Adaptive Fuzzy Model-Based Predictive Control Using Fuzzy Decision-Making

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Abstract: Motivated by the need to develop more effective methods of controlling uncertain non-linear systems, this paper focus on developing an adaptive fuzzy model-based controller, in which the optimisation variables remain in the fuzzy domain. The scheme uses an on-line fuzzy identification scheme, which is able to generate a fuzzy relational model using the training data from the system. The proposed control system is applied to supply air temperature control in the simulated cooling coil system of an air-conditioning system. Results are presented that demonstrate the improvement of the proposed scheme compare to the non-adaptive version of the controller.

Keywords: Fuzzy control, Adaptive control, Predictive control, Modelling & Simulation, Air-conditioning system.

1. INTRODUCTION

Nowadays non-linearity, uncertainties and complexity are increasing concerns in a wide spectrum of control problems in new technologies. An uncertain system can mean the variables and the parameters which characterise the system are ill-defined and/or have a significant amount of vagueness associated with them. Because uncertain systems are so ill-defined, modelling and controlling them tend to be a problem.

The existing literature shows that there is growing interest in fuzzy model-based control designs. These model-based fuzzy controllers share the same benefits as linear model-based algorithms and add the non-linear modelling/control capability that comes from fuzzy logic. A Fuzzy Decision-Making (FDM) approach to controlling temperature in air-conditioning systems has been suggested (Thompson and Dexter, 2005). This approach has the ability to deal successfully with the control of non-linear and uncertain systems. It is shown that the controller can maintain the temperature within a specified region with a minimum of control activity. The fuzzy model-based predictive controller is based on the use of a Fuzzy Relational Model (FRM) (Wong *et. al.*, 2000) with non-defuzzified outputs to describe the modelling uncertainties.

The FDM control scheme has two main parts: a FRM, which is identified off-line and generates the fuzzy predictions of the process output; a fuzzy decision-making scheme, which determines the optimal control signal to be applied to the process.

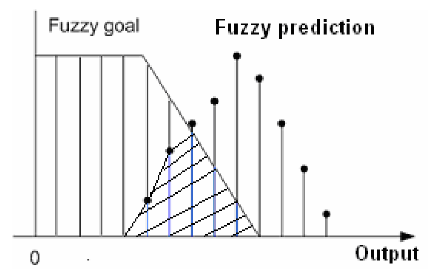

Fig. 1. Degree of satisfaction of fuzzy goal using overlap area

At each computational sample, the FRM generates a fuzzy prediction of process output for each candidate value of the current control signal. The prediction generated by the fuzzy relational model is not defuzzified in this scheme. The degree of similarity between the fuzzy set describing the prediction and the fuzzy set describing the goal is taken as the ratio of the common area between the two membership functions to the total area under the membership function describing the fuzzy prediction (see Fig. 1). The degrees of satisfaction of the goal are used as the membership values for the fuzzy optimal control signal. A crisp control signal is calculated using height defuzzification and applied to the actuator.

In the original scheme, the controller was based on a generic FRM, which was trained off-line using the test data generated by computer simulations of several different designs of the plant to be controlled. The behaviour of the generic FRM can be very different to the real system under control. Consequently, the control performance can be relatively poor due to the mismatch between the generic model and the process. These problems can be overcome by using on-line fuzzy identification to adapt the model using operating data from the actual plant. In addition, on-line adaptation has the potential to cope with the control of a time-varying process.

Several computational undemanding fuzzy identification schemes have been proposed in the literature. The version of the adaptive controller described in this paper uses a modified form of the well-known RSK fuzzy identification scheme (Ridley, Shaw and Kruger, 1988). The so called Rounded RSK fuzzy identification scheme (RRSK) rounds the input data to the closest centres of the fuzzy input sets before it identifies the fuzzy relational model (Wu, 2006). Consider a fuzzy relational model consisting of n inputs ($x_1,\ldots,x_n$) and one output y and where the input and output spaces are characterized by $r_1,\ldots,r_n$ and r_y fuzzy reference sets respectively. The element of the fuzzy relational array $\hat{R}_{s_{1k_1},\ldots,s_{nk_n},q}(t)$ is the estimated rule confidence obtained at time t, which represent the possibility of obtaining an output y in the fuzzy output set q (q is less than or equal to r_y) from inputs $x_1,\ldots x_n$ in the fuzzy input sets $s_{11},\ldots,s_{nk}$. If the training data that fires the rule is also a member of the output fuzzy set, then $f_{s_{1k_1},\ldots,s_{nk_n}}(x(t)) = 1$, and $\mu_q(y(t))$ is the degree of membership of the output set. The recursive updating scheme then becomes:

$$\hat{R}_{s_{1k_1},\ldots,s_{nk_n},q}(t) = \frac{\mu_q(y(t)) + \lambda \cdot \hat{R}_{s_{1k_1},\ldots,s_{nk_n},q}(t-1) \cdot F_{s_{1k_1},\ldots,s_{nk_n}}(t-1)}{F_{s_{1k_1},\ldots,s_{nk_n}}(t)} \quad (1)$$

where:

$$F_{s_{1k_1},\ldots,s_{nk_n}}(t) = 1 + \lambda F_{s_{1k_1},\ldots,s_{nk_n}}(t-1) \quad (2)$$

and λ is the forgetting factor. It should be noted that, the value of F matrix will converge to $1/(1-\lambda)$.

Section 2 discusses the problems that can occur when using FDM scheme on a less uncertain model. , A modification of the original FDM scheme using a weighting factor which indicates how much training has occurred is proposed in section 3. The results of simulations based on a simple non-linear first-order Hammerstein model of a cooling coil system are presented in section 4 to demonstrate the effectiveness of the weighting scheme and to examine the control performance when the model is fully trained and partially trained.

2. FDM BASED ON A FUZZY MODEL WITH LITTLE UNCERTAINTY

2.1 Limitations of the overlap area method of goal satisfaction

If there is only negligible measurement or disturbance noise on the training data, the output of the fuzzy model will become less and less fuzzy, as it learns the behaviour of the system on-line. However, implementation based on a discrete universe of discourse and the effect of rule aggregation will prevent the model from generating a crisp output, even when it is fully trained. In the original FDM scheme, the degree of similarity between the fuzzy predictions and the fuzzy goal is taken as the ratio of the overlap area. This method of fuzzy matching was developed to calculate the degree of similarity when the predictions are fuzzy. In general, in a noise free system the most crisp prediction has only two non-zero values of possibility since there is only finite number of fuzzy output sets and the crisp output of the model cannot be guaranteed to be at the centre of a fuzzy output set. In such a case, a method of estimating the crisp prediction is to defuzzify the fuzzy output using the height defuzzification scheme (Driankov *et. al*., 1993). Consequently, the value obtained by the overlap area method should be the same as the value obtained by calculating the degree to which the defuzzified output satisfies the goal.

It can be shown that the overlap area between the prediction and fuzzy goal will give the same result as the degree to which defuzzified output satisfies the fuzzy goal if only one group of rules is fired and the fuzzy prediction has two non-zero possibility values (Wu, 2006).

Consider the case if a fuzzy prediction has more than two non-zero possibility values and only two of the non-zero possibility values overlap with the core of fuzzy goal (see Fig. 2).

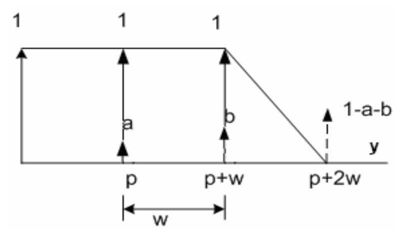

Fig. 2 Prediction and fuzzy goal when two of the three non-zero values of possibility overlap with the fuzzy goal

The ratio of the overlap area between the prediction and fuzzy goal is

$$\frac{a+b}{a+b+1-a-b} = a+b \quad (3)$$

In this case, the position of defuzzified output is given by:

$$\begin{aligned} y_{def} &= pa + (p+w)b + (p+2w)(1-a-b) \\ &= p + 2w - wb - 2wa \end{aligned} \quad (4)$$

If $y_{def} = p + 2w - wb - 2wa <= p + w$ i.e. $2a + b \geq 1$, the degree of the satisfaction of the fuzzy goal is one. The difference between these two methods is $1-a-b$.

If $y_{def} = p + 2w - wb - 2wa > p + w$ i.e. $2a + b < 1$, the degree of the satisfaction of the fuzzy goal is

$$\frac{P + 2w - y_{def}}{w} = \frac{p}{w} + 2 - \frac{p}{w} - 2 + b + 2a = b + 2a \quad (5)$$

The difference between these two methods is a.

Exactly the same result is obtained for the case when only one of the non-zero possibility values overlaps with the core of fuzzy goal. It can be seen these differences are caused by the smallest non-zero value of possibility.

The above analysis shows that the overlap area ratio method will give different values of satisfaction of the fuzzy goal if two groups of rules are fired and the fuzzy prediction three non-zero possibility values. If the prediction mainly satisfies the fuzzy goal, the small values of possibilities outside the goal will cause errors. On the other hand, if the prediction mainly does not satisfy the fuzzy goal, the small values of possibilities inside the fuzzy goal will also result in errors.

2.2 Problems with FDM when the output of the model is not very fuzzy

Some candidate values of the optimal control signal may have small non-zero membership values which should really be zero. Also, some control signal candidates will have smaller values of the membership of optimal control signal than the true values. Process non-linearity can make the problem worse. For example, consider the non-linearity associated with a cooling coil system shown in Fig. 3.

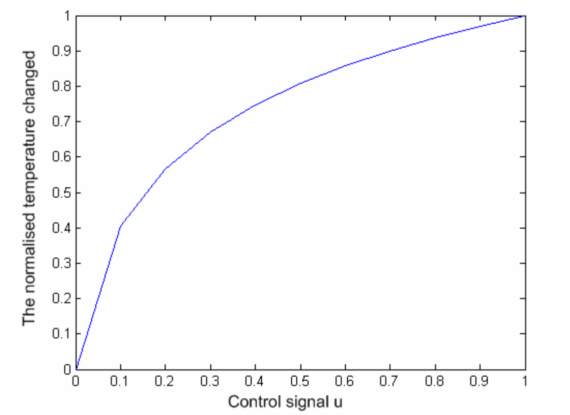

Fig. 3 Example non-linearity of a cooling coil

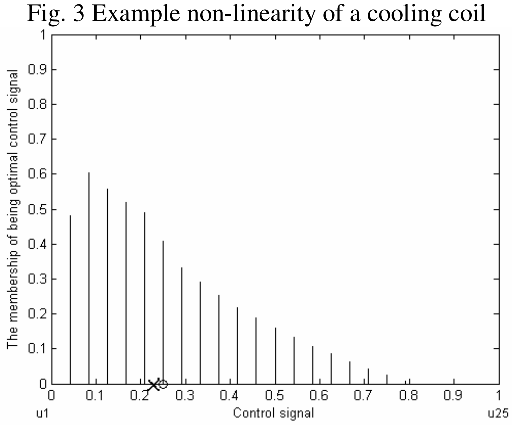

Fig. 4 Membership of the optimal control signal

The static gain of the system is low when the control signal has a high value. This causes the model predictions at high values of the control signal to have similar values and many candidate values of the control signal will have similar non-zero degrees of membership of the optimal control signal, which may result in an offset error on the defuzzified optimal control signal. Fig. 4 shows an example membership of the optimal control signal based on the above non-linearity by searching 25 equally spaced candidate values of control signal. The non-zero possibilities indicate the membership values. The circle and cross on the control signal axis show the value of the defuzzified control signal and the correct value of the optimal control signal required to achieve the set-point at steady state (this value was found by off-line simulation). It can be seen that, the height defuzzification scheme does not generate the correct value of the optimal control signal, because it makes use of all non-zero possibility values.

2.3 Problems with FDM when the model is only partially trained

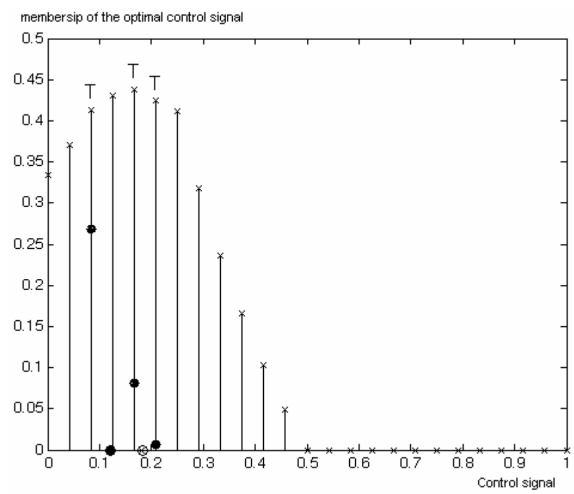

Fig. 5 Membership of the optimal control signal when the model has been partially trained

Fig. 5 shows an example of discrete membership function for the optimal control signal at a steady state condition when the model has been partially trained. The lines with cross ends indicate the membership values obtained from a subset of the generic model generated off-line. The circle and dark point on the control signal axis show the values of the defuzzified control signal and the correct steady state optimal control signal. The letter T indicates that the associated control signal results from a part of the model which has been updated on-line. It can be seen that only part of the membership function for the optimal control signal is obtained from rules in the fuzzy relational model which have been updated on-line. Other membership values which were obtained from rules, which have not been trained, are the same as those values have been obtained from the generic fuzzy model. The three membership grades (showing as three lines with dotted ends) obtained from the rules, which have been trained, are smaller than those obtained from the generic model. Therefore, the correct optimal control signal is unlikely to be one of these three values. Since the training has not occurred at the desired optimal control signal and the membership values at non-optimal values of the control signal are non-zero, the resulting defuzzified control signal is not the same as the correct optimal value. The fuzzy decision-maker will not give the correct optimal control signal in this case.

An alternative defuzzification method that is widely used in practical applications is the method of height defuzzification with a threshold. When using this method of defuzzification the elements of the control signal in the universe discourse U that have membership grades lower than a certain specified level α are completely discounted and the defuzzified value $\bar{u}(n)$ is calculated by the application of the height defuzzification only to those control signal candidates that have membership grades above the threshold. Hence:

$$\bar{u}(n) = \frac{\sum_{i \in H} \mu(u_i(n)) \cdot u_i(n)}{\sum_{i \in H} \mu(u_i(n))} \quad (6)$$

where, H is the set of elements in U which have membership grades $\mu(u(n)) \geq \alpha$. It is not difficult to see that when $\alpha = 0$ the scheme is the standard height defuzzification method and when $\alpha = \mu(u(n))_{\max}$ the scheme is equivalent to the mean of the maximum defuzzification. However, the use of a high value of the threshold can make the control scheme very sensitive to noise and identification errors.

3. MODIFIED FDM SCHEME WITH WEIGHTING

A modification to the original FDM scheme is proposed based on a variant of the method of height defuzzification with a threshold. A weight will be put on the membership values dependent upon how much the associated rules have been trained.

As discussed in section 1, the value of F matrix will converge to $1/(1-\lambda)$ in the RRSK fuzzy identification scheme. If the value of the element F matrix $F(t)$ converges to $1/(1-\lambda)$, that means $\hat{R}(t)$ has been trained fully. Therefore, the values of $F(t)$ and $1/(1-\lambda)$ are used to determine the extent to which a group of rule confidences $\hat{R}(t)$ has been updated.

The weights are calculated as follows:

1) The value of the control signal has a relatively high possibility of being optimal, if the control signal is associated with the rule that has been trained and $\mu(u(n)) \geq \alpha$.

The weight $w_1(u_i(n))$ can therefore be calculated as follows:

$$w_1(u_i(n)) = \frac{1}{2}\left(\frac{1+(1-\lambda)(F(t)-2F(0))}{1-F(0)(1-\lambda)}\right) \quad (7)$$

If the rule confidences have not been updated, the value of the element of the matrix $F(t)$ will remain at $F(0)$. Therefore, the weight $w_1(u_i(n))$ calculated is given a value of 0.5. This means only half weight is given to control signals which have not been trained. If the values of $F(t)$ converges to $1/(1-\lambda)$, the weight $w_1(u_i(n))$ calculated will be 1. Therefore, full weight is given to the membership grade of the control signal.

2) The candidate value of the control signal has a small possibility of being optimal if the control signal is associated with the rule that has been trained and $\mu(u(n)) < \alpha$.

The weight $w_2(u_i(n))$ can therefore be calculated as follows:

$$w_2(u_i(n)) = \frac{1}{2}\left(\frac{1-(1-\lambda)F(t)}{1-F(0)(1-\lambda)}\right) \quad (8)$$

If the rule confidences have not been updated, the value of the element of the F matrix $F(t)$ will remain at $F(0)$ and as before, a 0.5 weight is given to the control signals which have not been trained. If values of $F(t)$ converges to $1/(1-\lambda)$, the weight $w_2(u_i(n))$ calculated will give a value of 0. Therefore, zero weight is given to the control signals which have been fully trained and $\mu(u(n)) < \alpha$.

It should be noted that, if the fuzzy relational model has not been trained at all, all the membership grades will have the same value of weight (0.5). Therefore, the height defuzzification scheme will give the same result as the original FDM scheme which is designed for use with fuzzy models.

The variation of the weights during the training of the model in this example is shown in Fig. 6. The top curve shows the variation of w_1 against number of updates of the rule confidences and the bottom curve shows the variation of w_2 against number of updates.

As can be seen from Fig. 6, around 10 updates are required to update rule confidence fully. As the forgetting factor increases, more updates are required to fully train the model and those weights therefore coverage to their final value more slowly.

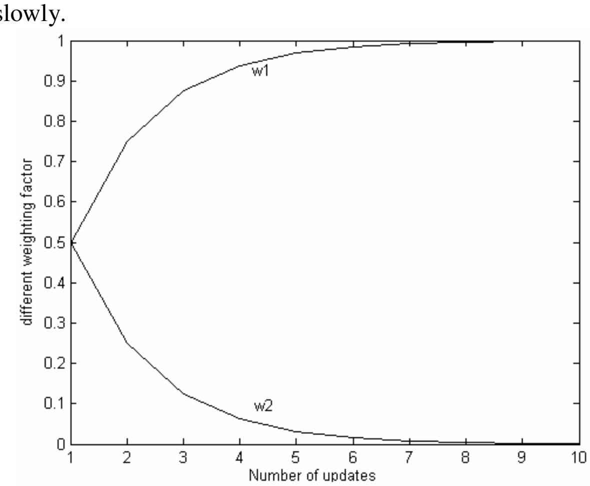

Fig. 6 Variation of the weights during training when $\lambda = 0.5, F(0) = 1$

In summary, the proposed scheme has three basic steps:

Step 1. Calculate the membership grades of the optimal control signal $\mu(u_i(n))$ using the overlap area method for each of the candidate values of the control signal.

Step 2. Pre-process the membership grades by comparing the values with a weighting threshold, α.

- If $\mu(u_i(n)) \geq \alpha$, $w_1(u_i(n))$ is used to calculate the new membership grade $\mu_1(u_i(n)) = w_1(u_i(n)) \cdot \mu(u_i(n))$.
- If $\mu(u_i(n)) < \alpha$, $w_2(u_i(n))$ is used to calculate the new membership grade
- $\mu_2(u_i(n)) = w_2(u_i(n)) \cdot \mu(u_i(n))$.

Step 3. Defuzzify the pre-processed membership grades to generate a crisp control signal:

$$\bar{u}(n) = \frac{\sum_{i\in H}\mu_1(u_i(n))\cdot u_i(n) + \sum_{i\notin H}\mu_2(u_i(n))\cdot u_i(n)}{\sum_{i\in H}\mu_1(u_i(n)) + \sum_{i\notin H}\mu_2(u_i(n))} \quad (9)$$

$$= \frac{\sum_{i\in H} w_1(u_i(n))\cdot\mu(u_i(n))\cdot u_i(n) + \sum_{i\notin H} w_2(u_i(n))\cdot\mu(u_i(n))\cdot u_i(n)}{\sum_{i\in H} w_1(u_i(n))\cdot\mu(u_i(n)) + \sum_{i\notin H} w_2(u_i(n))\cdot\mu(u_i(n))}$$

where, $i \in H$ are the values of the control signal that belong to the set of elements whose membership grades satisfy $\mu(u(n)) \geq \alpha$ and $i \notin H$ are the values of the control signal that belong to the set of elements whose membership grades $\mu(u(n)) < \alpha$.

It should be noted that the scheme has two limitations. Firstly, the value of forgetting factor, λ, cannot be unity. Otherwise the value of $1/(1-\lambda)$ will be infinity and the scheme is no longer applicable. In addition, the initial value of $F(t)$ $(F(0))$ cannot be greater or equal $1/(1-\lambda)$. Otherwise, $1-F(0)(1-\lambda) \leq 0$, and a negative or infinite weighting will be generated.

4. PERFORMANCE OF THE MODIFIED CONTROL SCHEME

The modification of the original FDM scheme has been proposed to deal with the problems arising when the output of the fuzzy relational model is not very fuzzy. In this section, the performance of the modified control scheme based on: (1) a fully trained model and (2) a partially trained model are examined to demonstrate the improvements and effectiveness of the new weighting scheme. The controllers are designed to control the supply air temperature of a simulated cooling coil system described by the first-order Hammerstein model, which does not consider the changes in the inlet air temperature and air mass flow rate. Here it is assumed that the inlet air temperature is approximately 26^{o}C, and the air flow rate is 70% of the design value. The goal of the controller is to control the supply air temperature within 1 ^{o}C of the set-point and the sampling time of the simulation is 15s.

4.1 Performance of the modified control scheme when the model is fully trained

Since the dynamics of the Hammerstein model are known, an "ideal" fuzzy relational model (Wu and Dexter, 2003), which guarantees there are no prediction errors at the centres of fuzzy input sets is used in the controller to represent a fully trained model. As the model is effectively fully trained. $w_1(u_i(n)) = 1$ when $\mu(u_i(n)) \geq \alpha$ and $w_2(u_i(n)) = 0$ when $\mu(u_i(n)) < \alpha$. The weighting threshold is set to 0.2 to eliminate the effect of small non-zero possibility values on the fuzzy decision-making. Therefore, the weighting scheme is equivalent to the height defuzzification with a threshold of 0.2. The simulated control results of the original and modified FDM schemes are shown in Fig. 7.

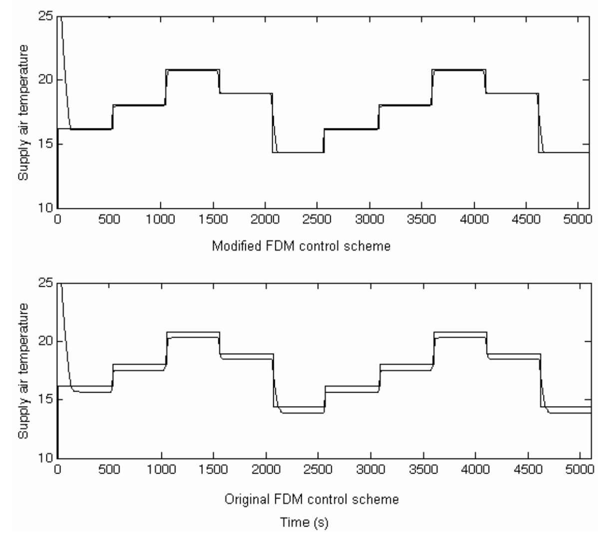

Fig. 7 Control of the supply air temperature using a fully trained "ideal" model

The Root-Mean-Squared-Error (RMSE) values obtained in the second cycle of the set-point changes from the original and modified control scheme are be used to demonstrate the improvement of the control performance as they are not sensitive to the initial conditions (see Table 1). The values of the RMSE show that the modified control scheme has a smaller tracking error even though it is based on the same model. It also can be seen from Fig. 7 that the steady state offsets are much smaller for all the set-points and the dynamic transient performance is much faster than that of the original FDM control scheme.

Table 1 Performance comparison when the model is fully trained

	RMSE (^{o}C)
Modified controller (2nd cycle)	0.4765
Original controller (2nd cycle)	0.854

4.2 Performance of the modified control scheme when the model is partially trained

A simulation is performed to determine the effect of the weighting scheme on the closed-loop control behaviour when the model has been partially trained. The sequence of step changes in the set-point is repeated ten times. The weighting threshold is again set to 0.2, the forgetting factor is set to 0.8 and *F(0)* is chosen 1. The closed-loop behaviour of different versions of the controller when tracking a repeated sequence of step changes in the set-point for the 1st, 2nd and 10th cycle are shown in Fig. 8. The dash dotted line is the set-point, the solid line is the supply air temperature obtained when using the adaptive version of the controller based on the modified FDM scheme, the dashed line is the supply air temperature obtained when using the adaptive version of the controller based on the original FDM scheme and the dotted line is the supply air temperature and the control signal obtained when using the non-adaptive version of the controller based on generic model.

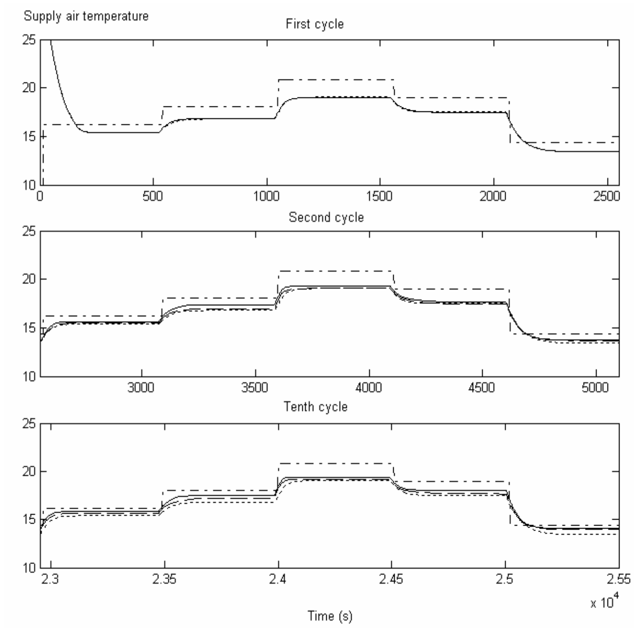

Fig. 8 Supply air temperatures when the model has been trained on-line to track repeated step changes

It can be seen from Fig. 8 that, the steady-state offsets with the adaptive controller based on the modified FDM scheme become smaller at all the set-points and the dynamic transient performance becomes faster after each cycle compared to the other two controllers. Therefore, both transient and steady-state closed-loop control behaviour of the adaptive version of the controller based on the modified FDM scheme improves gradually with repeated training. The value of the RMSE calculated over the tenth cycle is shown in Table 2. The improvement in the RMSE is defined as $\frac{RMSE_{non-adaptive} - RMSE_{adaptive}}{RMSE_{non-adaptive}} \times 100\%$,where $RMSE_{non-adaptive}$, the value of the RMSE obtained when using the non-adaptive controller based on the generic fuzzy relational model, is used as the baseline to compare the control performance. $RMSE_{adaptive}$ is the value of the RMSE obtained when using the adaptive fuzzy model-based controller. It should be noted that the values of RMSE obtained in the first cycle are not used to compare the two controllers and calculate the improvements in RMSE, since they will be sensitive to the initial condition of the supply air temperature.

Table 2 Improvements in control performance

	RMSE (^{o}C)	Relative Improvement.
Non-adaptive controller with generic model (2nd cycle)	1.727	0%
Adaptive controller with origin. FDM scheme (10th cycle)	1.447	16.2%
Adaptive controller with new FDM scheme (10th cycle)	1.094	36.7%

The transient and the steady-state closed loop performance of the adaptive version of the controller are almost the same as those of the non-adaptive version of the controller in the 1st cycle, because the amount of training is too small to make any significant differences in the closed-loop control behaviour. The closed-loop control performance becomes better after the 2nd cycle, as more training occurs. It can be seen from the third plots in Fig. 8, all of the set-points except 21^{o}C, can be controlled within the acceptable range in the 10th cycle by using the adaptive version of the controller based on the modified FDM scheme. By the tenth cycle there is a 36.7% improvement in the RMSE value obtained using the adaptive controller based on the modified FDM scheme compared to that of the non-adaptive version of the controller. On the other hand, only 16.2% improvement has been made by using the adaptive controller based on the original FDM scheme. Since the correct value of the defuzzified optimal control signal at a set-point of 21^{o}C is the smallest of the steady-state values of the control signal required at the five set-points, no training has occurred on the left-hand side of the correct optimal control signal. The fuzzy decision-maker is therefore unable to find the correct control signal and the closed-loop control behaviour at this set-point does not improve significantly.

5. CONCLUSIONS

Problems that can occur when the fuzzy decision-making scheme uses the overlap area method and height defuzzification have been considered. A weighting scheme has been proposed to deal with the problems arising when the model is trained on-line. The control performance of a simple first-order non-linear Hammerstein model of a cooling coil system is used to demonstrate the effectiveness of the weighting scheme when the model is fully trained and partially trained. Experiments on a real air-handling unit are planned to compare the performance of the proposed control scheme with that of a conventional controller.

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