Controller Design for Nonlinear Systems with Stochastic Time Delays Using Neural Networks and Information Entropy

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Abstract: In this paper, a novel controller is proposed for discrete-time nonlinear systems with uncertain output-channel time delays using RBF neural networks and information entropy. The controller is designed by minimizing the quadratic Renyi entropy. The probability density function (PDF) of tracking error is estimated by Parzen windowing technique. The Jacobian information is estimated by an RBF neural network. The convergent condition of the proposed control algorithm is given. A simulation example is given to show the effectiveness of the proposed algorithm.

1. INTRODUCTION

It is well known that time-delay systems have been an active research area for the last few decades. There have been a great number of research results concerning time-delay systems in existed literature. The importance of the study on time-delay systems was further highlighted by the recent survey paper (Richard, 2003; Gu and Niculescu, 2003) and monographs (Gu, 2003; Niculescu, 2001). Nonlinear systems with uncertain time delays have strong background in practical engineering systems, among which networked control system (NCS) has been recognized to be a typical example. Modelling, analysis and synthesis of NCSs have been summarized in (Antsaklis and Baillieul, 2004; Antsaklis and Baillieul, 2007; Tipsuwan and Chow, 2003; Yang, 2006; Hespanha and Xu, 2007).

In this paper, we aim at designing a controller for nonlinear systems with an uncertain output delay component. Since the nonlinearities of practical processes are usually very complex, the proposed control scheme uses an radial basis function (RBF) neural network to obtain Jacobian information. In addition, because the uncertain delays in reality may not obey Gaussian distribution, the controller is designed by minimizing the Renyi entropy of tracking error instead of mean square error (MSE).

The innovation of this scheme can be summarized as follows: (1) the controller is designed for nonlinear systems with uncertain output delays by maximizing the quadratic information potential of tracking error; (2) the convergent condition of the proposed control algorithm is given.

The remainder of this paper is organized as follows: Section 2 formulates the problem. Section 3 introduces an error-entropy minimization algorithm for nonlinear discrete-time systems with one random delay component; Section 4 presents a simulation example to illustrate the efficiency and feasibility of the proposed approach. The last section concludes this paper.

2. PROBLEM FORMULATION

Consider a nonlinear system with uncertain output delays shown in Fig. 1, in which a nonlinear plant is controlled by an incremental controller. τ_{sc} is a random delay from sensor to controller. In practice, the probability density function (PDF) of random variables can be estimated by nonparametric methods.

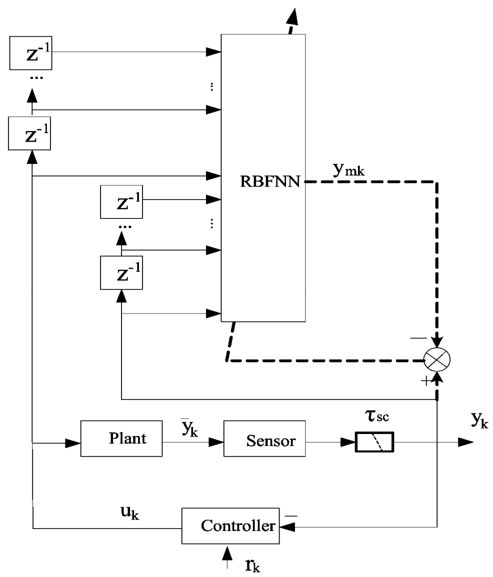

Fig. 1 Networked control system

In this paper, the discretized nonlinear ARMAX plant in NCSs can be described as

$$\bar{y}_k = f(\bar{y}_{k-1}, \bar{y}_{k-2}, \ldots, \bar{y}_{k-n_1}, u_k, u_{k-1}, \ldots, u_{k-m_1}) \tag{1}$$

The tracking error is $e_k = r_k - y_k$, r_k is the set-point of NCSs.

An LQG optimal controller was proposed in (Nilsson et al., 1998; Hu and Zhu, 2003). Under these linearity and Gaussianity assumptions, all possible statistical information of a signal can be extracted from its mean and variance. Although Gaussianity and linear modeling provide successful engineering solutions to most practical problems, it has become evident that when dealing with nonlinear systems, this approach needs to be refined (Pricipe et al. 2000; Erdogmus and Pricipe 2002a; Erdogmus and Pricipe2002b). Furthermore, the delays in practical engineering are usually non-Gaussian stochastic variables, such as the delays induced in NCSs with random access local area networks. Therefore, it is necessary to consider higher order statistical behavior of the systems and signals when design controllers. Recent papers have addressed this issue both in control literature (Wang, 2002; Yue and Wang, 2003; Feng et al., 1997) and in the signal processing and machine learning literature (Erdogmus and Pricipe, 2002; Fisher et al., 2000). It has been pointed out that minimizing the error entropy is equivalent to minimizing the distance between the probability distributions of the desired and system outputs (Erdogmus and Pricipe, 2002).

Renyi entropy with parameter α for random tracking error e with PDF $\gamma_e(e)$ is

$$H_\alpha(e) = \frac{1}{1-\alpha} \log \int_{-\infty}^{\infty} \gamma_e^\alpha(e) de \tag{2}$$

Renyi entropy shares the same extreme points with Shannon's definition for all values of α, i.e. its minimum value occurs at the δ-distribution, and the maximum value occurs at a uniform PDF (Erdogmus and Pricipe, 2001). It is necessary to non-parametrically estimate the PDF of tracking error from the samples when evaluating its entropy, since an analytical expression is not available in most cases.

The PDF of the tracking error in feedback control systems is estimated by Parzen windowing directly from the samples $\{e_1, e_2, \ldots, e_N\}$ using Gaussian kernel function, whose size is specified by the parameter σ, with the following expression:

$$\hat{\gamma}_e(\xi) = \frac{1}{N} \sum_{i=1}^{N} \kappa(\xi - e_i, \sigma^2) \tag{3}$$

where κ denotes the multidimensional Gaussian function with a radically symmetric variance σ^2 for simplicity, $\kappa(x, \sigma^2) = \frac{1}{\sqrt{2\pi}\sigma} \exp(\frac{-x^2}{2\sigma^2})$.

In this research, the quadratic entropy ($\alpha = 2$) is employed due to the computational efficiency of its nonparametric estimator. Substituting the Parzen Estimator in (2) with the quadratic entropy expression, we obtain

$$H_2(e) = -\log V(e) \tag{4}$$

$$V(e) = \int (\frac{1}{N} \sum_{i=1}^{N} \kappa(\xi - e_i, \sigma^2))^2 d\xi \tag{5}$$

where $V(e)$ named information potential in analogy with the potential field in physics (Erdogmus and Pricipe, 2002b), it can be calculated from the samples using Gaussian kernel.

$$V(e) = \frac{1}{N^2} \sum_{i=1}^{N} \sum_{j=1}^{N} \kappa(e_i - e_j, 2\sigma^2) \tag{6}$$

For the same reason, the derivative of $V(e)$ is named the information force $F(e) = \frac{\partial V(e)}{\partial e}$ (Erdogmus and Pricipe, 2002b).

Since Renyi's quadratic entropy is a monotonic function of the information potential $V(e)$, we can equivalently maximizing the information potential $V(e)$ instead of minimizing Renyi's entropy. As such, we present a novel performance index shown in (7) so as to design controller

$$J_k(\Delta u_k) = \frac{1}{V(e_k)} + \alpha(\Delta u_k)^2 \rightarrow \min \tag{7}$$

where $\Delta u_k = u_k - u_{k-1}$, $\alpha > 0$ is the control effort weighting factor.

3. CONTROLLER DESIGN

The control law can be obtained by minimizing the cost function in (7) using the gradient descent optimizing technique, i.e.

$$\Delta u_k = -\eta \frac{\partial J_k}{\partial u_k} \tag{8}$$

where $\eta > 0$ is the optimizing step.

$$\frac{\partial J_k}{\partial u_k} = -\frac{1}{V(e_k)^2} \frac{\partial V(e_k)}{\partial u_k} + 2\alpha \Delta u_k \tag{9}$$

$$\Delta u_k = \frac{\eta}{(1 + 2\eta\alpha) V(e_k)^2} \frac{\partial V(e_k)}{\partial u_k} \tag{10}$$

Due to $\frac{\partial e_k}{\partial u_k} = -\frac{\partial y_k}{\partial u_k}$, hence

$$\frac{\partial V(e_k)}{\partial u_k} = \frac{\partial V(e_k)}{\partial e_k} \frac{\partial e_k}{\partial u_k} = -\frac{\partial V(e_k)}{\partial e_k} \frac{\partial y_k}{\partial u_k}$$

$$\frac{\partial V(e_k)}{\partial u_k}=\frac{1}{2N^2\sigma^2}\sum_{i=1}^{N}\sum_{j=1}^{N}(e_j-e_i)\kappa(e_i-e_j,2\sigma^2)[\frac{\partial e_i}{\partial u_k}-\frac{\partial e_j}{\partial u_k}]$$
$$=\frac{1}{2N^2\sigma^2}\sum_{i=1}^{N}\sum_{j=1}^{N}(e_j-e_i)\kappa(e_i-e_j,2\sigma^2)[\frac{\partial y_j}{\partial u_k}-\frac{\partial y_i}{\partial u_k}] \quad (11)$$

The Jacobian information $\frac{\partial y_k}{\partial u_k}$ can be calculated by an RBF neural network shown in Fig. 1. The input vector of the RBF neural networks is $\boldsymbol{X}=[x_1,x_2,...,x_n]^T=[u_k,u_{k-1},...,u_{k-\bar{n}},y_k,y_{k-1},...,y_{k-\bar{m}}]^T$. The radial basis vector is $\boldsymbol{H}=[h_1,h_2,...,h_m]^T$, where the Gaussian basis function is

$$h_j=\exp(-\frac{\left\|\boldsymbol{X}-\boldsymbol{C}_j\right\|^2}{2b_j^2}),\ j=1,2...,m \quad (12)$$

The central vector of the jth node is $\boldsymbol{C}_j=[c_{j1},c_{j2},...,c_{ji},...,c_{jn}]^T,\ i=1,2,...,n$.

The output of RBF neural network is

$$y_{mk}=\sum_{j=1}^{m}w_jh_j \quad (13)$$

The performance index of training the RBF neural network is

$$J_1=\frac{1}{2}(y_k-y_{mk})^2 \quad (14)$$

The training algorithm of the RBF neural network can be obtained using gradient descent optimizing technique.

$$w_{j,k}=w_{j,k-1}+\bar{\eta}(y_k-y_{mk})h_j+\bar{\alpha}(w_{j,k-1}-w_{j,k-2}) \quad (15)$$

$$\Delta b_j=(y_k-y_{mk})w_{j,k-1}h_j\frac{\left\|\boldsymbol{X}-\boldsymbol{C}_j\right\|^2}{b_{j,k-1}^3} \quad (16)$$

$$b_{j,k}=b_{j,k-1}+\bar{\eta}\Delta b_j+\bar{\alpha}(b_{j,k-1}-b_{j,k-2}) \quad (17)$$

$$\Delta c_{ji}=(y_k-y_{mk})w_{j,k-1}h_j\frac{x_i-c_{ji}}{b_{j,k-1}^2} \quad (18)$$

$$c_{ji,k}=c_{ji,k-1}+\bar{\eta}\Delta c_{ji}+\bar{\alpha}(c_{ji,k-1}-c_{ji,k-2}) \quad (19)$$

where $\bar{\eta}$ is the learning rate, $\bar{\alpha}$ is the momentum factor. The Jacobian information is

$$\frac{\partial y_k}{\partial u_k}\approx\frac{\partial y_{mk}}{\partial u_k}=\sum_{j=1}^{m}w_{j,k}h_j\frac{c_{j1,k}-x_1}{b_{j,k}^2} \quad (20)$$

It should be noted that equation (20) is obtained under the condition $x_1=u_k$, i.e., u_k is the first node in the input layer of RBF neural networks.

The algorithm can therefore be summarized as follows:

Step 1: Perform Monte Carlo test at instant k and obtain the entropy of the tracking error e_k, the collected stochastic time delays is used to calculate a series of tracking error $\{e_1(k),e_2(k),...,e_N(k)\}$. The information potential of tracking error can be obtained by (6).

Step 2: Calculate the Jacobian information and solve the incremental controller. The Jacobian information $\frac{\partial y_j}{\partial u_k}$ in (11) can be calculated by (12-20). Δu_k is solved by (10) and (11) .

Step 3: Apply the control input $u_k=u_{k-1}+\Delta u_k$ to actuate the plant, set $k\leftarrow k+1$, and go back to step 1.

Remark 1 The kernel size (the window width of Parzen estimator) can be set experimentally after a preliminary analysis of the dynamic range of the tracking error.

Remark 2 Although the RBF neural network is trained by MSE criterion in this research, the performance index in (14) can be replaced by error-entropy minimization like that one in (7), the training rule of the RBF neural network can be obtained by minimizing the entropy of training error $\bar{e}_k=y_k-y_{mk}$.

Remark 3 The proposed controller is very simple. Even the Jacobian information $\frac{\partial y_k}{\partial u_k}$ can be approximated by $\mathrm{sgn}\frac{\partial y_k}{\partial u_k}$, obviously, the design of the controller is straightforward based on the optimization of maximizing the information potential of tracking error.

Remark 4 Since entropy does not change with the mean of the distribution, the proposed control algorithm may not yield zero-mean error. However, the problem could be solved if the proposed control law combines with integral control law.

Remark 5 Since the controller is directly designed based on minimizing entropy of the tracking error, it is not necessary to know the PDF of τ_{sc}. Moreover, the controller can deal with non-Gaussian noises existed in plants.

Theorem: If the parameter satisfies

$$\eta>0, \quad (21)$$

the control algorithm given by (10) will be convergent.

Proof:

Select the performance index in (7) to be a Lyapunov function, i.e.,

$$\pi = J(t) = \frac{1}{V(e(t))+\varepsilon} + \alpha(\Delta u(t))^2 , \tag{22}$$

where ε is an arbitrarily small positive constant to ensure the slightly modified performance index to be a Lyapunov function, then

$$\frac{\partial \pi}{\partial t} = \frac{\partial J(t)}{\partial u(t)} \frac{\partial u(t)}{\partial t} \tag{23}$$

Approximately, we have

$$\frac{\Delta \pi}{\Delta t} = \frac{\partial J(t)}{\partial u(t)} \frac{\Delta u(t)}{\Delta t} \tag{24}$$

Since the controller is designed by minimizing the performance index (7) with steepest descent approach, the above approach to design controller can be denoted in short

$$\Delta u(t) = -\eta \frac{\partial J(t)}{\partial u(t)} \tag{25}$$

From (24) and (25), we have

$$\Delta \pi = -\eta (\frac{\partial J(t)}{\partial u(t)})^2 \tag{26}$$

$$E(\Delta \pi) = E\left\{ -\eta (\frac{\partial J(t)}{\partial u(t)})^2 \right\} \tag{27}$$

Therefore, we obtain the condition for convergence:

4. SIMULATIONS

In order to show the applicability of the proposed control algorithm, let us consider the following feedback control system, which consists of the nonlinear plant represented as follows

$$\bar{y}_k = \frac{-0.15\bar{y}_{k-1} + u_{k-1}}{1+\bar{y}_{k-1}^2} \tag{28}$$

and an stochastic output delay τ_{sc} whose PDF is given to produce a non-Gaussian tracking error clearly. In practice, the PDF of tracking error can be calculated directly without knowing the PDF of τ_{sc}

$$\gamma_{\tau_{sc}}(x) = \begin{cases} [4^{\tilde{\alpha}+\tilde{\lambda}+1}\beta(\tilde{\alpha}+1,\tilde{\lambda}+1)]^{-1} x^{\tilde{\alpha}}(4-x)^{\tilde{\lambda}}, & x \in (0,4) \\ 0, & otherwise \end{cases} \tag{29}$$

where $\beta(\tilde{\alpha}+1,\tilde{\lambda}+1) = \int_0^1 x^{\tilde{\alpha}}(1-x)^{\tilde{\lambda}}dx \quad \tilde{\lambda},\tilde{\alpha} \geq -1$. In this simulation, $\tilde{\alpha}=1, \tilde{\lambda}=2$, the set-point of the feedback control system is set to $r(t)=1$, the sampling period $T=1$. In the simulation, the weights in (7) is $\alpha = 0.02$, the optimizing step in (8) η =0.0001, the kernel size used to estimate the entropy is experimentally set at σ = 0.5., the samples for controller is trained with a segment of N =500 at each instant. For the training of the 8-10-1 RBF neural network, set the learning rate $\bar{\eta} = 0.25$, the momentum factor $\bar{\alpha} = 0.05$, $\bar{n} = 1$ and $\bar{m} = 5$.

The simulation results are shown in Figs. 2-5. The control input and incremental control signal are illustrated in Fig. 2 respectively, it can be seen that the incremental control signal converges. The quadratic information potential of tracking error which is shown in Fig. 3 increases with time. From Fig. 2 and Fig. 3, it can be shown that the performance index decreases with time. The tracking error is shown in Fig. 4. In Fig. 5, both the range of tracking error and the PDF of tracking error are given, where it can be seen that the shape of PDF becomes narrower along with the increasing time. Both Figs. 4 and 5 indicate that the control system has a small uncertainty in its closed loop operation.

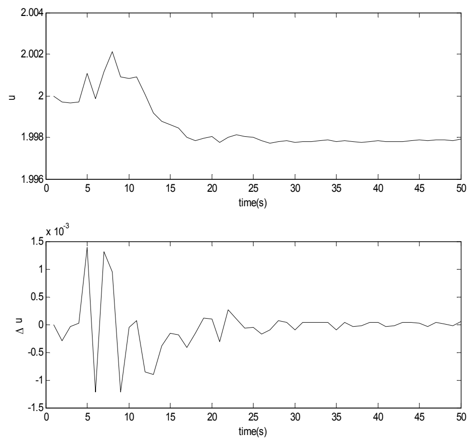

Fig. 2 The control input and incremental control signal

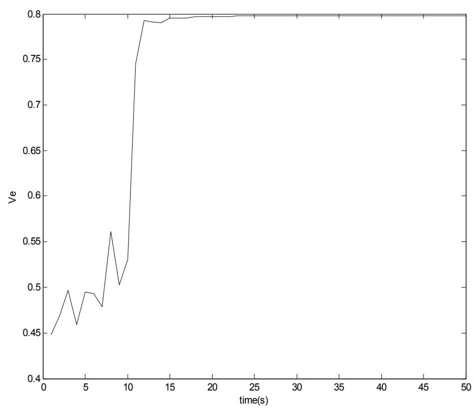

Fig. 3 The quadratic information potential of tracking error

5. CONCLUSIONS

The main results in this paper have the following features: (1) The controller is designed for nonlinear systems with uncertain output delays; (2) the design of the controller is straightforward based on maximizing the quadratic information potential of tracking error; (3) The convergent condition of the proposed control algorithm is given. Finally, a simulation example is given to show the effectiveness of the proposed algorithm.

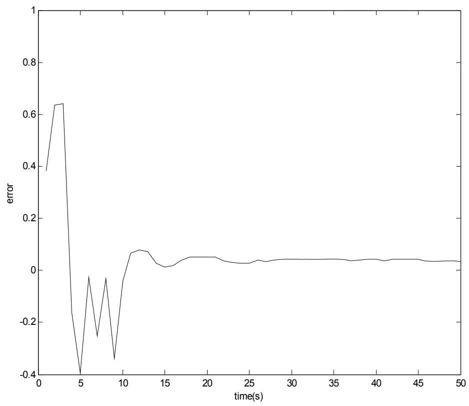

Fig. 4 The tracking error

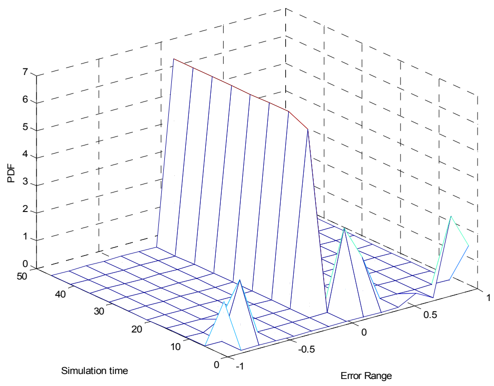

Fig. 5 PDF of the tracking error

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