU by opening a tap directly after the pump and before the flow rate sensor. The LOC tries to keep the flow rate constant as the reference input. Fig.11 (the upper figure) shows a constant output flow rate during application of different leakage input, the lower

figure shows the changes of the first sub-system model parameters with each change of the leakage. A leakage with a small value is applied first at 480 s and then this leakage is increase at 543 s. The parameters changes can be seen and so the controller parameters are changed in turn. In this case the advantage of the time varying LOC can be observed.

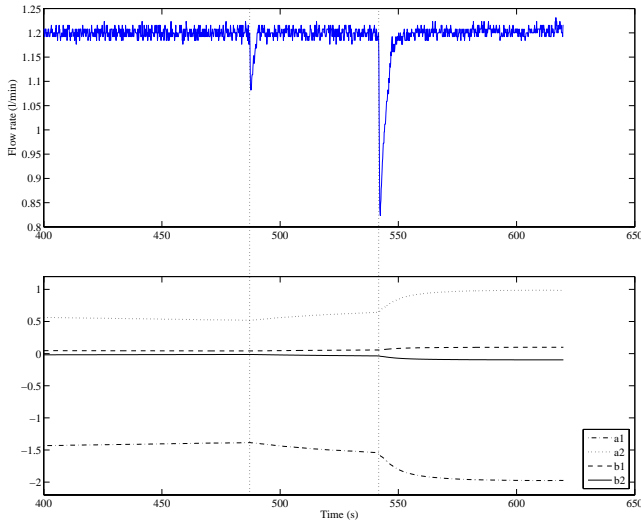


Fig.11. varying of model parameters with disturbance

5. CONCLUSIONS

A time varying LOC associated with on-line closed-loop system identification has been represented in this paper. The on-line closed-loop system identification using RLS algorithm with the direct method gives similar results to that of the off-line identification with excitation inputs. When the system under consideration has time-invariant model parameters, the time varying LOC gives similar results to that obtained using the normal LOC. The advantage of the time-varying LOC over the normal one appears when dealing with time-variant model parameters. This has been verified and confirmed in the lab by implementation on a practical multivariable system (PCU).

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