Robust H_∞ Control For Neutral Systems Via Dynamic Output Feedback

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Abstract: This paper deals with the design problem for dynamic output feedback robust H_∞ control problem for a class of uncertain linear neutral systems in delay dependent case. Sufficient conditions for the existence of H_∞ controller is derived based on the delay dependent Bounded Real Lemma (BRL) in terms of linear matrix inequalities (LMIs) with inverse constraints which is obtained without resorting to any model transformations. A convex optimization algorithm is used to satisfy these constraints. A numerical example is given to illustrate the effectiveness of the proposed results.

1. INTRODUCTION

In recent years the stability and stabilization problem for time delay systems has received considerable attention and a lot of papers have been published dealing with this problem because physical or biological many control systems have time delay in their model representations (Mahmoud, 2000). The existence of delays is frequently is a source of instability and bad performance of the system. Therefore, the analysis of the stability of dynamic control systems with delays and synthesis of controllers for them in terms of state and output feedback has been largely treated in the literature (Niculescu and Verriest, 1998; Wu and She, 2004; Zhang *et al.*, 2005; Park, 2004).

Neutral systems are the general form of time delay systems which contain delays on the derivatives of some systems components such as states, inputs and outputs. Recently, sufficient conditions for asymptotic stability of such systems are given by various choice of Lyapunov-Krasovskii functional (Ivanescu *et al.*, 2003; Han *et al.*, 2004).

The standard H_∞ control problem is to obtain a controller which internally stabilizes the closed-loop system. Furthermore, it is required that the cost function (performance index) is less than zero. Since late 1980s, this problem has attracted much attention due to its both practical and theoretical importance and a great number of results have been reported for linear systems in terms of state feedback (Jiang and Han, 2005; Xu *et al.*, 2006) and in terms of output feedback (Baser, 2003; Fridman and Shaked 2002; Başer and Kızılsaç, 2007; Kızılsaç, 2007).

On another front of research, robust estimation methods have been recently developed for time delay systems which involves uncertain parameters in their components such states, inputs and outputs. Therefore, the stability analysis and stabilization of such systems has been largely treated in the literature (Wu and She, 2004 ; Park and Ko, 2007; Jiang and Han, 2006; Chen *et al.,* 2007; Palhares *et al.*, 2005; Xu *et al.*, 2003; Chen, 2007a). There are also great number of state feedback robust H_∞ control methods (Jiang and Han, 2005; Xu *et al.*, 2006) and robust H_∞ filtering methods (Park *et al.*, 2004; Xu *et al.*, 2005; Sun, 2006).

Recently there have been reported some results in terms of output feedback robust H_∞ control of linear delay systems in the case of a system which is not neutral with constant delay (Suplin and Shaked, 2007; Chen, 2007b), and with time varying delay (Xu and Chen, 2004). In (Başer and Kızılsaç, 2007) dynamic output feedback H_∞ control problem for linear neutral systems with time varying delays is considered but the system does not include uncertainties.

In this paper dynamic output feedback robust H_∞ control problem is considered for a class of uncertain linear neutral systems with time varying state delay in delay dependent case and no restriction on the derivative of the time varying delay is needed, which allows the time delay to be a fast time varying function. The results given in (Başer and Kızılsaç, 2007) are improved by adding uncertain parameters in to the system components. In section II, problem statement and preliminaries are presented. Extension of the ideas of the Bounded Real Lemma (Boyd *et al.*, 1994) is obtained for closed loop system in section III. It is noted that the BRL is obtained without resorting to any model transformations. Based on this BRL the delay dependent sufficient conditions for the existence of a dynamic output feedback H_∞ controller is obtained in terms of LMIs with inverse constraints in section IV, by using the method introduced in (Gahinet and Apkarian, 1994). An iterative algorithm involving convex optimization methods is stated in section V. Finally, a numerical example is presented to illustrate the results.

Through out the paper R^n denotes n-dimensional linear vector space and $R^{n\times m}$ is the set of all $n\times m$ matrices. $\mathbf{M}>0$

($\mathbf{M}<0$) stands for a symmetric and positive (negative) definite matrix $\mathbf{M}$. The space of functions in R^n that are square integrable over $[0,\infty]$ is denoted by $L_2^n[0,\infty)$ with the norm $\|\cdot\|_{L_2}$. The space of continuous functions $\varphi:[-\tau,0]\to R^n$ with the supremum norm $\|\cdot\|$ is denoted by $C_n[-\tau,0]$. Denote $\mathbf{x}_t(\theta)=\mathbf{x}(t+\theta)$, $\theta\in[-\tau,0]$.

2. PROBLEM STATEMENT AND PRELIMINARIES

Consider the n^{th} order uncertain linear time-invariant generalized neutral system described by the following equation:

$$\begin{aligned}&\dot{\mathbf{x}}(t)=\mathbf{A}_\Delta x(t)+\mathbf{A}_{1\Delta}\mathbf{x}(t-h(t))+\mathbf{E}_\Delta\dot{x}(t-d)+\mathbf{B}_1 w(t)+\mathbf{B}_2 u(t)\\&\mathbf{z}(t)=\mathbf{C}_1\mathbf{x}(t)+\mathbf{D}_{11}\mathbf{w}(t)+\mathbf{D}_{12}\mathbf{u}(t)\\&\mathbf{y}(t)=\mathbf{C}_2\mathbf{x}(t)+\mathbf{D}_{21}\mathbf{w}(t)+\mathbf{D}_{22}\mathbf{u}(t)\\&\mathbf{x}(t_0+\theta)=\varphi(\theta),\quad\forall\theta\in[-\tau,0],\quad(t_0,\varphi)\in R^+\times C_{\tau,n}\end{aligned}\tag{1}$$

$$\mathbf{A}_\Delta=\mathbf{A}+\Delta\mathbf{A}(t),\quad\mathbf{A}_{1\Delta}=\mathbf{A}_1+\Delta\mathbf{A}_1(t),\quad\mathbf{E}_\Delta=\mathbf{E}+\Delta\mathbf{E}(t)\tag{2}$$

where $\mathbf{x}\in R^n$ is the plant state, $\mathbf{w}\in R^q$ is the exogenous input, $\mathbf{u}\in R^m$ is the control input, $\mathbf{z}\in R^p$ is the regulated output, $\mathbf{y}\in R^l$ is the measured output, $\mathbf{A},\mathbf{E},\mathbf{A}_1,\mathbf{B}_i,\mathbf{C}_i,\mathbf{D}_{ij}$ for $i,j=1,2$ are known real constant matrices of appropriate dimensions, $\Delta\mathbf{A}(t),\Delta\mathbf{E}(t)$ and $\Delta\mathbf{A}_1(t)$ are the matrices of uncertain parameters of the form:

$$\left[\,\Delta\mathbf{A}(t)\;\Delta\mathbf{A}_1(t)\;\Delta\mathbf{E}(t)\right]=\mathbf{M}\mathbf{F}(t)\left[\,\mathbf{N}_\mathbf{A}\;\;\mathbf{N}_\mathbf{h}\;\;\mathbf{N}_\mathbf{E}\,\right]\tag{3}$$

$\mathbf{M},\mathbf{N}_\mathbf{A},\mathbf{N}_\mathbf{h},\mathbf{N}_\mathbf{E}$ are approximately dimensioned constant matrices and $\boldsymbol{F}(t)$ is an unknown real time varying matrix with Lebesque measurable elements satisfying

$$\mathbf{F}(t)^T\mathbf{F}(t)\le\mathbf{I}.\tag{4}$$

$h(t)$ is a time varying differentiable function satisfying

$$0<h(t)\le\bar{h}<\infty,\qquad\dot{h}(t)\le\mu<\infty\tag{5}$$

for all $t\ge0$. d is a positive constant delay $\tau:=\max\left\{\bar{h},d\right\}$ and $\varphi(\cdot)$ is a given continuously differentiable function on $[-\tau,0]$. $\mathbf{D}_{22}=0$ and this involves no loss of generality, while considerably simplifying algebraic manipulations (Doyle *et al.*, 1989). Consider a difference operator $\mu:C[-\tau,0]\to R^n$ given by $\mu(\mathbf{x}_t)=\mathbf{x}(t)-\mathbf{E}x(t-d)$. Also assume that:

Assumption 1. The triple $(\mathbf{A},\mathbf{B}_2,\mathbf{C}_2)$ is stablizable and detectable.

Assumption 2. $\|\mathbf{E}\|<1$, where $\|\cdot\|$ denotes any matrix norm. This assumption is the sufficient condition for the asymptotic stability of $\mu(\mathbf{x}_t)=0$, independently of all delays, see (Hale and Lunel, 1991).

In order to stabilize the system (1) consider the n_c^{th} order linear time invariant dynamic $(n_c>0)$ and static $(n_c=0)$ controller

$$\begin{aligned}&\dot{\mathbf{x}}_c(t)=\mathbf{K}_{21}\mathbf{y}(t)+\mathbf{K}_{22}\mathbf{x}_c(t)\\&\mathbf{u}(t)=\mathbf{K}_{11}\mathbf{y}(t)+\mathbf{K}_{12}\mathbf{x}_c(t)\end{aligned}\tag{6}$$

where $\mathbf{x}_c\in R^{n_c}$ is the controller state $\mathbf{K}_{11},\mathbf{K}_{12},\mathbf{K}_{21},\mathbf{K}_{22}$ are the constant matrices of appropriate dimensions. Let $\mathbf{x}_e(t)=[\mathbf{x}^T(t)\quad\mathbf{x}_c^T(t)]^T$. Then the closed loop system is obtained as:

$$\begin{aligned}&\dot{\mathbf{x}}_e(t)=\bar{\mathbf{A}}_\Delta\mathbf{x}_e(t)+\bar{\mathbf{A}}_{1\Delta}\mathbf{F}\mathbf{x}_e(t-h(t))+\bar{\mathbf{E}}_\Delta\mathbf{F}\dot{\mathbf{x}}_e(t-d)+\bar{\mathbf{B}}\mathbf{w}(t)\\&\mathbf{z}(t)=\bar{\mathbf{C}}\mathbf{x}_e(t)+\bar{\mathbf{D}}\mathbf{w}(t)\end{aligned}$$

where (7)

$$\begin{aligned}&\bar{\mathbf{A}}_\Delta=\hat{\mathbf{A}}+\hat{\mathbf{M}}\mathbf{F}(t)\hat{\mathbf{N}}_\mathbf{A}+\hat{\mathbf{B}}_2\mathbf{K}\hat{\mathbf{C}}_2,\ \bar{\mathbf{B}}=\hat{\mathbf{B}}_1+\hat{\mathbf{B}}_2\mathbf{K}\hat{\mathbf{D}}_{21},\\&\bar{\mathbf{C}}=\hat{\mathbf{C}}_1+\hat{\mathbf{D}}_{12}\mathbf{K}\hat{\mathbf{C}}_2,\ \bar{\mathbf{D}}=\mathbf{D}_{11}+\hat{\mathbf{D}}_{12}\mathbf{K}\hat{\mathbf{D}}_{21},\\&\bar{\mathbf{E}}_\Delta=\bar{\mathbf{E}}+\hat{\mathbf{M}}\mathbf{F}(t)\mathbf{N}_\mathbf{E},\quad\bar{\mathbf{A}}_{1\Delta}=\bar{\mathbf{A}}_1+\hat{\mathbf{M}}\mathbf{F}(t)\mathbf{N}_\mathbf{h},\\&\bar{\mathbf{A}}_1=\begin{bmatrix}\mathbf{A}_1^T&0\end{bmatrix}^T,\quad\bar{\mathbf{E}}=\begin{bmatrix}\mathbf{E}^T&0\end{bmatrix}^T,\quad\mathbf{F}=[\,\mathbf{I}\;\;0\,],\\&\hat{\mathbf{A}}=\begin{bmatrix}\mathbf{A}&0\\0&0\end{bmatrix},\quad\hat{\mathbf{B}}_1=\begin{bmatrix}\mathbf{B}_1\\0\end{bmatrix},\quad\hat{\mathbf{B}}_2=\begin{bmatrix}\mathbf{B}_2&0\\0&\mathbf{I}\end{bmatrix},\\&\hat{\mathbf{C}}_2=\begin{bmatrix}\mathbf{C}_2&0\\0&\mathbf{I}\end{bmatrix},\quad\hat{\mathbf{D}}_{21}=\begin{bmatrix}\mathbf{D}_{21}\\0\end{bmatrix},\quad\mathbf{K}=\begin{bmatrix}\mathbf{K}_{11}&\mathbf{K}_{12}\\\mathbf{K}_{21}&\mathbf{K}_{22}\end{bmatrix},\\&\hat{\mathbf{M}}=\begin{bmatrix}\mathbf{M}^T&0\end{bmatrix}^T,\hat{\mathbf{N}}_\mathbf{A}=\begin{bmatrix}\mathbf{N}_\mathbf{A}&0\end{bmatrix},\hat{\mathbf{C}}_1=[\mathbf{C}_1\;0],\hat{\mathbf{D}}_{12}=[\mathbf{D}_{12}\;0].\end{aligned}\tag{8}$$

For a prescribed scalar $\gamma>0$, define the performance index

$$J(\mathbf{w})=\int_0^\infty(\mathbf{z}^T\mathbf{z}-\gamma^2\mathbf{w}^T\mathbf{w})\,d\tau.\tag{9}$$

Definition 1: Given a scalar $\gamma>0$, the controller (6) for system (1) is said to be an H_∞- controller if the following conditions hold:

(i). The closed loop system (7) is asymptotically stable

(ii). Under zero initial condition, the closed-loop system satisfies $J(\mathbf{w})<0$, for all nonzero $\mathbf{w}\in L_2^q[0,\infty)$.

Before proceeding further, the following preliminary results should be given.

Lemma 1. (Park, 2004) For any constant symmetric positive matrix Θ, a positive scalar σ, and the vector function $\mathbf{w}:[0,\sigma]\to R^m$ such that the integrations in the following are well defined, then:

$$\sigma\int_0^\sigma\mathbf{w}^T(s)\Theta\,\mathbf{w}(s)ds\ge\left(\int_0^\sigma\mathbf{w}(s)\,ds\right)^T\Theta\left(\int_0^\sigma\mathbf{w}(s)\,ds\right).\tag{10}$$

Lemma 2. (Xu *et al.*, 2003) For any scalar $\varepsilon>0$, matrices $\mathbf{U}$ and $\mathbf{V}$ with appropriate dimensions

$$\mathbf{U}\mathbf{V}+\mathbf{V}^T\mathbf{U}^T\le\varepsilon\mathbf{U}\mathbf{U}^T+\varepsilon^{-1}\mathbf{V}^T\mathbf{V}.\tag{11}$$

Lemma 3. (Gahinet and Apkarian, 1994; Iwasaki and Skelton, 1994) Given a symmetric matrix Ω and two matrices Γ and Σ with appropriate dimensions. The inequality

$$\boldsymbol{\Omega}+\boldsymbol{\Sigma}^T\mathbf{K}\,\boldsymbol{\Gamma}+\boldsymbol{\Gamma}^T\mathbf{K}^T\,\boldsymbol{\Sigma}<0\tag{12}$$

is solvable for $\mathbf{K}$ if and only if

$$\mathbf{N}_\Gamma^T\Omega\mathbf{N}_\Gamma<0,\quad\mathbf{N}_\Sigma^T\Omega\mathbf{N}_\Sigma<0\tag{13}$$

where $\mathbf{N}_\Gamma$ and $\mathbf{N}_\Sigma$ denote the orthogonal complements of $\boldsymbol{\Gamma}$ and $\boldsymbol{\Sigma}$ respectively.

3. THE BOUNDED REAL LEMMA

Lemma 4. Suppose that Assumptions 1 and 2 hold. Given $\gamma > 0, \bar{h} > 0$ and μ, if there exist matrices $\bar{\mathbf{P}}^T = \bar{\mathbf{P}} > 0, \mathbf{W}^T = \mathbf{W} > 0, \mathbf{R}^T = \mathbf{R} > 0, \mathbf{Q}^T = \mathbf{Q} > 0$, $\mathbf{N}_1^T = \mathbf{N}_1 > 0$ and $\mathbf{N}_2^T = \mathbf{N}_2 > 0$ satisfying the following LMI, then the Closed Loop System given by (7) is asymptotically stable and the cost function (9) achieves $J(\mathbf{w}) < 0$ for all nonzero $\mathbf{w} \in L_2^q[0, \infty)$.

$$\bar{\mathbf{\Omega}}_\Delta = \begin{bmatrix} \bar{\mathbf{\Theta}}_1 & \bar{\mathbf{\Theta}}_2 & \bar{\mathbf{P}}\bar{\mathbf{E}}_\Delta & -\bar{h}\mathbf{F}^T\mathbf{N}_1 & \bar{\mathbf{P}}\bar{\mathbf{B}} & \bar{\mathbf{C}}^T & \bar{\mathbf{A}}_\Delta^T\mathbf{F}^T\boldsymbol{\phi} \\ * & \bar{\mathbf{\Theta}}_3 & 0 & -\bar{h}\mathbf{N}_2 & 0 & 0 & \bar{\mathbf{A}}_{1\Delta}^T\mathbf{F}^T\boldsymbol{\phi} \\ * & * & -\mathbf{R} & 0 & 0 & 0 & \bar{\mathbf{E}}_\Delta^T\mathbf{F}^T\boldsymbol{\phi} \\ * & * & * & -\bar{h}\mathbf{W} & 0 & 0 & 0 \\ * & * & * & * & -\gamma^2\mathbf{I} & \bar{\mathbf{D}}^T & \bar{\mathbf{B}}^T\mathbf{F}^T\boldsymbol{\phi} \\ * & * & * & * & * & -\mathbf{I} & 0 \\ * & * & * & * & * & * & -\boldsymbol{\phi} \end{bmatrix} < 0 \tag{14}$$

with

$$\bar{\mathbf{\Theta}}_1 = \bar{\mathbf{P}}\bar{\mathbf{A}}_\Delta + \bar{\mathbf{A}}_\Delta^T\bar{\mathbf{P}} + \bar{\mathbf{Q}} + \bar{\mathbf{N}}_1 + \bar{\mathbf{N}}_1^T, \quad \bar{\mathbf{\Theta}}_2 = \bar{\mathbf{P}}\bar{\mathbf{A}}_{1\Delta} - \mathbf{F}^T\mathbf{N}_1 + \mathbf{F}^T\mathbf{N}_2^T$$

$$\bar{\mathbf{\Theta}}_3 = -(1-\mu)\mathbf{Q} - \mathbf{N}_2 - \mathbf{N}_2^T, \quad \boldsymbol{\phi} = \mathbf{R} + \bar{h}\mathbf{W}$$

$\boldsymbol{\phi} = \mathbf{R} + \bar{h}\mathbf{W}$ with $\bar{\boldsymbol{\phi}} = \mathbf{F}^T\boldsymbol{\phi}\mathbf{F}$ and $\bar{\mathbf{N}}_1 = \mathbf{F}^T\mathbf{N}_1\mathbf{F}$.

Proof. Consider the following Lyapunov-Krasovskii functional

$$V(\mathbf{x}_t) = \mathbf{x}_e^T(t)\bar{\mathbf{P}}\mathbf{x}_e(t) + \int_{t-h(t)}^{t} \mathbf{x}_e^T(s)\bar{\mathbf{Q}}\mathbf{x}_e(s)ds + \int_{t-d}^{t} \dot{\mathbf{x}}_e^T(s)\bar{\mathbf{R}}\dot{\mathbf{x}}_e(s)ds$$
$$+ \int_{-\bar{h}}^{0}\int_{t+\theta}^{t} \dot{\mathbf{x}}_e^T(s)\bar{\mathbf{W}}\dot{\mathbf{x}}_e(s)dsd\theta$$

$$\bar{\mathbf{Q}} = \mathbf{F}^T\mathbf{Q}\mathbf{F}, \quad \bar{\mathbf{W}} = \mathbf{F}^T\mathbf{W}\mathbf{F}, \quad \bar{\mathbf{R}} = \mathbf{F}^T\mathbf{R}\mathbf{F} \tag{15}$$

the time derivative of this functional is given as follows:

$$\dot{V}(\mathbf{x}_t) = 2\mathbf{x}_e^T(t)\bar{\mathbf{P}}\dot{\mathbf{x}}_e(t) + \mathbf{x}_e^T(t)\bar{\mathbf{Q}}\mathbf{x}_e(t) - \int_{t-\bar{h}}^{t} \dot{\mathbf{x}}_e^T(s)\bar{\mathbf{W}}\dot{\mathbf{x}}_e(s)ds$$
$$-(1-\dot{h}(t))\mathbf{x}_e^T(t-h(t))\bar{\mathbf{Q}}\mathbf{x}_e(t-h(t)) + \dot{\mathbf{x}}_e^T(t)\bar{\mathbf{R}}\dot{\mathbf{x}}_e(t)$$
$$+\bar{h}\dot{\mathbf{x}}_e^T(t)\bar{\mathbf{W}}\dot{\mathbf{x}}_e(t) - \dot{\mathbf{x}}_e^T(t-d)\bar{\mathbf{R}}\dot{\mathbf{x}}_e(t-d). \tag{16}$$

Since $0 < h(t) \le \bar{h}$ it is clear that

$$-\int_{t-\bar{h}}^{t} \dot{\mathbf{x}}_e^T(s)\bar{\mathbf{W}}\dot{\mathbf{x}}_e(s)ds \le -\int_{t-h(t)}^{t} \dot{\mathbf{x}}_e^T(s)\bar{\mathbf{W}}\dot{\mathbf{x}}_e(s)ds. \tag{17}$$

By considering Lemma 1, system (7), the inequality (17) and $\dot{h}(t) \le \mu$, the inequality

$$\dot{V}(\mathbf{x}_t) \le 2\mathbf{x}_e^T(t)\bar{\mathbf{P}}(\bar{\mathbf{A}}_\Delta\mathbf{x}_e(t) + \bar{\mathbf{A}}_{1\Delta}\mathbf{F}\mathbf{x}_e(t-h(t)) + \bar{\mathbf{E}}_\Delta\mathbf{F}\dot{\mathbf{x}}_e(t-d)$$
$$+\bar{\mathbf{B}}\mathbf{w}(t)) + \mathbf{x}_e^T(t)\bar{\mathbf{Q}}\mathbf{x}_e(t) - (1-\mu)\mathbf{x}_e^T(t-h(t))\bar{\mathbf{Q}}\mathbf{x}_e(t-h(t))$$
$$-\dot{\mathbf{x}}_e^T(t-d)\bar{\mathbf{R}}\dot{\mathbf{x}}_e(t-d) - \frac{1}{\bar{h}}\int_{t-h(t)}^{t} \dot{\mathbf{x}}_e^T(s)ds(\bar{h}\bar{\mathbf{W}})\frac{1}{\bar{h}}\int_{t-h(t)}^{t} \dot{\mathbf{x}}_e(s)ds$$
$$+(\bar{\mathbf{A}}_\Delta\mathbf{x}_e(t) + \bar{\mathbf{A}}_{1\Delta}\mathbf{F}\mathbf{x}_e(t-h(t)) + \bar{\mathbf{E}}_\Delta\mathbf{F}\dot{\mathbf{x}}_e(t-d)$$
$$+\bar{\mathbf{B}}\mathbf{w}(t))^T(\bar{h}\bar{\mathbf{W}} + \bar{\mathbf{R}}) \times (\bar{\mathbf{A}}_\Delta\mathbf{x}_e(t) + \bar{\mathbf{A}}_{1\Delta}\mathbf{F}\mathbf{x}_e(t-h(t))$$
$$+\bar{\mathbf{E}}_\Delta\mathbf{F}\dot{\mathbf{x}}_e(t-d) + \bar{\mathbf{B}}\mathbf{w}(t)) \tag{18}$$

can be obtained.

It is required that the associated Hamiltonian $H(\mathbf{x}_t, \mathbf{w}, t)$ satisfies

$$H(\mathbf{x}_t, \mathbf{w}, t) = \dot{V}(\mathbf{x}_t) + \mathbf{z}^T(t)\mathbf{z}(t) - \gamma^2\mathbf{w}^T(t)\mathbf{w}(t) < 0. \tag{19}$$

By adding the following term on the right hand side of (18)

$$2(\mathbf{x}_e^T(t)\mathbf{F}^T\mathbf{N}_1 + \mathbf{x}^T(t-h(t))\mathbf{N}_2)(\mathbf{x}(t) - \mathbf{x}(t-h(t))$$
$$-\int_{t-h(t)}^{t} \dot{\mathbf{x}}(s)ds) = 0$$

and the fact $\mathbf{F}\mathbf{x}_e(t) = \mathbf{x}(t)$ it is easy to obtain the right hand side of the equation (19) in matrix form in terms of the vector

$$\boldsymbol{\eta}^T(t) = \left[\ \mathbf{x}_e^T(t), \mathbf{x}^T(t-h(t)), \dot{\mathbf{x}}^T(t-d), \frac{1}{\bar{h}}\int_{t-h(t)}^{t} \dot{\mathbf{x}}^T(s)ds, \mathbf{w}^T(t)\ \right]. \tag{20}$$

and to obtain the result by well known Schur Complement (Baser, 2003).

4. H_∞ CONTROLLER DESIGN

In this section a computational method will be introduced to obtain static or dynamic output feedback H_∞ controller.

Theorem 1. Suppose that Assumptions 1 and 2 hold. Given $\gamma > 0, \bar{h} > 0, \mu$, ε, the matrices $\mathbf{N}_1^T = \mathbf{N}_1 > 0$, $\mathbf{N}_2^T = \mathbf{N}_2 > 0$ and the matrices $\mathbf{N}_\mathbf{A}, \mathbf{N}_\mathbf{h}, \mathbf{N}_\mathbf{E}, \mathbf{M}$, if there exist matrices

$$\mathbf{X}^T = \mathbf{X} > 0, \mathbf{Y}^T = \mathbf{Y} > 0, \mathbf{W}^T = \mathbf{W} > 0, \boldsymbol{\phi}^T = \boldsymbol{\phi} > 0,$$
$$\tilde{\boldsymbol{\phi}}^T = \tilde{\boldsymbol{\phi}} > 0, \mathbf{Q}^T = \mathbf{Q} > 0, \tilde{\mathbf{Q}}^T = \tilde{\mathbf{Q}} > 0$$

satisfying the following LMIs, then the closed loop system (7) is asymptotically stable and the cost function (9) achieves $J(\mathbf{w}) < 0$ for all nonzero $\mathbf{w} \in L_2^q[0, \infty)$.

$$\mathbf{N}_\Gamma^T\tilde{\mathbf{\Omega}}_\Delta\mathbf{N}_\Gamma < 0, \quad \mathbf{N}_\Sigma^T\hat{\mathbf{\Omega}}_\Delta\mathbf{N}_\Sigma < 0, \tag{21}$$

$$\begin{bmatrix} \mathbf{Y} & I \\ I & \mathbf{X} \end{bmatrix} > 0 \tag{22}$$

where

$$\mathbf{\Gamma} = \begin{bmatrix} \mathbf{C}_2 & 0 & 0 & 0 & \mathbf{D}_{21} & 0 & 0 & 0 & 0 \end{bmatrix}, \tag{23}$$

$$\mathbf{\Sigma} = \begin{bmatrix} \mathbf{B}_2^T & 0 & 0 & 0 & 0 & \mathbf{D}_{12}^T & \mathbf{B}_2^T & 0 & 0 & 0 & 0 \end{bmatrix}, \tag{24}$$

$\mathbf{N}_\Gamma$ and $\mathbf{N}_\Sigma$ denote the orthogonal complements of $\mathbf{\Gamma}$ and $\mathbf{\Sigma}$ respectively,

$$\tilde{\mathbf{\Omega}}_\Delta = \begin{bmatrix} \tilde{\mathbf{\Theta}}_1 & \tilde{\mathbf{\Theta}}_2 & \mathbf{X}\mathbf{E} & -\bar{h}\mathbf{N}_1 & \mathbf{X}\mathbf{B}_1 & \mathbf{C}_1^T & \mathbf{A}^T\boldsymbol{\phi} & \varepsilon\mathbf{N}_\mathbf{A}^T & \mathbf{X}\mathbf{M} \\ * & \tilde{\mathbf{\Theta}}_3 & 0 & -\bar{h}\mathbf{N}_2 & 0 & 0 & \mathbf{A}_1^T\boldsymbol{\phi} & \varepsilon\mathbf{N}_\mathbf{h}^T & 0 \\ * & * & -\mathbf{R} & 0 & 0 & 0 & \mathbf{E}^T\boldsymbol{\phi} & \varepsilon\mathbf{N}_\mathbf{E}^T & 0 \\ * & * & * & -\bar{h}\mathbf{W} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -\gamma^2\mathbf{I} & \mathbf{D}_{11}^T & \mathbf{B}_1^T\boldsymbol{\phi} & 0 & 0 \\ * & * & * & * & * & -\mathbf{I} & 0 & 0 & 0 \\ * & * & * & * & * & * & -\boldsymbol{\phi} & 0 & \boldsymbol{\phi}\mathbf{M} \\ * & * & * & * & * & * & * & -\varepsilon\mathbf{I} & 0 \\ * & * & * & * & * & * & * & * & -\varepsilon\mathbf{I} \end{bmatrix} \tag{25}$$

$$\tilde{\mathbf{\Theta}}_1 = \mathbf{XA} + \mathbf{A}^T\mathbf{X} + \mathbf{Q} + \mathbf{N}_1 + \mathbf{N}_1^T,$$
$$\tilde{\mathbf{\Theta}}_2 = \mathbf{XA}_1 - \mathbf{N}_1 + \mathbf{N}_2^T,$$
$$\tilde{\mathbf{\Theta}}_3 = -(1-\mu)\mathbf{Q} - \mathbf{N}_2 - \mathbf{N}_2^T,$$

$$\hat{\mathbf{\Omega}}_\Delta = \begin{bmatrix} \hat{\mathbf{\Theta}}_1 & \hat{\mathbf{\Theta}}_2 & \mathbf{E} & -\bar{h}\mathbf{YN}_1 & \mathbf{B}_1 & \mathbf{YC}_1^T & \mathbf{YA}^T & \varepsilon\mathbf{YN}_A^T & \mathbf{M} & \mathbf{Y} & \mathbf{Y} \\ * & \hat{\mathbf{\Theta}}_3 & 0 & -\bar{h}\mathbf{N}_2 & 0 & 0 & \mathbf{A}_1^T & \varepsilon\mathbf{N}_h^T & 0 & 0 & 0 \\ * & * & -\mathbf{R} & 0 & 0 & 0 & \mathbf{E}^T & \varepsilon\mathbf{N}_E^T & 0 & 0 & 0 \\ * & * & * & -\bar{h}\mathbf{W} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -\gamma^2\mathbf{I} & \mathbf{D}_{11}^T & \mathbf{B}_1^T & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -\mathbf{I} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -\tilde{\phi} & 0 & \mathbf{M} & 0 & 0 \\ * & * & * & * & * & * & * & -\varepsilon\mathbf{I} & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & -\varepsilon\mathbf{I} & 0 & 0 \\ * & * & * & * & * & * & * & * & * & -\tilde{\mathbf{Q}} & 0 \\ * & * & * & * & * & * & * & * & * & * & -\frac{1}{2}\mathbf{N}_1^{-1} \end{bmatrix} \tag{26}$$

$$\hat{\mathbf{\Theta}}_1 = \mathbf{AY} + \mathbf{YA}^T,$$
$$\hat{\mathbf{\Theta}}_2 = \mathbf{A}_1 - \mathbf{YN}_1 + \mathbf{YN}_2^T,$$
$$\hat{\mathbf{\Theta}}_3 = -(1-\mu)\mathbf{Q} - \mathbf{N}_2 - \mathbf{N}_2^T,$$

and

$$\mathbf{XY} = \mathbf{I}, \quad \mathbf{Q}\tilde{\mathbf{Q}} = \mathbf{I}, \quad \phi\tilde{\phi} = \mathbf{I}\,. \tag{27}$$

In that case, an explicit description of output feedback $\mathbf{K}$ corresponding to a feasible solution $\mathbf{X}^T = \mathbf{X} > 0$, $\mathbf{Y}^T = \mathbf{Y} > 0$, $\mathbf{W}^T = \mathbf{W} > 0$, $\phi^T = \phi > 0$, $\tilde{\phi}^T = \tilde{\phi} > 0$, $\mathbf{Q}^T = \mathbf{Q} > 0$, $\tilde{\mathbf{Q}}^T = \tilde{\mathbf{Q}} > 0$ to (21) and (22) is given as

$$\mathbf{K} = -\rho\,\hat{\mathbf{\Sigma}}\mathbf{V}\hat{\mathbf{\Gamma}}^T(\hat{\mathbf{\Gamma}}\,\mathbf{V}\hat{\mathbf{\Gamma}}^T)^{-1} + \mathbf{S}^{\frac{1}{2}}\mathbf{L}(\hat{\mathbf{\Gamma}}\,\mathbf{V}\hat{\mathbf{\Gamma}}^T)^{-\frac{1}{2}} \tag{28}$$

where

$$\mathbf{V} = (\hat{\mathbf{\Sigma}}^T\hat{\mathbf{\Sigma}} - \frac{1}{\rho}\mathbf{\Omega}_\Delta)^{-1} > 0,$$
$$\mathbf{S} = \mathbf{I} - \hat{\mathbf{\Sigma}}(\mathbf{V} - \mathbf{V}\hat{\mathbf{\Gamma}}^T(\hat{\mathbf{\Gamma}}\,\mathbf{V}\hat{\mathbf{\Gamma}}^T)^{-1}\hat{\mathbf{\Gamma}}\mathbf{V})\hat{\mathbf{\Sigma}}^T, \tag{29}$$
$$\hat{\mathbf{\Gamma}} = \begin{bmatrix} \hat{\mathbf{C}}_2 & 0 & 0 & 0 & \hat{\mathbf{D}}_{21} & 0 & 0 & 0 & 0 \end{bmatrix},$$
$$\hat{\mathbf{\Sigma}} = \begin{bmatrix} \hat{\mathbf{B}}_2^T\bar{\mathbf{P}} & 0 & 0 & 0 & 0 & \hat{\mathbf{D}}_{12}^T & \hat{\mathbf{B}}_2^T\mathbf{F}^T\phi & 0 & 0 \end{bmatrix},$$

$$\mathbf{\Omega}_\Delta = \begin{bmatrix} \mathbf{\Theta}_1 & \mathbf{\Theta}_2 & \bar{\mathbf{P}}\mathbf{E} & -\bar{h}\mathbf{F}^T\mathbf{N}_1 & \bar{\mathbf{P}}\hat{\mathbf{B}}_1 & \hat{\mathbf{C}}_1^T & \hat{\mathbf{A}}^T\mathbf{F}^T\phi & \varepsilon\hat{\mathbf{N}}_A^T & \bar{\mathbf{P}}\hat{\mathbf{M}} \\ * & \mathbf{\Theta}_3 & 0 & -\bar{h}\mathbf{N}_2 & 0 & 0 & \bar{\mathbf{A}}_1^T\mathbf{F}^T\phi & \varepsilon\mathbf{N}_h^T & 0 \\ * & * & -\mathbf{R} & 0 & 0 & 0 & \bar{\mathbf{E}}^T\mathbf{F}^T\phi & \varepsilon\mathbf{N}_E^T & 0 \\ * & * & * & -\bar{h}\mathbf{W} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -\gamma^2\mathbf{I} & \mathbf{D}_{11}^T & \hat{\mathbf{B}}_1^T\mathbf{F}^T\phi & 0 & 0 \\ * & * & * & * & * & -\mathbf{I} & 0 & 0 & 0 \\ * & * & * & * & * & * & -\phi & 0 & \phi\mathbf{F}\hat{\mathbf{M}} \\ * & * & * & * & * & * & * & -\varepsilon\mathbf{I} & 0 \\ * & * & * & * & * & * & * & * & -\varepsilon\mathbf{I} \end{bmatrix} \tag{30}$$

$$\mathbf{\Theta}_1 = \bar{\mathbf{P}}\hat{\mathbf{A}} + \hat{\mathbf{A}}^T\bar{\mathbf{P}} + \bar{\mathbf{Q}} + \bar{\mathbf{N}}_1 + \bar{\mathbf{N}}_1^T,$$
$$\mathbf{\Theta}_2 = \bar{\mathbf{P}}\bar{\mathbf{A}}_1 - \mathbf{F}^T\mathbf{N}_1 + \mathbf{F}^T\mathbf{N}_2^T,$$
$$\mathbf{\Theta}_3 = -(1-\mu)\mathbf{Q} - \mathbf{N}_2 - \mathbf{N}_2^T,$$

ρ and $\boldsymbol{L}$ are free parameters satisfying $\|\mathbf{L}\| < \rho$. Note that the parametrization of $\mathbf{K}$ is given in (Gahinet and Apkarian, 1994; Iwasaki and Skelton, 1994).

Proof. By Lemma 4., the system (6) defines H_∞ - controller if $\bar{\mathbf{\Omega}}_\Delta < 0$. By the closed loop system matrices given in (8) it is easy to see that $\bar{\mathbf{\Omega}}_\Delta < 0$ is equivalent to

$$\boldsymbol{\psi} + \mathbf{UV} + \mathbf{V}^T\mathbf{U}^T + \boldsymbol{\alpha}^T\mathbf{K}\boldsymbol{\beta} + \boldsymbol{\beta}^T\mathbf{K}^T\boldsymbol{\alpha} < 0 \tag{31}$$

where

$$\boldsymbol{\psi} = \begin{bmatrix} \boldsymbol{\psi}_1 & \boldsymbol{\psi}_2 & \bar{\mathbf{P}}\mathbf{E} & -h\mathbf{F}^T\mathbf{N}_1 & \bar{\mathbf{P}}\hat{\mathbf{B}}_1 & \hat{\mathbf{C}}_1^T & \hat{\mathbf{A}}^T\mathbf{F}^T\phi \\ * & \boldsymbol{\psi}_3 & 0 & -\bar{h}\mathbf{N}_2 & 0 & 0 & \bar{\mathbf{A}}_1^T\mathbf{F}^T\phi \\ * & * & -\mathbf{R} & 0 & 0 & 0 & \bar{\mathbf{E}}^T\mathbf{F}^T\phi \\ * & * & * & -\bar{h}\mathbf{W} & 0 & 0 & 0 \\ * & * & * & * & -\gamma^2\mathbf{I} & \mathbf{D}_{11}^T & \hat{\mathbf{B}}_1^T\mathbf{F}^T\phi \\ * & * & * & * & * & -\mathbf{I} & 0 \\ * & * & * & * & * & * & -\phi \end{bmatrix} \tag{32}$$

$$\boldsymbol{\psi}_1 = \bar{\mathbf{P}}\hat{\mathbf{A}} + \hat{\mathbf{A}}^T\bar{\mathbf{P}} + \bar{\mathbf{Q}} + \bar{\mathbf{N}}_1 + \bar{\mathbf{N}}_1^T,$$
$$\boldsymbol{\psi}_2 = \bar{\mathbf{P}}\bar{\mathbf{A}}_1 - \mathbf{F}^T\mathbf{N}_1 + \mathbf{F}^T\mathbf{N}_2^T,$$
$$\boldsymbol{\psi}_3 = -(1-\mu)\mathbf{Q} - \mathbf{N}_2 - \mathbf{N}_2^T,$$

$$\mathbf{U} = \begin{bmatrix} \hat{\mathbf{M}}^T\bar{\mathbf{P}} & 0 & 0 & 0 & 0 & 0 & \hat{\mathbf{M}}^T\mathbf{F}^T\phi \end{bmatrix}^T,$$
$$\mathbf{V} = \mathbf{F}(t)\begin{bmatrix} \hat{\mathbf{N}}_A & \mathbf{N}_h & \mathbf{N}_E & 0 & 0 & 0 & 0 \end{bmatrix}, \tag{33}$$

and

$$\boldsymbol{\beta} = \begin{bmatrix} \hat{\mathbf{C}}_2 & 0 & 0 & 0 & \hat{\mathbf{D}}_{21} & 0 & 0 \end{bmatrix},$$
$$\boldsymbol{\alpha} = \begin{bmatrix} \hat{\mathbf{B}}_2^T\bar{\mathbf{P}} & 0 & 0 & 0 & 0 & \hat{\mathbf{D}}_{12}^T & \hat{\mathbf{B}}_2^T\mathbf{F}^T\phi \end{bmatrix}, \tag{34}$$

By using Lemma 2., inequality (4) and well known Shur Complement it can be shown that the inequality (31) is equivalent to

$$\mathbf{\Omega}_\Delta + \hat{\mathbf{\Sigma}}^T\mathbf{K}\,\hat{\mathbf{\Gamma}} + \hat{\mathbf{\Gamma}}^T\mathbf{K}^T\,\hat{\mathbf{\Sigma}} < 0 \tag{35}$$

where $\mathbf{\Omega}_\Delta$, $\hat{\mathbf{\Gamma}}$ and $\hat{\mathbf{\Sigma}}$ are the matrices given in (29) and (30). According to Lemma 3. the inequality (35) is equivalent to the following inequalities

$$\mathbf{N}_{\hat{\Gamma}}^T\mathbf{\Omega}_\Delta\mathbf{N}_{\hat{\Gamma}} < 0, \quad \mathbf{N}_{\hat{\Sigma}}^T\mathbf{\Omega}_\Delta\mathbf{N}_{\hat{\Sigma}} < 0 \tag{36}$$

where $\mathbf{N}_{\hat{\Gamma}}$ and $\mathbf{N}_{\hat{\Sigma}}$ denote the orthogonal complements of $\hat{\mathbf{\Gamma}}$ and $\hat{\mathbf{\Sigma}}$, respectively.

The inequalities (36) are in the same form of with the inequalities (42) in the proof of the Theorem 1 in (Başer and Kızılsaç, 2007). At this step the proof can be completed by the same method in the proof of the Theorem 1 in (Başer and Kızılsaç, 2007).

It should be noted that since the LMIs in (21) are convex, but LMI (22) with inverse constraints in (27) is non-convex, so the cone complementary linearization (CCL) algorithm given in (Ghaoui *et al.*, 1997) should be used. Thus, let

$$\begin{bmatrix} \mathbf{X} & \mathbf{I} \\ \mathbf{I} & \mathbf{Y} \end{bmatrix} \geq 0, \quad \begin{bmatrix} \mathbf{Q} & \mathbf{I} \\ \mathbf{I} & \tilde{\mathbf{Q}} \end{bmatrix} \geq 0, \quad \begin{bmatrix} \phi & \mathbf{I} \\ \mathbf{I} & \tilde{\phi} \end{bmatrix} \geq 0. \qquad (37)$$

By CCL algorithm, suggest the following nonlinear minimization problem:

$$\underset{\text{subject to (21) and (37)}}{Minimize\ trace}\ (\mathbf{X}\mathbf{Y} + \mathbf{Q}\tilde{\mathbf{Q}} + \phi\tilde{\phi})$$

If the solution of the above minimization problem is $3n$, that is

$$trace\ (\mathbf{X}\mathbf{Y} + \mathbf{Q}\tilde{\mathbf{Q}} + \phi\tilde{\phi}) = 3n$$

then the solution is feasible and

$$\mathbf{XY} = \mathbf{Q}\tilde{\mathbf{Q}} = \phi\tilde{\phi} = \mathbf{I}_n .$$

Although it is impossible to find the global optimal solution, the proposed nonlinear minimization problem is easier than the non-convex feasibility problem in Theorem 1. In terms of CCL method the suboptimal maximal delay can easily be found using an iterative algorithm presented in section V.

Remark1. It is well known that the stability results are less conservative as the number of parameters increase in the LMIs which take part in the sufficient conditions. To design an output feedback is more complicated than to design a state feedback as the parameters increase. In the dynamic output feedback solutions, the number of LMIs are greater than the number of LMIs in the state feedback solutions. That's why the results related to state feedback is less conservative than the results related to output feedback H_∞ control problem. On the other hand it is more practical to use dynamic output feedback because of the use of measured output data.

Remark2 .Here, the Lyapunov-Krasovskii functional given in (Başer and Kızılsaç, 2007) is revised in order to remove the model transformation and add uncertain parameters in to the system components. This allows to obtain better results than (Başer and Kızılsaç, 2007) as in the following example.

5. ALGORITHM AND EXAMPLE

(i). Find a feasible set

$$\left(\mathbf{X}_0, \mathbf{Y}_0, \mathbf{Q}_0, \tilde{\mathbf{Q}}_0, \phi_0, \tilde{\phi}_0, \mathbf{W}_0 \right)$$

satisfying the inequalities in (21) and (37). Set $k = 0$.

(ii). Solve the following LMI problem for the variables $\left(\mathbf{X}, \mathbf{Y}, \mathbf{Q}, \tilde{\mathbf{Q}}, \phi, \tilde{\phi}, \mathbf{W} \right)$

$$\underset{\text{Subject to (21) and (37)}}{Minimize\ trace} \left(\mathbf{X}_k\mathbf{Y} + \mathbf{Y}_k\mathbf{X} + \mathbf{Q}_k\tilde{\mathbf{Q}} + \tilde{\mathbf{Q}}_k\mathbf{Q} + \phi_k\tilde{\phi} + \tilde{\phi}_k\phi \right)$$

(iii). If a stopping criterion is satisfied, exit. Otherwise set $\mathbf{X}_{k+1} = \mathbf{X},\ \mathbf{Y}_{k+1} = \mathbf{Y},\ \mathbf{Q}_{k+1} = \mathbf{Q},\ \tilde{\mathbf{Q}}_{k+1} = \tilde{\mathbf{Q}},\ \phi_{k+1} = \phi,\ \tilde{\phi}_{k+1} = \tilde{\phi}$ set k=k+1 and go to step 2.

Example. Consider the system (1) with

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad \mathbf{E} = \begin{bmatrix} 0.36 & 0.25 \\ -0.1 & 0.2 \end{bmatrix}, \quad \mathbf{A}_1 = \begin{bmatrix} -0.1 & 0.1 \\ 0.2 & 0 \end{bmatrix},$$

$$\mathbf{B}_1 = \begin{bmatrix} 0 \\ 2 \end{bmatrix}, \quad \mathbf{B}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad \mathbf{C}_1 = \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad \mathbf{C}_2 = \begin{bmatrix} 1 & 1 \end{bmatrix},$$

$$\begin{bmatrix} \mathbf{D}_{11} & \mathbf{D}_{12} \\ \mathbf{D}_{21} & \mathbf{D}_{22} \end{bmatrix} = \begin{bmatrix} 0.5 & 0 \\ 0.25 & 0 \end{bmatrix},$$

$$\mathbf{M} = \begin{bmatrix} 0 \\ 0.5 \end{bmatrix}, \quad \mathbf{N}_A = \begin{bmatrix} 0.1 & 0 \end{bmatrix}, \quad \mathbf{N}_h = \begin{bmatrix} 0.1 & 0.2 \end{bmatrix}, \quad \mathbf{N}_E = \begin{bmatrix} 0.2 & 0.2 \end{bmatrix}.$$

Let $\bar{h} = 3.7, \quad \varepsilon = 2.89, \quad \mathbf{N}_1 = \begin{bmatrix} 1.5 & 1 \\ 1 & 1.5 \end{bmatrix}, \quad \mathbf{N}_2 = \begin{bmatrix} 2.5 & 1 \\ 1 & 2.5 \end{bmatrix}.$

By the algorithm given above the controller $\mathbf{K}$ is obtained as follows

$$\mathbf{K} = \begin{bmatrix} -7.98 & -0.12 & 0.2 \\ -0.14 & -94.9 & -2.05 \\ 0.83 & -2.1 & -80.5 \end{bmatrix}$$

$\gamma = 0.51$ is the maximum value for feasible solution. In (Başer and Kızılsaç, 2007), for the same example without uncertainties $\gamma = 0.54$ is obtained for the same value of $\bar{h}$. Here $\gamma = 0.52$ is obtained without uncertainties for the same value of $\bar{h}$.

6. CONCLUSIONS

This paper presents the solvability conditions of delay dependent dynamic output feedback robust H_∞ control problem for a class of uncertain linear neutral systems, where the delay is considered as a time varying continuous or differentiable uniformly bounded function and no restriction on the derivative of the delay is needed, which allows the time delay to be a fast time varying function. Based on a BRL which is obtained without resorting to any model transformations, delay dependent sufficient conditions for the existence of the dynamic output feedback robust H_∞ controller is obtained in terms of LMIs with inverse constraints. These inequalities are solved by an iterative algorithm involving convex optimization methods. The results obtained here are the extensions of (Başer and Kızılsaç, 2007). Finally a numerical example is given to illustrate the results.

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