Reduced-order Local Optimal Controller for a Higher Order System

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Abstract:

In this paper, a reduced order local optimal controller is designed for a higher order system. A reduced order model is obtained for the higher order system and its parameters are used for the reduced order local optimal controller. Also, genetic algorithm is used with the reduced order local optimal controller structure to design the controller parameters instead of obtaining them from the reduced order model. These results obtained are compared with the results obtained from full order local optimal controller. Finally, analogy between reduced order local optimal controller and PI controller parameters is represented. As such, this reduced order local optimal controller can be used for tuning PI controller parameters. Experimental results on a lab-based test rig confirm the effectiveness of the reduced order local optimal controller.

Keywords: Local optimal controller, genetic algorithm, and PI controller.

1. INTRODUCTION

Local Optimal Control is a new control approach was first introduced by Lyantsev in 2004 [1]. It is used to control multivariable systems as well as Single Input-Single Output (SISO) systems. Local optimal controller (LOC) design is based on the model parameters of the system to be controlled. In all the previous work done full order models are used to design the LOC. In this paper, reduced order model parameters are used to design the LOC. Also, Genetic Algorithm (GA) is used with the reduced order LOC structure to design the controller parameters instead of obtaining them from the reduced order model. These results obtained are compared with the results from LOC designed using full order model parameters. Finally, using this reduced order LOC an analogy is held with the conventional digital PI controller. As such, this reduced order LOC can be used for tuning PI controller parameters. Experimental results on a lab-based test rig confirm the effectiveness of the reduced order local optimal controller. This paper is organised as follows: Section 2 contains a brief description of the system rig to be controlled. Section 3 introduces system identification of the system to obtain the full order and reduced order models. In section 4 full order and reduced order local optimal controllers are designed and a comparison between them is held. In section 5 the GA is used with the reduced order LOC structure to design the controller parameters. In section 6 a relation between the reduced order LOC parameters and the digital PI controller parameters is introduced. As such a digital PI controller is designed based on the reduced order LOC and the results compared with the genetically tuned digital PI controller. Concluding remarks are made in section 7.

2. THE TEST RIG PROCESS DESCRIPTION

The system rig is the Bytronic Process Control Unit (PCU) which is based around a fluid flow process, where flow and temperature can be controlled. This system is a multivariable system [2]. In the case studied SISO system only is considered. In this system a fluid is pumped from a sump in a closed path and then drained back to the sump while the flow rate of the fluid is monitored. As such, the system input is the DC voltage to the pump, and the system output is the fluid flow rate.

The system is connected to a power supply unit that provides the input power to all the system elements. Also the process inputs and outputs are connected to a computer control module that works as an interface between the PCU and PC-based controller. The system was modified by replacing the existing I/O interface module with a NI PCI-DIO-96 digital I/O to enable working in Matlab environment [3]. The overall block diagram of the system with I/O interface is illustrated in Fig.1.

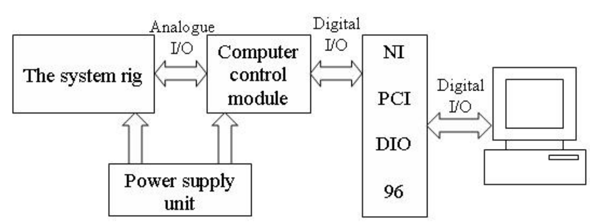

Fig.1. Overall block diagram of the I/O interface system

3. OPEN LOOP SYSTEM IDENTIFICATION

For this system under consideration, the input is the DC voltage to the pump and the output is the fluid flow rate.

Prior to the system identification several initial open loop tests must be performed to determine the characteristics of the system and design the excitation signal [4], based on which the system's time constant is 700 ms and its cut-off frequency is 6 rad/sec. As such, the sampling time is chosen as 125 ms and the frequency band for the excitation signal is chosen accordingly.

The input voltage to the pump can vary between 0-12 V. The I/O curve for this system is shown in Fig.2. From this curve it is obvious that the system can be considered as linear when working between 3-9 V.

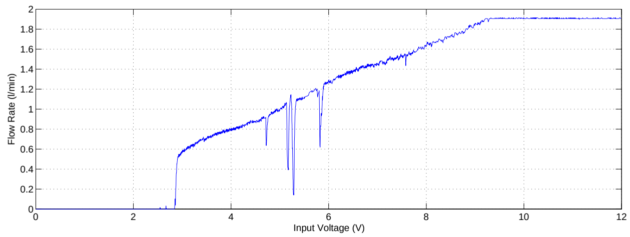

Fig.2. input-output curve for the pump

According to Fig.2., the excitation signal must vary between 3-9 V in amplitude and its power spectrum must be flat for frequencies from 0-6 rad/sec. Chirp and multi-sine signals can be considered as the excitation signals satisfying mentioned requirements [4].

Two sets of I/O data are obtained for system identification using two different inputs as excitation signals. Fig.3. and Fig.4. show the I/O data using chirp and multi-sine as excitation signals respectively.

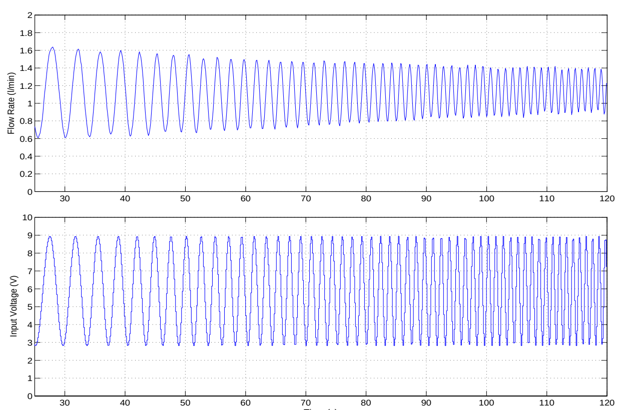

Fig.3. The input voltage to the pump (lower) and the output flow rate

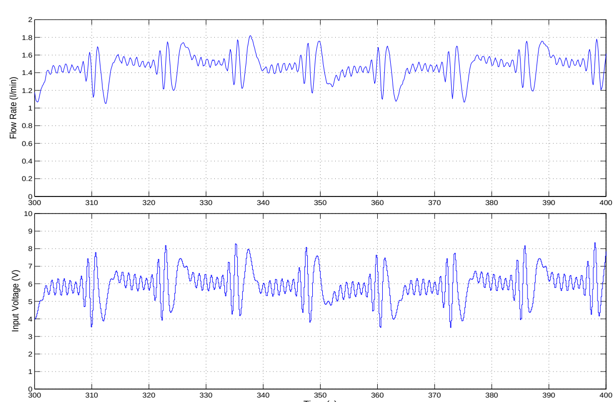

Fig.4. The input voltage to the pump (lower) and the output flow rate

3.1 Full order system model

Using these two I/O data sets the system model is obtained. System identification toolbox of Matlab and Output Error (OE) method [4, 5] are used with data set 1 (chirp input) to give the model m_1 and with data set 2 (multi-sine input) to give the model m_2. The model structure is given in (1) and the models estimated parameters are given in table 1.

$$y(i) = -a_1 y(i-1) - a_2 y(i-2) + b_1 u(i-1) + b_2 u(i-2) \quad (1)$$

Also, GA proposed in [6] is used to estimate the model parameters of the same model structure given in (1). This GA is used with data set 1 and data set 2 to give models m_3 and m_4 respectively. The models estimated parameters are also given in table 1.

Table 1. Second order models parameters

	a_1	a_2	b_1	b_2
m_1	-0.9078	0.1557	0.0136	0.0336
m_2	-0.8323	0.1378	0.0167	0.0472
m_3	-0.8348	0.1318	0.0042	0.0462
m_4	-0.8359	0.1360	0.0187	0.0448

Model validation

The two sets of output data generated by each experiment mentioned above are used to validate the different models obtained. The models fitness with each set of output data are listed in table 2. For each model two fitness values are presented, one is using the same data used to produce the model and the other is the cross validation. The four models are subjected to a cross validation, i.e., to validate the model generated using certain set of I/O data with the output data of the other set. The cross validation for each model is represented as gray box in table 2.

Table 2. Fitness for each model with each output data

	Fitness with Set 1	Fitness with Set 2
m_1	91.89%	80.18%
m_2	72.92%	90.95%
m_3	92.39%	78.04%
m_4	72.84%	90.9%

From table 2, m_1 is chosen to be the full order system model.

3.2 Reduced order system model

The same I/O data sets are used to obtain a reduced order model (first order). Models M_1 and M_2 are obtained using OE method of system identification toolbox for I/O data set 1 and I/O data set 2 respectively. Models M_3 and M_4 are obtained using GA proposed in [6] for I/O data set 1 and I/O data set 2 respectively. The model structure for all reduced order models is given in (2). The models parameters are given in table 3.

$$y(i) = ay(i-1) + bu(i-1) \quad (2)$$

Table 3. First order models parameters

Model	a	b
M_1	0.8065	0.0461
M_2	0.7484	0.0581
M_3	0.8019	0.0455
M_4	0.7563	0.0585

Model validation
As in section 3.1 the two sets of output data generated by each experiment are used to validate the different models obtained. The models fitness for each set of output data are listed in table 4. The cross validation for each model is represented as gray box in table 4.

Table 4. Fitness for each model with each output data

	Fitness with Set 1	Fitness with Set 2
M_1	75.07%	72.46%
M_2	66.33%	77.37%
M_3	74.79%	72.5%
M_4	66.43%	77.47%

From table 4, M_1 is chosen to be the reducer order system model.

4. SYSTEM CONTROL

The main purpose of this section is to design a reduced order LOC for the pump system under consideration and to compare its response with the full order LOC response. Fig.5. shows the closed loop step response of the pump system without any controller when it is subjected to the reference input shown.

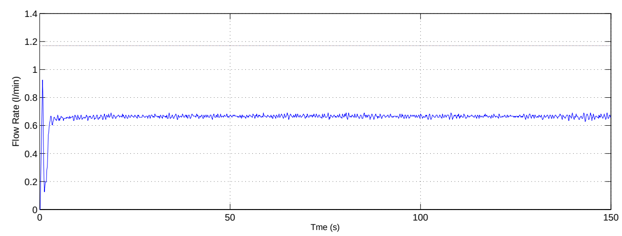

Fig.5. The reference input and the closed loop response of the pump

4.1 The Full Order LOC Design

The LOC is designed and implemented as proposed in [1]. For the pump model, two states $x_1 = y(i\text{-}1)$, $x_2 = y(i\text{-}2)$ are considered and the following equation is driven from the model in (1).

$$y(i+1) - y(i) = h[\alpha_1 \Delta y(i) + \alpha_2 \Delta y(i-1) + \beta_1 \Delta u(i) + \beta_2 \Delta u(i-1)] \quad (3)$$

where:

- $\alpha_1, \alpha_2, \beta_1, \beta_2$ are constants related to the model parameters in (1).
- $\Delta y(i) = y(i) - y(i-1), \quad \Delta u(i) = u(i) - u(i-1)$
- h is a weighting coefficient indicating the level of uncertainty involved in the plant dynamics [1].

From (3), the full order LOC for the pump system which is a non-minimum phase system is designed as in the block diagram shown in Fig.6.

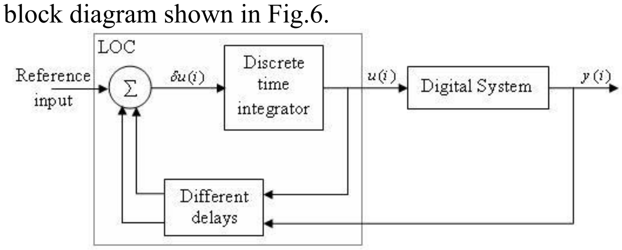

Fig. 6. Block diagram of the full order local optimal controller.

Fig.7. shows the response of the model with the proposed full order LOC using Simulink for different values of the controller parameter, namely *h (h=2, 3, 4, and 5).*

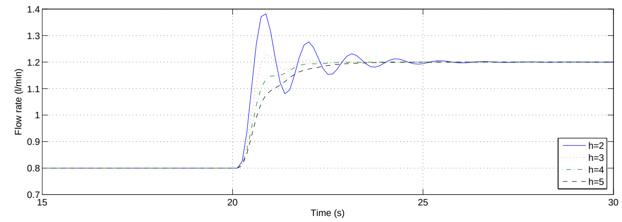

Fig.7. Simulated output response of full order LOC for different h

For *h=5* the controller is designed for pump system and Fig.8. shows the output response for both real and simulated system. From this Fig.8. it is clear that the real and simulated outputs are approximately identical.

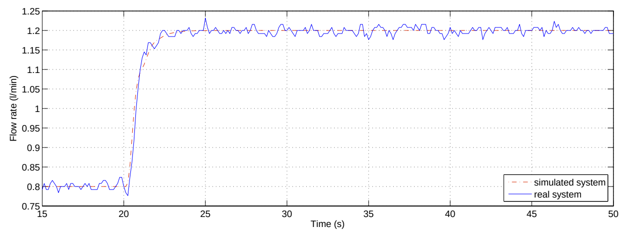

Fig.8. The output response for both real and simulated system at h=5

4.2 The Reduced Order LOC Design

The reduced order LOC is designed and implemented also as proposed in [1] but the model used is the reduced order model represented in (2). As such the following equation is obtained.

$$y(i+1) - y(i) = h[a\Delta y(i) + b\Delta u(i)] \quad (4)$$

where:

- a, b are the reduced order model parameters.
- $\Delta y(i) = y(i) - y(i-1), \quad \Delta u(i) = u(i) - u(i-1)$
- h is a weighting coefficient indicating the level of uncertainty involved in the plant dynamics [1].

From (4), the reduced order LOC of the pump system is designed as in the general block diagram shown in Fig.6. The detailed block diagram of the first order LOC is illustrated in Fig.9.

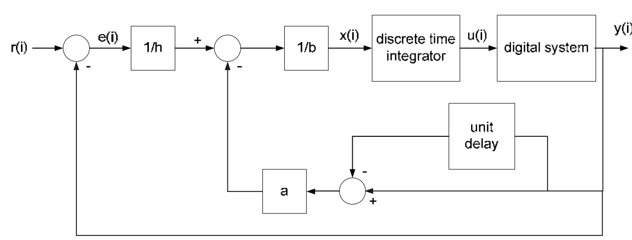

Fig.9. detailed block diagram of reduced order LOC

Fig.10. shows the simulated response of the full order system model with the proposed reduced order LOC using Simulink for different values of the controller parameter, namely *h (h=1, 2, and 3).* It is clear from this Fig.10. that the reduced order LOC gives acceptable response at *h*=3 which is different from that of the full order LOC (*h*=5).

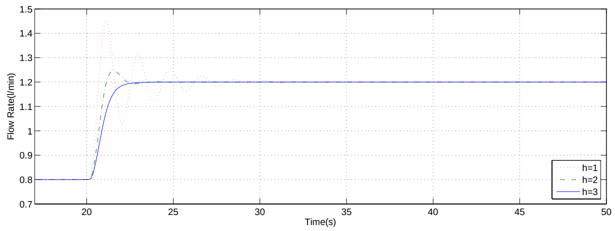

Fig.10. Simulated output response of reduced order LOC for different h

For $h=3$ the controller is designed for the pump system and Fig.11. shows the output response for both real and simulated systems. From this Fig. it is clear that the real and simulated outputs are approximately identical.

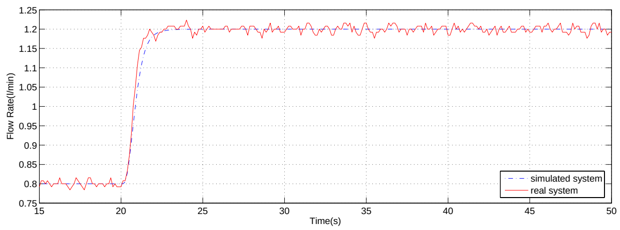

Fig.11. The output response for both real and simulated reduced order LOC at h=3

4.3 Comparison between full and reduced order LOC

Fig.12. shows the simulated output response of the system with full and reduced order LOC as obtained from Simulink.

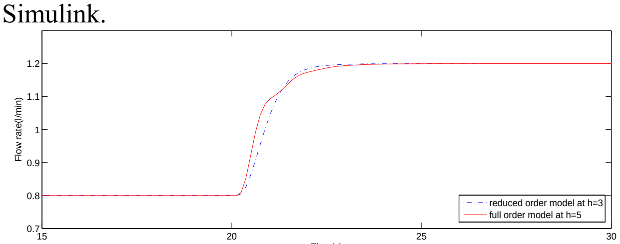

Fig.12. Simulated output response of full and reduced order LOC

Fig.13. shows the real output response as obtained from the test rig for both full and reduced order LOC.

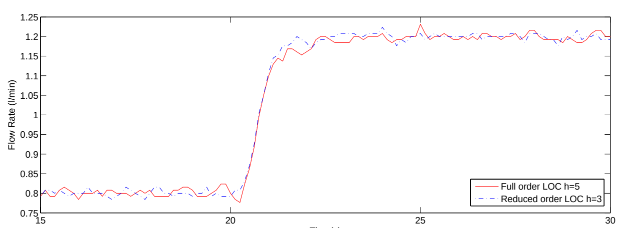

Fig. 13. Real output response of full and reduced order LOC

From these two Figures it is clear that the results obtained using reduced order LOC approximately the same as that of the full order LOC.

5. PARAMETERS ESTIMATION OF REDUCED ORDER LOC STRUCTURE USING GA

In this section GA is used to estimate the controller parameters (a, b, and h) for the reduced order LOC structure shown in Fig.9.

Minimization of the Mean Squared Error (MSE) between the reference input and the system's output and to have no over shoot is used as a multi-objective function to estimate these three controller parameters genetically [7, 8].

GA toolbox of Matlab [9] is used to estimate the three controller parameters. It is found that for each value of h there are unique values of a and b that satisfy the objective function. Similar results are obtained using GA proposed in [6]. Table 5 represents the values of a and b for different values of h ($h=1, 2, 3$) and the value of the objective function (Z_{min}) obtained by the two GA approaches stated.

Table 5. Controller parameters calculated genetically of the reduced order LOC structure

	GA toolbox			GA proposed in [6]		
	a	b	Z_{min}	a	b	Z_{min}
$h=1$	2.9623	0.1178	0.0142	3.2098	0.088	0.0126
$h=2$	1.5662	0.0446	0.0126	1.6047	0.044	0.0126
$h=3$	0.8303	0.0347	0.0132	0.9863	0.0304	0.0127

Fig.14. shows the simulated output response for the controller parameters calculated genetically.

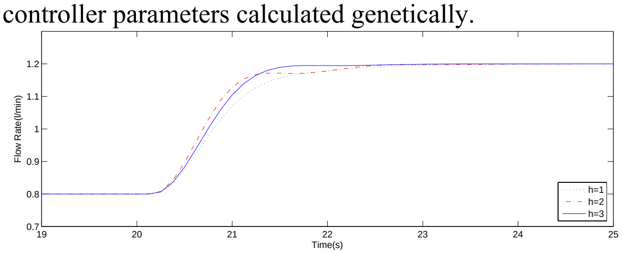

Fig.14. Simulated output response for h=1, 2, 3

From table 5 and Fig.14. it is clear that, there are values of a and b satisfy the objective function for each value of h. As such, infinite number of solutions satisfying the objective function may be found. But for the normal reduced order LOC design, fixed values of a and b are exist according to the reduced order model parameters and controller parameter h is the only value tuned to get the required response. Always, the value of h is increased gradually until obtaining overshoot free output response.

6. ANALOGY BETWEEN REDUCED ORDER LOC AND DIGITAL PI CONTROLLER

In this section a relation between the reduced order (first order) LOC parameters and the digital PI controller parameters is deduced. As such, the LOC parameters can be used for tuning the digital PI controller parameters.

Fig.15. shows the digital PI controller of the system.

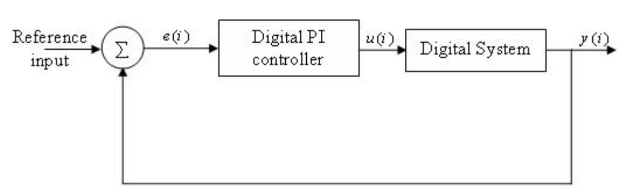

Fig.15. block diagram of digital PI controller.

According to discrete time control theory [10], the digital PI controller in Fig.15. can be represented in Z-domain as in (5).

$$\frac{u(i)}{e(i)} = k_p + \frac{k_i}{1-Z^{-1}} = \frac{k_p + k_i - k_p Z^{-1}}{1-Z^{-1}} \quad (5)$$

And so, the following relation is obtained.

$$\Delta u(i) = (k_p + k_i)e(i) - k_p e(i-1) \quad (6)$$

where $\Delta u(i) = u(i) - u(i-1)$.

From Fig.9. the following relation can be obtained for the first order LOC.

$$\begin{aligned} x(i) &= \frac{1}{b}[\frac{1}{h}e(i) - a(y(i) - y(i-1))] \\ &= \frac{1}{b}[\frac{1}{h}e(i) - a(y(i) - y(i-1) + r(i) - r(i-1))] \end{aligned} \quad (7)$$

where $r(i) = r(i-1)$ is the reference input which is constant value. So the next equation can be obtained.

$$x(i) = \frac{1}{b}[\frac{1}{h}e(i) + ae(i) - ae(i-1)] \quad (8)$$

And also from Fig.9. it is clear that:

$$u(i) = \frac{T_s}{1-Z^{-1}} x(i) \quad (9)$$

where T_s is the sampling time. So the following relation can be obtained.

$$\Delta u(i) = [\frac{T_s}{bh} + \frac{aT_s}{b}]e(i) - \frac{aT_s}{b}e(i-1) \quad (10)$$

From (6), (10) the following relation can be deduced.

$$k_p = \frac{aT_s}{b}, \qquad k_i = \frac{T_s}{bh} \quad (11)$$

From the relation deduced in (11), the digital PI controller parameters can be calculated from the reduced order LOC parameters (a, b, and h).

6.1 Design of digital PI controller based on the reduced order LOC

In this subsection (11) is used to calculate the digital PI controller parameters (k_p and k_i) based on the reduced order LOC parameters (a, b, and h). Fig.16. shows the output response for the real and simulated systems controlled by the digital PI controller tuned using (11).

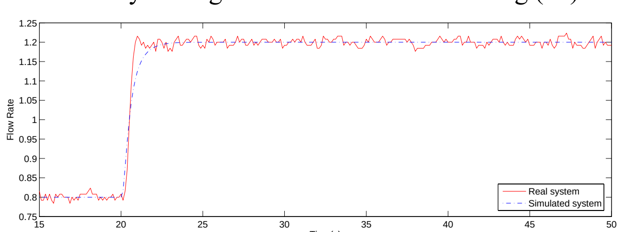

Fig.16. The output response for the real and simulated systems controlled by digital PI controller tuned using the reduced order LOC.

It is clear from this Fig.16. that the output response of the real system is slightly different from that of the simulated one. Fig.17. shows the simulated output responses of the reduced order LOC and that of the digital PI controller designed using (11). From this Fig.17. it is clear that they are approximately the same.

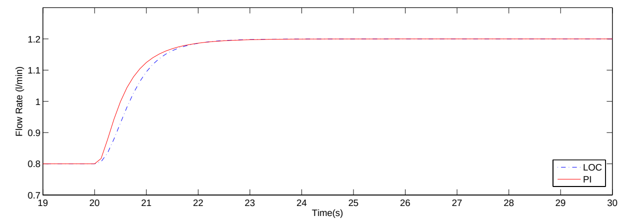

Fig.17. The simulated output responses of the reduced order LOC and the digital PI controller deduced.

Fig.18. shows the output responses obtained from the test rig for both reduced order LOC and digital PI controller. In this case, the responses are not exactly the same and a faster response of PI controller can be noticed.

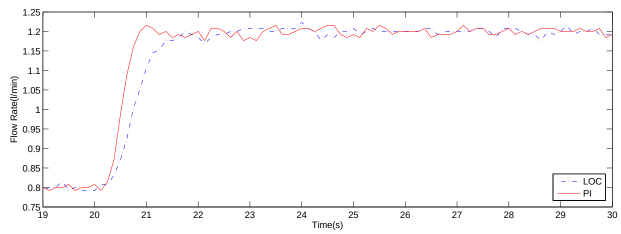

Fig.18. The real output responses of the reduced order LOC and the digital PI controller deduced.

6.2 Comparison between digital PI tuned using reduced order LOC and digital PI tuned genetically

GA is used for tuning the controller parameters (K_P, K_I) [6-8]. The multi-objective function is used to minimize the MSE of the tracking error and eliminate the overshoot. Fig.19. shows the response of the pump controlled by genetically tuned digital PI controller. Two curves illustrated are obtained from simulated model, and the real PCU system.

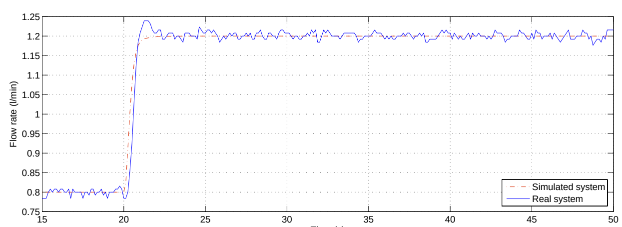

Fig.19. The pump response with PI controller (simulation, real).

The difference observed between the simulated model and the real system is because of model uncertainty is involved in the controller parameters tuning made by GA. Table 6 represents the digital PI controller parameters obtained by both tuning techniques discussed.

Table 6. PI controller parameters tuned by GA and reduced order LOC

	k_p	k_i
Tuning using reduced order LOC	2.185	0.903
Tuning using GA	3.15	1.33

Fig.20. shows the simulated output responses of the digital PI controller tuned using the two techniques discussed.

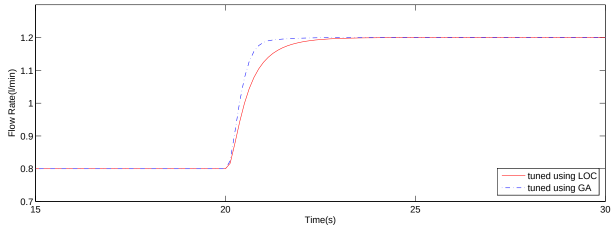

Fig.20. Simulated output response of PI controller tuned genetically and using reduced order LOC.

From this Fig.20, it can be seen that the response of the genetically tuned PI is slightly faster than the response of the PI tuned using the reduced order LOC. The real system responses obtained from the test rig for the PI controller tuned by the two techniques discussed are shown in Fig.21.

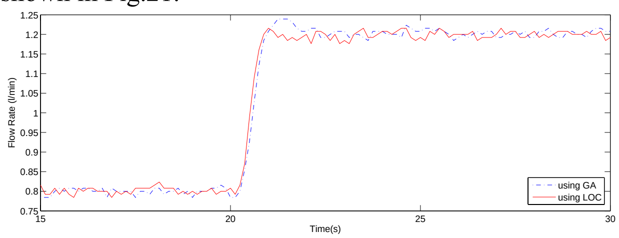

Fig.21. Real output response of PI controller tuned genetically and using reduced order LOC.

From this Fig.21. it has been noticed that both controllers approximately have the same speed. Also, it has been found that the PI controller tuned using LOC has better response from the overshoot point of view. This means that PI tuned using LOC has a good robust performance compared with the PI tuned genetically.

But in general, the digital PI controller does not show an excellent robust performance like that of the full or reduced order LOC. Fig.8. and Fig.11. confirm the excellent robust performance of the full and reduced order LOC. While Fig.16. and Fig.19. confirm the weakness of the digital PI controller to have excellent robust performance.

7. CONCLUSION

A reduced order (first order) LOC is designed for a higher order (second order) system. This reduced order LOC gives approximately similar results to that obtained using full order LOC. Moreover, it is simpler and faster in design.

Using this first order LOC structure, the controller parameters are obtained genetically and so, no need to obtain the model parameters using system identification. In this case, a solution can be obtained for each value of h and so, an infinite number of solutions may be obtained.

Finally, a relation is obtained between the first order LOC and the digital PI controller. As such, the first order LOC parameters can be easily used for tuning the digital PI controller parameters. This method gives similar and may be more robust results to that obtained by digital PI controller parameters tuned genetically.

The experimental results obtained confirm the effectiveness of the reduced order LOC and also the effectiveness of the digital PI controller tuned using the reduced order LOC.

Also, the experimental results confirm that the full and reduced order LOC is better than digital PI controller tuned genetically or using the reduced order LOC from robust performance point of view.

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